Solutions Attached

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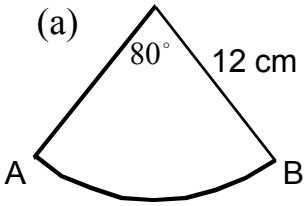
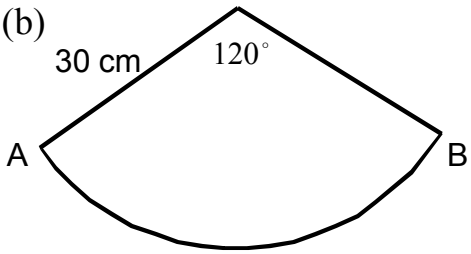
Arcs & Sectors

1. Calculate the length of arc AB in each question below

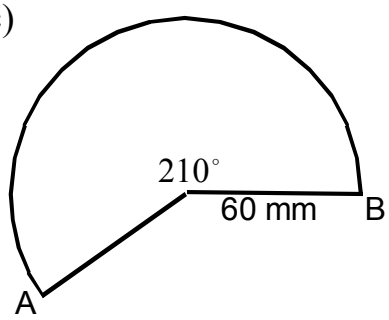
(a)



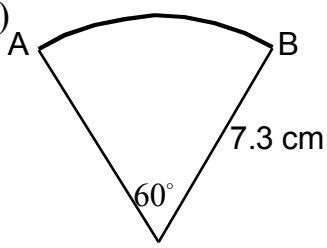
(b)



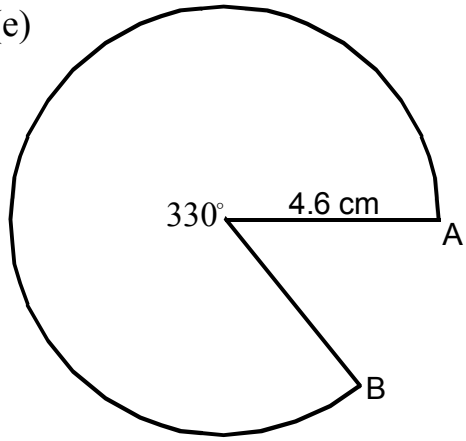
(c)



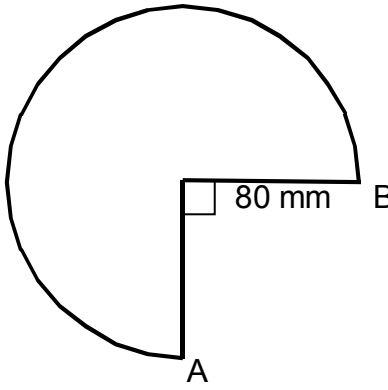
(d)



(e)

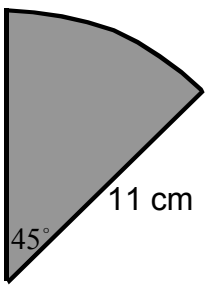


(f)

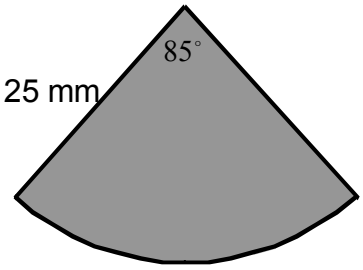


2. Calculate the area of each sector below.

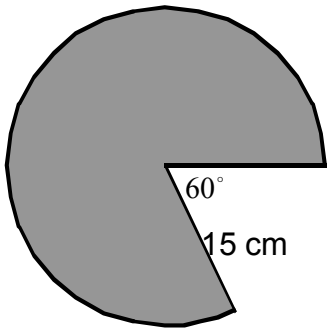
(a)



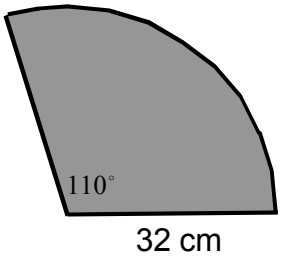
(b)



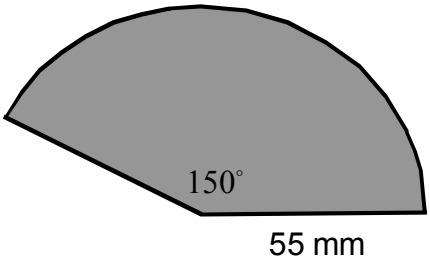
(c)



