Area of a Triangle

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2} ab \sin C$
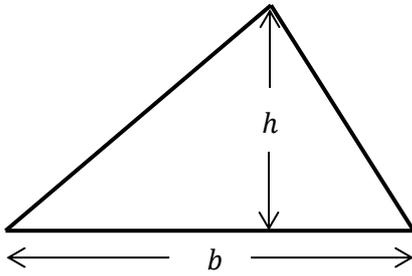
Volume of a sphere: Volume $= \dfrac{4}{3} \pi r^3$

Volume of a cone: Volume $= \dfrac{1}{3} \pi r^2 h$

Volume of a Pyramid: Volume $= \dfrac{1}{3} Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.
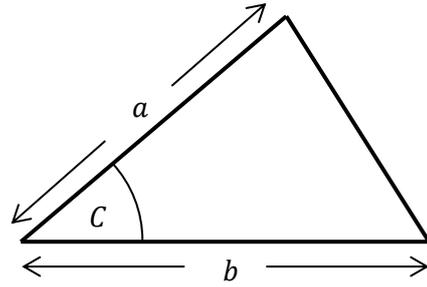
Area of a Triangle

There are two methods we can use to find the area of a triangle.
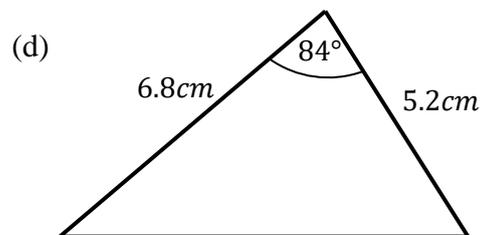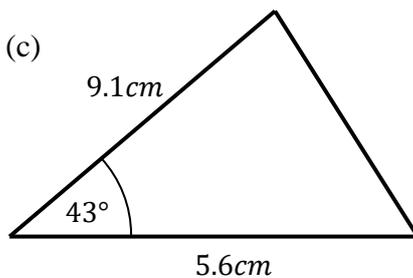

$$Area = \frac{1}{2}bh \qquad \text{OR} \qquad Area = \frac{1}{2}ab\sin C$$
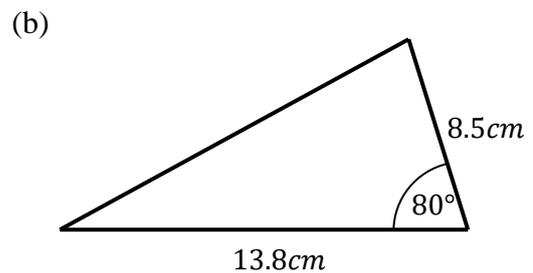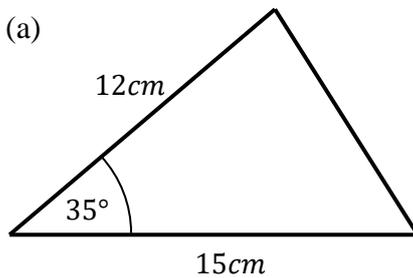
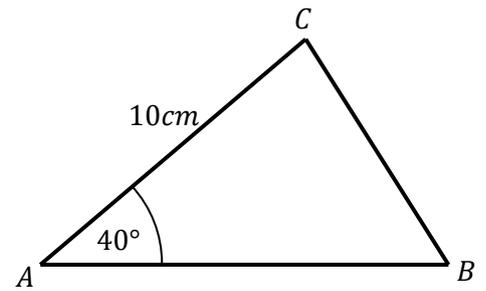To use $Area = \frac{1}{2}ab\sin C$ to find the area we need to know two sides and the angle inbetween

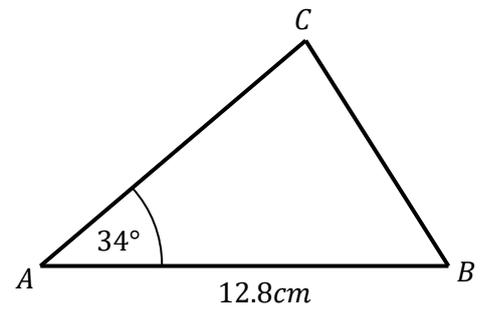If we are given the area we can use $Area = \frac{1}{2}ab\sin C$ to find a length or angle.

