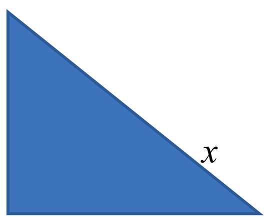
National 5 Checklist

Check your readiness for National 5 Maths and tick the relevant box to show how confident you feel answering questions on each topic.

Chapter	Topic	Skills			
1 Working with surds					
1A	Identify and simplify surds	Surds are numbers which, when left in root form, cannot be simplified e.g. $\sqrt{5}$. To simplify a surd: express the surd as a product of two smaller surds, where one of the smaller surds is the root of a square number. e.g. $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ (Square numbers: 4, 9, 16, 25, 36, 49, 64,.....) Add and subtract surds: only like terms can be added e.g. $\sqrt{3} + 2\sqrt{5} + 4\sqrt{3} + \sqrt{5} = 5\sqrt{3} + 3\sqrt{5}$ Simplify first, then add like terms e.g. $\sqrt{50} + \sqrt{8} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$			
1B	Multiplying and Dividing surds	Use the rules $\sqrt{a \times b} = \sqrt{ab}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ e.g. $\sqrt{3} \times \sqrt{6} = \sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$ $\frac{\sqrt{21}}{\sqrt{3}} = \sqrt{\frac{21}{3}} = \sqrt{7}$			
1C	Calculating and Manipulating Surds	A number written as a surd is described as an **exact** value e.g. Calculate the length of x, leaving your answer as an exact value  $x^2 = 5^2 + 7^2$ $= 74$ $x = \sqrt{74}$			
1D	Rationalising the denominator	Remove the irrational denominator by multiplying both the numerator and denominator by the same surd, then simplify e.g. $\frac{4}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$			
1E	Expand brackets and simplify	Combine algebraic skills with surd rules e.g. $\sqrt{2}\left(3+\sqrt{2}\right) = 3\sqrt{2} + \sqrt{4} = 3\sqrt{2} + 2$			

Chapter	Topic	Skills	🟩	🟨	🟥
Simplifying Expressions using the laws of indices					
2A	Writing and Using Index form	The index number (also known as the "power") tells you the number of times the base number is multiplied by itself e.g. $2^5 = 2\times2\times2\times2\times2$ Rule 1: $a^0 = 1$ e.g. $9^0 = 1$			
2B	Writing and calculating negative indices	Rule 2: $a^{-m} = \frac{1}{a^m}$ e.g. $x^{-3} = \frac{1}{x^3}$ $\frac{2m^{-4}}{3} = \frac{2}{3m^4}$			
2C	Simplify expressions using the laws of indices	Rule 3: $a^m \times a^n = a^{m+n}$ e.g. $5x^3 \times 6x^4 = 30x^7$ Rule 4: $\frac{a^m}{a^n} = a^{m-n}$ e.g. $\frac{12y^{10}}{3y^4} = 4y^6$			
2D	Raising a power to a further power	Rule 5: $\left(a^m\right)^n = a^{mn}$ e.g. $\left(x^4\right)^3 = x^{12}$ $\left(4y^5\right)^2 = 4^2\left(y^5\right)^2 = 16y^{10}$			
2E	Fractional Indices	Rule 6: $a^{\frac{1}{n}} = \sqrt[n]{a}$ e.g. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ Rule 7: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ e.g. $16^{\frac{3}{4}} = \sqrt[4]{16^3} = 2^3 = 8$			
2F	Multiplying out brackets	Combine algebraic skills with indices rules e.g. $x^{\frac{1}{2}}\left(x^{\frac{3}{2}} + x^{-\frac{1}{2}}\right) = x^2 + x^0 = x^2 + 1$			
2G	Writing numbers using Scientific Notation	In scientific notation, all numbers are written in the form $a \times 10^b$ e.g. $53000 = 5.3 \times 10^4$ $2.7 \times 10^{-3} = 0.0027$			
2H	Calculations using scientific notation	e.g. the mass of one atom of oxygen is 2.7×10^{-23}. What is the mass of 5×10^{30} atoms? $\left(2.7\times10^{-23}\right)\times\left(5\times10^{30}\right) = 1.35\times10^8$			