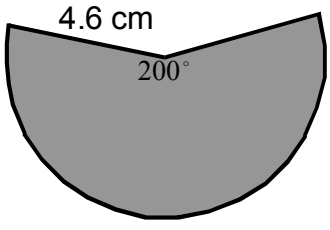
(d)



(e)

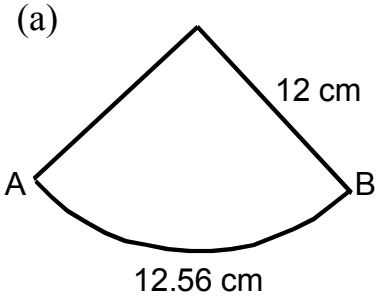


(f)

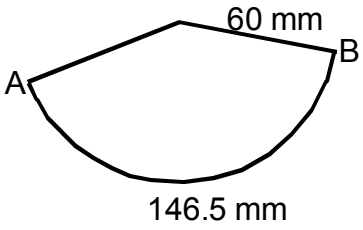


3. In each diagram below the length of arc AB is given. Calculate the size of the angle at the centre of the sector.

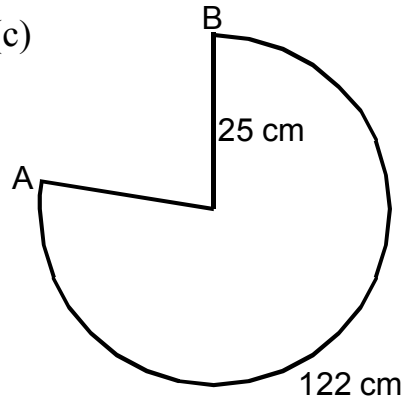
(a)



(b)

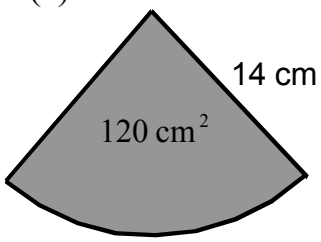


(c)

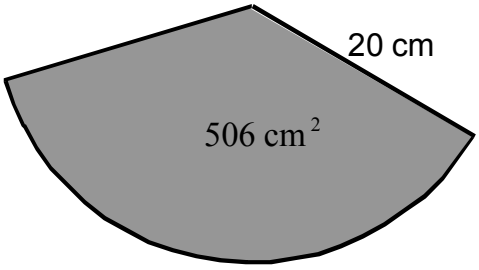


4. In each diagram below the area of the sector is given. Calculate the size of the angle at the centre of the sector.

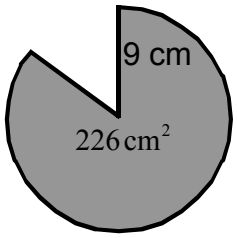
(a)



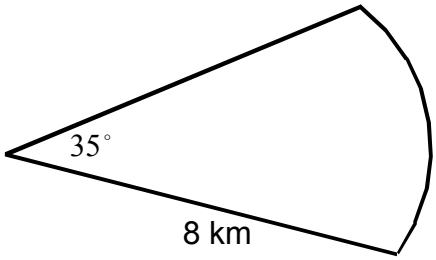
(b)



(c)

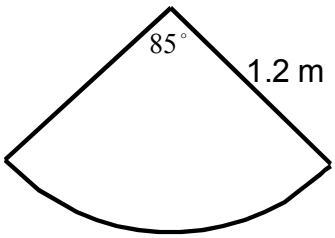


5. The beam from a lighthouse reaches a distance of 8 kilometres and spreads to an angle of 35°.



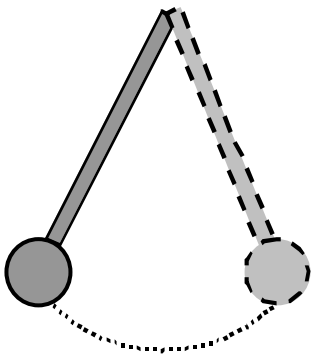
Calculate the area covered by the beam from the lighthouse.