Examples

1. Find the area of each triangle, giving your answer to 3 significant figures.

(a)
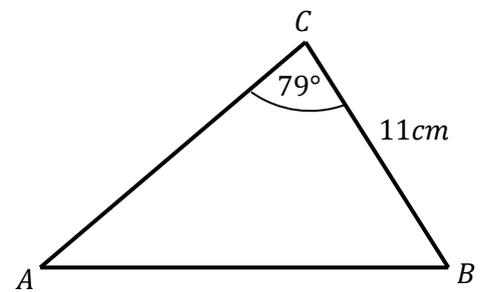

12cm

35°

15cm

(b)

8.5cm

80°

13.8cm

(c)

9.1cm

43°

5.6cm

(d)

84°

6.8cm

5.2cm

2. Triangle ABC has area $45cm^2$. Find the length AB.

C

$10cm$

$40°$

A B

3. Triangle ABC has area $39cm^2$. Find the length AC.

C

$34°$

A B

$12.8cm$

4. Triangle ABC has area $94cm^2$. Find the length AC.

C

$79°$

$11cm$

A B

5. Triangle ABC has area $168cm^2$. Find the length AB.

C

$17cm$

$64°$

A B

6. Triangle ABC has area $49cm^2$. Find angle CAB.

C

10cm

A   16cm   B

7. Triangle ABC has area $52.1cm^2$. Find angle ABC.

C

8.7cm

A   12.4cm   B

8. Triangle ABC has area $205cm^2$. Find angle ACB.

C

23cm   18cm

A   B

9. Triangle ABC has area $88cm^2$. Find angle BAC.

C

15cm

A   17cm   B

Answers

1. (a) $51.6\ cm^2$ (b) $57.8\ cm^2$ (c) $17.4\ cm^2$ (d) $17.6\ cm^2$

2. $14\ cm$

3. $10.9\ cm$

4. $17.4\ cm$

5. $22\ cm$

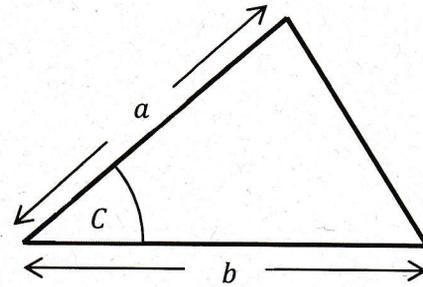6. $37.8°$

7. $75°$

8. $82°$

9. $43.6°$

Area of a triangle
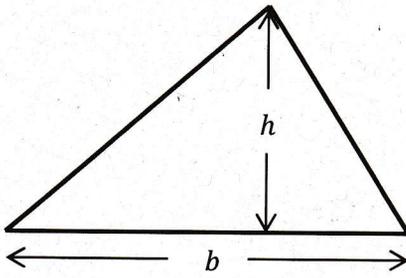
There are two methods we can use to find the area of a triangle.

$$Area = \frac{1}{2}bh \qquad \text{OR} \qquad Area = \frac{1}{2}ab \sin C$$
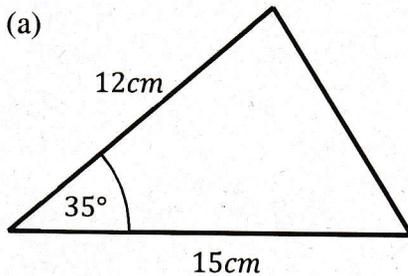
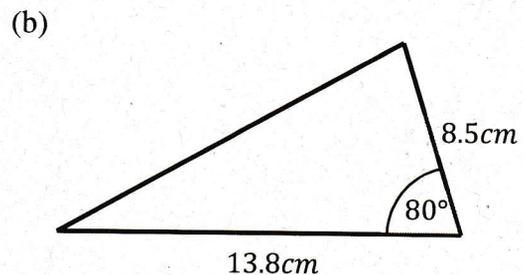To use $Area = \frac{1}{2}ab \sin C$ to find the area we need to know two sides and the angle inbetween

If we are given the area we can use $Area = \frac{1}{2}ab \sin C$ to find a length or angle.

