

Worked Solutions Included!

# Trig Identities Worksheet

Worksheet & Solutions courtesy of Mr R Milton

Notes:

1. You must remember the two Trig Identities:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

2. Re-arranging equation 2 above you also get:

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

3. In this worksheet you only work on the **left hand side** of the equation. This means that you try to get the left hand side of the equation equal to the right hand side.

1. Show that  $\tan x \cos x = \sin x$

2. Show that  $\frac{\sin x}{\tan x} = \cos x$

3. Show that  $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

4. Show that  $\frac{\sin^2 x}{\tan x} = \sin x \cos x$

5. Show that  $\frac{1 - \cos^2 A}{\cos^2 A} = \tan^2 A$

6. Show that  $\frac{1 - \sin^2 A}{\cos^2 A} = 1$

7. Show that  $(\cos x + \sin x)^2 = 1 + 2\sin x \cos x$

8. Show that  $(\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x = 1$

9. Show that  $\sin^3 x + \sin x \cos^2 x = \sin x$

10. Show that  $\cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$

# Worked Solutions

Courtesy of Mr R Milton

1.  $\tan x \cos x = \sin x$

LHS: Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\sin x}{\cos x} \times \cos x$

Cancel the  $\cos x$  diagonally:  $= \sin x$

2.  $\frac{\sin x}{\tan x} = \cos x$

LHS: Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\sin x}{\frac{\sin x}{\cos x}}$

$$= \frac{\sin x}{1} \div \frac{\sin x}{\cos x}$$

Cancel the  $\sin x$  diagonally:  $= \frac{\sin x}{1} \times \frac{\cos x}{\sin x}$

$$= \cos x$$

3.  $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

LHS Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\frac{\sin x}{\cos x}}{\sin x}$

$$= \frac{\sin x}{\cos x} \div \frac{\sin x}{1}$$

Cancel the  $\sin x$  diagonally:  $= \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$$= \frac{1}{\cos x}$$

$$4. \quad \frac{\sin^2 x}{\tan x} = \sin x \cos x$$

$$\text{LHS Substitute } \tan x = \frac{\sin x}{\cos x}:$$

$$\frac{\sin^2 x}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{1} \div \frac{\sin x}{\cos x}$$

$$= \frac{\sin^2 x}{1} \times \frac{\cos x}{\sin x}$$

$$\text{Cancel the } \sin x \text{ diagonally:} \quad = \sin x \cos x$$

$$5. \quad \frac{1 - \cos^2 A}{\cos^2 A} = \tan^2 A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\text{Re-arrange:} \quad \sin^2 A = 1 - \cos^2 A \quad \text{--- (1)}$$

$$\text{Substitute (1) into the top line, LHS:} \quad \frac{1 - \cos^2 A}{\cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$\text{Since } \tan x = \frac{\sin x}{\cos x}:$$

$$= \tan^2 A$$

$$6. \quad \frac{1 - \sin^2 A}{\cos^2 A} = 1$$

$$\sin^2 A + \cos^2 A = 1$$

Re-arrange:  $\cos^2 A = 1 - \sin^2 A \quad \text{--- (1)}$

Substitute (1) into the top line, LHS:  $\frac{1 - \sin^2 A}{\cos^2 A}$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

$$7. \quad (\cos x + \sin x)^2 = 1 + 2\sin x \cos x$$

LHS:  $(\cos x + \sin x)^2$

$$= (\cos x + \sin x)(\cos x + \sin x)$$

$$= \cos^2 x + \cos x \sin x + \sin x \cos x + \sin^2 x$$

$$= \cos^2 x + 2\sin x \cos x + \sin^2 x$$

$$= 1 + 2\sin x \cos x \quad (\text{since } \sin^2 x + \cos^2 x = 1)$$

$$8. \quad (\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x = 1$$

LHS:  $(\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x$

Multiply out brackets:  $= \cos x(\cos x - \sin x) + \sin x(\cos x - \sin x) + 2\sin^2 x$

$$= \cos^2 x - \cos x \sin x + \sin x \cos x - \sin^2 x + 2\sin^2 x$$

Simplify:  $= \cos^2 x - \sin^2 x + 2\sin^2 x$

Simplify again:  $= \cos^2 x + \sin^2 x$

$$= 1 \quad (\text{since } \sin^2 x + \cos^2 x = 1)$$

$$9. \sin^3 x + \sin x \cos^2 x = \sin x$$

LHS, factorise:

$$\sin x (\sin^2 x + \cos^2 x)$$

Substitute:  $\sin^2 A + \cos^2 A = 1$

$$= \sin x (1)$$

$$= \sin x$$

$$10. \cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$$

LHS, factorise:

$$\cos^2 x (\sin^2 x + \cos^2 x)$$

Substitute:  $\sin^2 A + \cos^2 A = 1$

$$= \cos^2 x (1)$$

$$= \cos^2 x$$

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**Answers Included!**

For marking schemes, please go to N5 Maths Past Paper link [HERE](#)

# Trig Identities

## SQA Questions

### FORMULAE LIST

The roots of  $ax^2 + bx + c = 0$  are  $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$  or  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle:  $\text{Area} = \frac{1}{2} ab \sin C$

Volume of a sphere:  $\text{Volume} = \frac{4}{3} \pi r^3$

Volume of a cone:  $\text{Volume} = \frac{1}{3} \pi r^2 h$

Volume of a Pyramid:  $\text{Volume} = \frac{1}{3} Ah$

Standard deviation:  $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$ , where n is the sample size.

## Trig Identities SQA Questions

Source: SQA 2022 N5 Maths P2 Q13

13. Simplify  $\frac{\sin x^\circ + 2 \cos x^\circ}{\cos x^\circ}$ .

2

Answer:

$\tan x + 2$

2019 P2	<p>17. Expand and simplify</p> $(\sin x^\circ + \cos x^\circ)^2.$ <p>Show your working.</p>	2
Ans	$1 + 2 \sin x \cos x$	
2018 P1	<p>18. Express <math>\sin x^\circ \cos x^\circ \tan x^\circ</math> in its simplest form.</p> <p>Show your working.</p>	2
Ans	$\sin^2 x$	
2016 P1	<p>11. Simplify</p> $\tan^2 x^\circ \cos^2 x^\circ.$ <p>Show your working.</p>	2
Ans	$\sin^2 x^\circ$	
PPA P2	<p>9.</p> <p>(b) Show that</p> $\tan x \cos x = \sin x.$	2
Ans	b. Proof using $\tan x = \frac{\sin x}{\cos x}$	
PPC P2	<p>(b) Prove that</p> <p>11.</p> $\sin^3 x + \sin x \cos^2 x = \sin x.$	3
Ans	b $\sin^3 x + \sin x \cos^2 x = \sin x (\sin^2 x + \cos^2 x) = \sin x$ (Since $\sin^2 x + \cos^2 x = 1$ )	
PPF P2	<p>Simplify</p> <p>Q11.</p> $\tan x^\circ \cos x^\circ.$	2
Ans	Proof using the fact that $\tan x = \frac{\sin x}{\cos x}$	