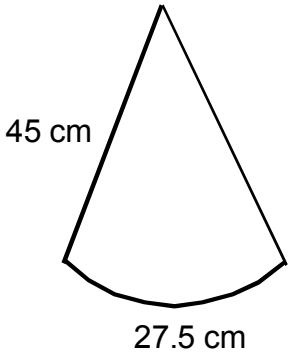
6. The curved part on an anchor is in the shape of an arc of a circle which has radius 1.2 metres.



Calculate the length of this arc.

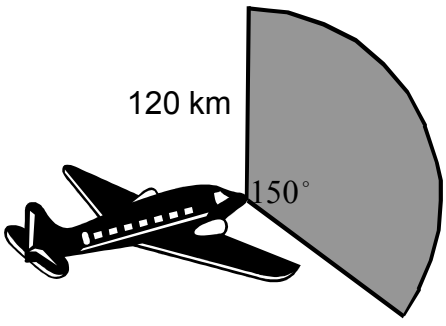
7. A pendulum is 45 centimetres long. When the pendulum swings it travels along the arc of a circle and covers a distance of 27.5 centimetres.





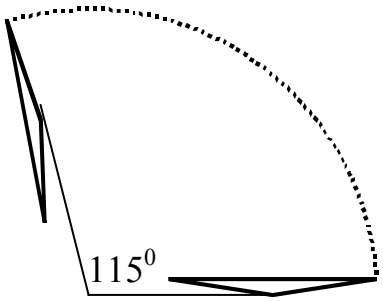
Calculate the size of the angle through which the pendulum travels.

8. The radar beam sent out by an aeroplane reaches a distance of 120 kilometres and covers an angle of 150°.



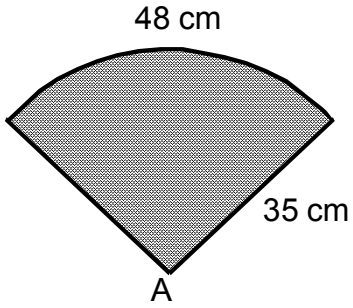
Calculate the area covered by the beam.

9. A windscreen wiper is 45 centimetres long. In one sweep it turns through an angle of 115°.



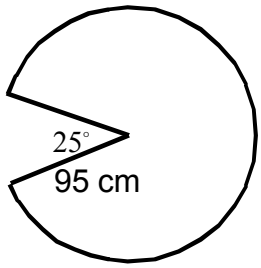
Calculate the distance it covers in one sweep.

10. A fan is in the shape of an arc of a circle with radius 35 centimetres.

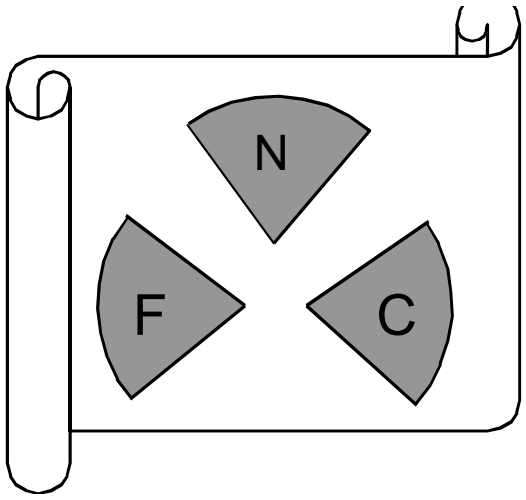


Calculate the size of the angle at A.

11. The path traced by a golfer's club when he hits the ball is an arc of a circle. If the golf club is 95 centimetres long, calculate the distance travelled when the golfer swings his club.

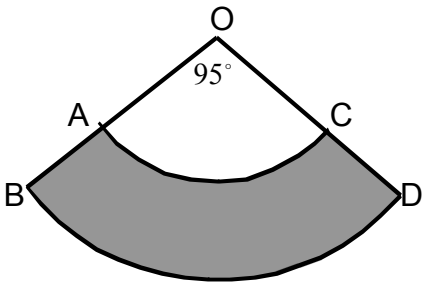


12. The diagram below shows the emblem for a sports club. The emblem consists of 3 identical sectors of a circle, each of radius 16 millimetres.