<u>Examples</u>
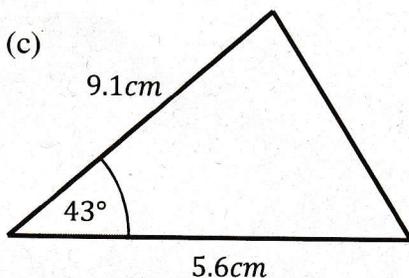
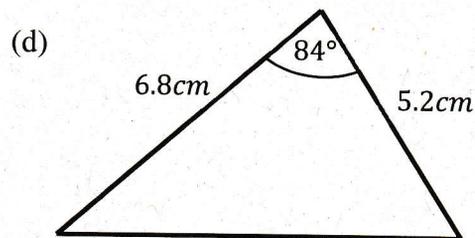1. Find the area of each triangle, giving your answer to 3 significant figures.

(a)
12cm
35°
15cm

$$A = \frac{1}{2} \times 12 \times 15 \times \sin 35$$
$$= 51.6 cm^2 ✓$$

(b)
8.5cm
80°
13.8cm

$$A = \frac{1}{2} \times 8.5 \times 13.8 \times \sin 80$$
$$= 57.7589 \ldots$$
$$= 57.8 cm^2 ✓$$

(c)
9.1cm
43°
5.6cm

$$A = \frac{1}{2} \times 9.1 \times 5.6 \times \sin 43$$
$$= 17.377 \ldots = 17.4 cm^2$$ ✓

(d)
84°
6.8cm   5.2cm

$$A = \frac{1}{2} \times 6.8 \times 5.2 \times \sin 84$$
$$= 17.583 \ldots$$
$$= 17.6 cm^2 ✓$$

2. Triangle ABC has area $45cm^2$. Find the length AB.

$$45 = \frac{1}{2} \times 10 \times b \times \sin 40$$

$$45 = b \times 5 \sin 40$$

$$b = \frac{45}{5\sin 40} = \underline{14cm} \ \checkmark \ (2 \, SF)$$
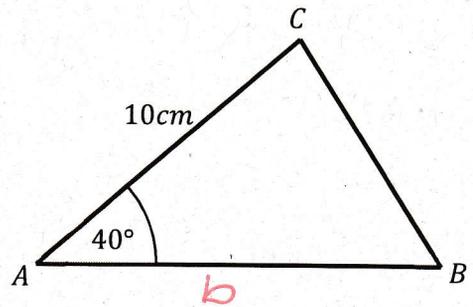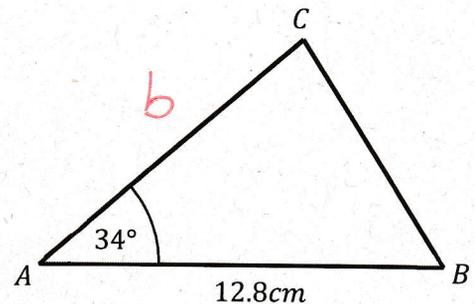

3. Triangle ABC has area $39cm^2$. Find the length AC.

$$39 = \frac{1}{2} \times 12.8 \times b \times \sin 34$$

$$39 = 6.4 \sin 34 \times b$$

$$b = \frac{39}{6.4\sin 34} = 10.897... = \underline{10.9cm} \ (3 \, SF)$$
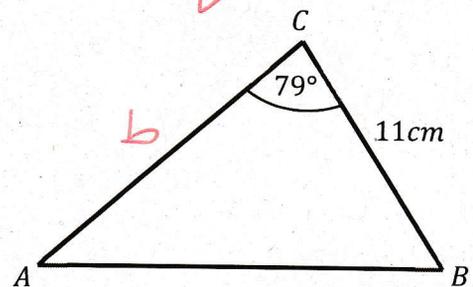
$\checkmark$