Chapter	Topic	Skills	☐	☐	☐
3 Working with algebraic expressions involving expansion of brackets					
3A	Collecting like terms	Add and subtract like terms e.g. $2a+3b-4a+7b$ $=2a-4a+3b+7b$ $=-2a+10b$			
3B	Using the distributive law to expand brackets	Expand brackets and simplify where necessary e.g. $7(a+2b-3c)=7a+14b-21c$ $3(2x+1)-2(x-3)$ $=6x+3-2x+6$ $=6x-2x+3+6$ $=4x+9$			
3C	Multiplying pairs of brackets	Multiply every term in the second bracket by every term in the first bracket e.g $(2x-1)(x+4)$ $=2x^2+4x-x-4$ $=2x^2+3x-4$			
3D	Multiplying a binomial expression by a trinomial expression	e.g. $(x+4)(2x^2-3x-1)$ $=x(2x^2-3x-1)+4(2x^2-3x-1)$ $=2x^3-3x^2-x+8x^2-12x-4$ $=2x^3-3x^2+8x^2-x-12x-4$ $=2x^3+5x^2-13x-4$			

Chapter	Topic	Skills	☐	☐	☐
4 Factorise an algebraic expression					
4A	Factorise by finding a common factor	Find the highest common factor of each term e.g. $9m+15n=3(3m+5n)$ $2x^2+6x=2x(x+3)$			
4B	Factorise an expression with a difference of two squares	Squared term minus a squared term e.g. $x^2-y^2=(x-y)(x+y)$ $4a^2-25=(4a-5)(4a+5)$			
4C	Factorise by finding a common factor followed by a difference of two squares	Find highest factor first e.g. $3x^2-48=3(x^2-16)$ $=3(x-4)(x+4)$			

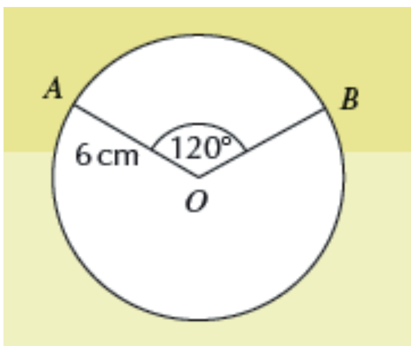
Chapter	Topic	Skills			
4D	Factorising a trinomial with a unitary x^2 coefficient	• Identify pairs of factors which multiply to make the first term • Identify pairs of factors which multiply to make the first term • Use trial and improvement to identify the combination of factors that multiply to give the correct middle term e.g. $x^2-5x+6=(x-2)(x-3)$ $x^2+2x-3=(x+3)(x-1)$			
4E	Factorising more complex trinomials with a non-unitary x^2 coefficient	Similar to process above, but consider all combinations of factors e.g. $3x^2+x-2=(3x+1)(x-2)$ ✗ (would produce middle term of $-5x$) $3x^2+x-2=(3x-2)(x+1)$ ✓			

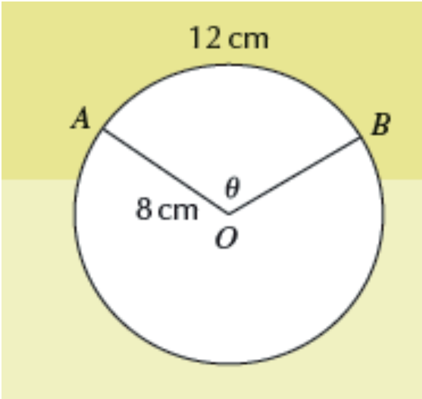
Chapter	Topic	Skills			
5 Completing the square in a quadratic expression with a unitary x^2 coefficient					
5A	Completing the square in a quadratic expression with a unitary x^2 coefficient	Divide the x-coefficient by 2, then subtract the square of this to maintain the value of the expression e.g. $x^2+4x+7=(x+2)^2-4+7$ $=(x+2)^2+3$			
5B	Solve equations by completing the square	An alternative to solving equations using the quadratic formula. e.g. $x^2+6x-5=0$ $(x+3)^2-14=0$ $(x+3)^2=14$ $x+3=\pm\sqrt{14}$ $x=-3+\sqrt{14},\ -3-\sqrt{14}$ $=0.7,\ -6.7$			