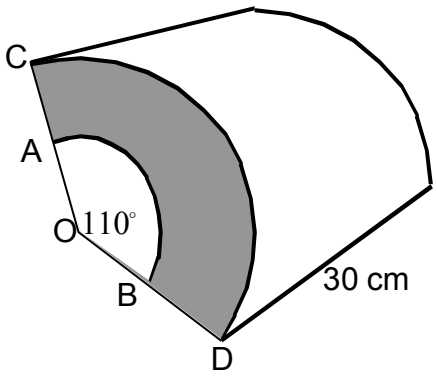
The sectors have a combined area of 335 square millimetres. Calculate the size of angle at the centre of each sector.

13. In the diagram below AC and BD are arcs of circles with centres at O. The radius, OA, is 10 centimetres and the radius, OB, is 16 centimetres.

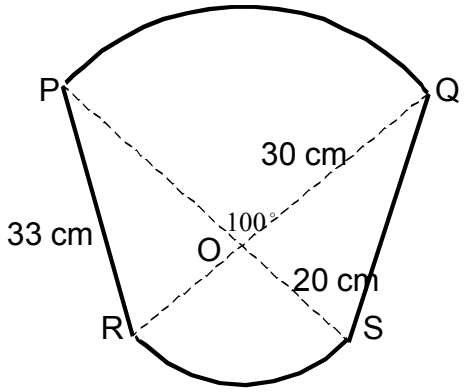


Find the shaded area.

14. The diagram shows a prism whose cross-section is the area between two sectors. One sector has radius OA = 12 centimetres and the other has radius OC = 15 centimetres. Calculate the volume of this prism.

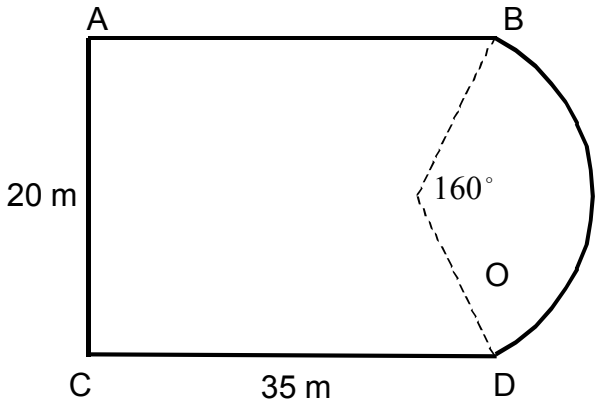


15. In the diagram PQ and RS are arcs of circles with centre O.
The radius, OQ, is 30 centimetres long and the radius, OS, is 20 centimetres long.



Calculate the perimeter of the shape.

16. The diagram below shows an ornamental garden. The garden is in the shape of a rectangle with a sector of a circle added at one end.
The length of the garden is 35 metres and its breadth is 20 metres.



(a) Calculate OB the radius of the sector.
(b) Find the perimeter of the garden.

Worked Solutions

(1) (a) $\text{ARC}_L = \frac{80}{360} \times \pi \times 24 = 16.75\text{ cm}$

(b) $\text{ARC}_L = \frac{120}{360} \times \pi \times 60 = 62.8\text{ cm}$

(c) $\text{ARC}_L = \frac{210}{360} \times \pi \times 120 = 219.8\text{ cm}$

(d) $\text{ARC}_L = \frac{60}{360} \times \pi \times 14.6 = 7.64\text{ cm}$

(e) $\text{ARC}_L = \frac{330}{360} \times \pi \times 9.2 = 26.48\text{ cm}$

(f) $\text{ARC}_L = \frac{270}{360} \times \pi \times 160 = 376.8\text{ cm}$

(2) (a) $\text{AREA} = \frac{45}{360} \times \pi \times 11^2 = 47.49\text{ cm}^2$