4. Triangle ABC has area $94cm^2$. Find the length AC.

$$94 = \frac{1}{2} \times 11 \times b \times \sin 79$$

$$94 = 5.5 \sin 79 \times b$$

$$b = \frac{94}{5.5\sin 79} = 17.410$$
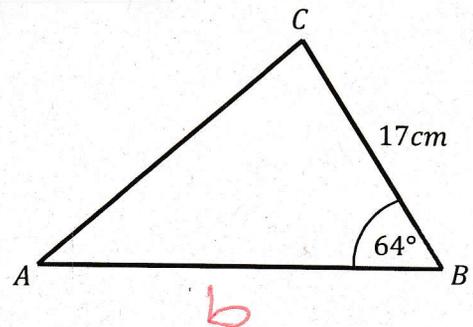$$= \underline{17.4cm} \ (3 \, SF) \ \checkmark$$

5. Triangle ABC has area $168cm^2$. Find the length AB.

$$168 = \frac{1}{2} \times 17 \times b \times \sin 64$$

$$b = \frac{168}{8.5 \times \sin 64}$$
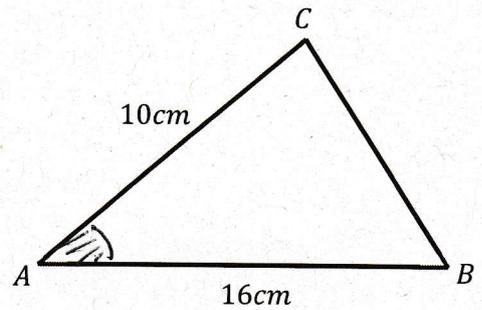
$$= 21.99 = \underline{22cm} \ (2 \, SF)$$

6. Triangle ABC has area $49cm^2$. Find angle CAB.

$49 = \frac{1}{2} \times 10 \times 16 \times \sin A$

$49 = 80 \times \sin A$

$\sin A = \frac{49}{80}$

$A = \sin^{-1}\left[\frac{49}{80}\right] = 37.8°$ (3 SF) ✓

```
          C
   10cm  /\
        /  \
       /    \  16cm... 
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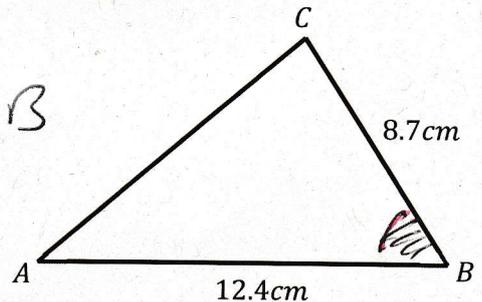10cm

16cm

A ⌣ B

7. Triangle ABC has area $52.1cm^2$. Find angle ABC.

$52.1 = \frac{1}{2} \times 8.7 \times 12.4 \times \sin B$

$52.1 = 53.94 \times \sin B$

$B = \sin^{-1}\left[\frac{52.1}{53.94}\right] = 75°$ (2 SF) ✓
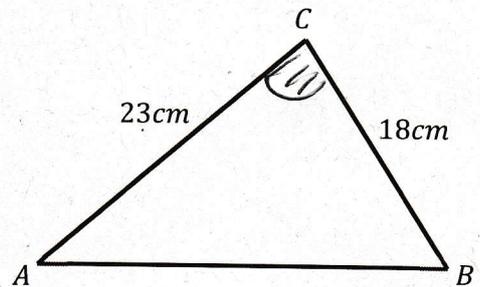
C

8.7cm

A   12.4cm   B

8. Triangle ABC has area $205cm^2$. Find angle ACB.

$205 = \frac{1}{2} \times 23 \times 18 \times \sin C$

$205 = 207 \sin C$

$C = \sin^{-1}\left[\frac{205}{207}\right] = 82°$ (2 SF) ✓

C

23cm   18cm

A   B

9. Triangle ABC has area $88cm^2$. Find angle BAC.

$88 = \frac{1}{2} \times 15 \times 17 \times \sin A$

$88 = 127.5 \times \sin A$

$A = \sin^{-1}\left[\frac{88}{127.5}\right] = 43.6°$ (3 SF) ✓

C

15cm

A   17cm   B