(b) $\text{AREA} = \frac{85}{360} \times \pi \times 25^2 = 463.4\text{ cm}^2$

(c) $\text{AREA} = \frac{300}{360} \times \pi \times 15^2 = 588.75\text{ cm}^2$

(d) $\text{AREA} = \frac{110}{360} \times \pi \times 32^2 = 982.47\text{ cm}^2$

(e) $\text{AREA} = \frac{150}{360} \times \pi \times 55^2 = 3957.71\text{ cm}^2$

(f) $\text{AREA} = \frac{200}{360} \times \pi \times 4.6^2 = 36.91\text{ cm}^2$

3 (a) $\text{SECTOR LENGTH} = \text{ANGLE FRACTION} \times \pi D$

$12.56 = \frac{x}{360} \times 3.14 \times 24$

$12.56 = x \times 0.209$

$x = 60^\circ$

(b) $146.5 = \frac{x}{360} \times 3.14 \times 120$

$146.5 = x \times 1.05$

$x = 139.5^\circ$

(c) $122 = \frac{x}{360} \times 3.14 \times 50$

$122 = x \times 0.436$

$x = 280^\circ$

4 (a) $\text{SECTOR AREA} = \text{ANGLE FRACTION} \times \pi \times r^2$

$120 = \frac{x}{360} \times 3.14 \times 14^2$

$120 = x \times 1.71$

$x = 70^\circ$

(b) $506 = \frac{x}{360} \times 3.14 \times 20^2$

$560 = x \times 3.49$

$x = 160^\circ$

(c) $226 = \frac{x}{360} \times 3.14 \times 9^2$

$226 = x \times 0.7065$

$x = 320^\circ$

(5) AREA $= \frac{35}{360} \times \pi \times 8^2 = 19.5\ km^2$

(6) LENGTH $= \frac{85}{360} \times \pi \times 2.4 = 1.78\ m$

(7) $27.5 = \frac{x}{360} \times \pi \times 90$

$27.5 = x \times 0.785$

$x = 35^\circ$

(8) AREA $= \frac{150}{360} \times \pi \times 120^2 = 18840\ km^2$

(9) LENGTH $= \frac{115}{360} \times \pi \times 90 = 90.275\ cm$

(10) $48 = \frac{x}{360} \times \pi \times 70$

$x \times 0.61 = 48$

$x = 78.7^\circ$

(11) LENGTH $= \frac{335}{360} \times \pi \times 190 = 555.2\ cm$

(12) $335 \div 3 = 111.7$ AREA EACH SECTOR

$111.7 = \frac{x}{360} \times \pi \times 16^2$

$x \times 2.23 = 111.7$

$x = 50^\circ$

(13) SHADED AREA $= \left[\frac{95}{360}\right] \times \pi \times 16^2 - \left[\frac{95}{360}\right] \times \pi \times 10^2$

$= 212.12 - 82.86$

$= 129.3\ cm^2$

(14) AREA $= \frac{110}{360} \times \pi \times 15^2 - \frac{110}{360} \times \pi \times 12^2$

$= 215.9 - 138.2$

$= 77.7\ cm^2$

VOLUME PRISM $= A \times L$

$= 77.7 \times 30$

$= 2331\ cm^3$

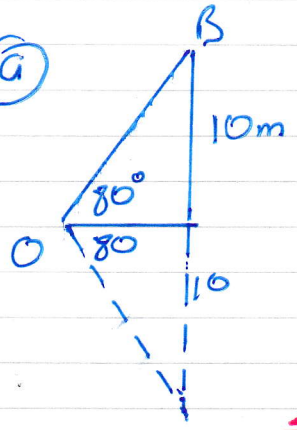
(15) LENGTH PQ $= \left(\frac{100}{360}\right) \times 3.14 \times 60 = 52.33\ cm$

LENGTH RS $= \left[\frac{100}{360}\right] \times 3.14 \times 40 = 34.89\ cm$

PERIMETER $= 33 + 52.33 + 33 + 34.89$

$= 153.22\ cm$

(16) (a)



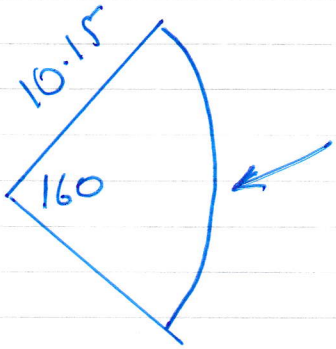
SOH-CAH-TOA

$\sin 80 = \frac{10}{HYP}$

$HYP = \frac{10}{\sin 80} = 10.15$

$OB = 10.15\ m$

(5)



$$\text{ARC}_{L} = \frac{160}{360} \times \pi \times 10.15^2$$

$$= 143.77\text{ m}$$

$$\text{PERIMETER} = 35 + 20 + 35 + 143.77$$

$$= 233.77\text{ m}$$