Chapter	Topic	Skills			
6 Reducing an algebraic fraction to its simplest form					
6A	Using Factorisation to simply an algebraic fraction	Factorise expressions first, then simplify fractions e.g. $\frac{10a^2}{12ab}=\frac{2a\times5a}{2a\times6b}=\frac{5a}{6b}$ $\frac{x^2-9}{x^2+x-12}=\frac{(x-3)(x+3)}{(x-3)(x+4)}=\frac{x+3}{x+4}$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
7 Applying one of the four operations to algebraic fractions					
7A	Adding and subtracting fractions with one term in the numerator or denominator	Express each fraction with the same common denominator (lowest common multiple) and add/subtract the numerators e.g. $\frac{2}{x}+\frac{3}{y}=\frac{2y}{xy}+\frac{3x}{xy}=\frac{2y+3x}{xy}$			
7B	Adding and subtracting fractions with more than one term in the numerator or denominator	Same method as above: make sure numerators are simplified e.g. $\frac{2}{(x+3)}-\frac{3}{(x-1)}$ $=\frac{2(x-1)}{(x+3)(x-1)}-\frac{3(x+3)}{(x+3)(x-1)}$ $=\frac{2(x-1)-3(x+3)}{(x+3)(x-1)}$ $=\frac{2x-2-3x-9}{(x+3)(x-1)}=\frac{-x-11}{(x+3)(x-1)}$			
7C	Multiplying algebraic fractions	Multiply numerators and denominators together. Simplify as before (see exercise 6A). e.g. $\frac{3x}{4y^2}\times\frac{5}{6x}=\frac{3x\times5}{4y^2\times6x}$ $=\frac{5\times3x}{4y^2\times2\times3x}$ $=\frac{5}{8y^2}$			
7D	Dividing algebraic fractions	Multiply first fraction by reciprocal of second fraction e.g. $\frac{4}{3a}\div\frac{2}{9a^2}=\frac{4}{3a}\times\frac{9a^2}{2}$ $=\frac{4\times9a^2}{3a\times2}=\frac{3a\times3a\times2\times2}{3a\times2}$ $=\frac{6a}{1}=6a$			

Chapter	Topic	Skills			
8 Determine the gradient of a straight line, given two coordinates					
8A	Gradient and the gradient formula	$\text{gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$ Apply this process to the general points (x_1, y_1) and (x_2, y_2) produces this formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ e.g. The gradient between $(-2,1)$ and $(4,-2)$: $m = \frac{-2-1}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2}$			

Chapter	Topic	Skills			
9 Calculating the length of an arc of the area of a sector					
9A	Calculating arc lengths and sector areas	An arc of a circle is part of its circumference. $\text{arc} = \frac{\text{angle}}{360} \times \pi d$ where d is the diameter of the circle A sector is part of the area of a circle $\text{sector} = \frac{\text{angle}}{360} \times \pi r^2$ where r is the diameter of the circle			
9B	Expressing arc lengths and sector areas in terms of π	Calculate as above, but leave π in calculation. e.g.  $arc = \frac{120}{360} \times \pi \times 12$ $= \frac{1}{3} \times \pi \times 12 = 4\pi \text{ cm}$			

Chapter	Topic	Skills			
9C	Finding the sector angle	Use rule for calculating arc length or sector area, and re-arrange  e.g. $12 = \frac{\vartheta}{360} \times \pi \times 16$ $\vartheta = \frac{12 \times 360}{\pi \times 16}$ $\vartheta = 86°$			

Chapter	Topic	Skills			
10 Calculating the volume of a standard solid					
10A	Volume of a sphere	Use the formula $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere			
10B	Volume of a pyramid	Use the formula $V = \frac{1}{3}Ah$ where A is the area of the base of the pyramid, and h is the perpendicular height			
10C	Volume of a cone	Use the formula $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the circular base and h is the perpendicular height			
10D	Volumes of composite solids	Two or more solids combine to create a new solid. Calculate the volume of each, then add/subtract as necessary			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
11 Rounding to a given number of significant figures					
11A	Rules when using significant figures	• Trailing zeroes are not significant e.g. 35000 has 2 significant figures • Leading zeroes are not significant e.g. 0.0000047 has 2 significant figures • All other zeroes are significant e.g. 904.05 has 5 significant figures			
11B	Rounding to a given number of significant figures	Round to nearest significant figure: use the subsequent digit to determine if it rounds up or is truncated e.g. 36410 = 36000 to 2 significant figures 2.1578 = 2.16 to 3 significant figures 4267.153 = 4270 to 3 significant figures			

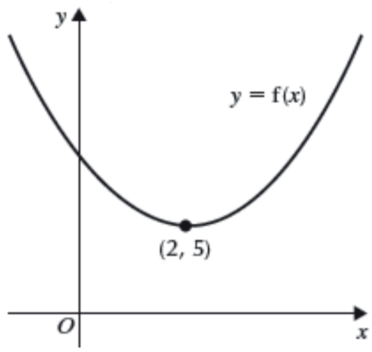
Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
12 Determining the equation of a straight line, given the gradient					
12A	Find the equation of a straight line in the form $y = mx + c$	• Find the gradient (m) from a graph (using skills from chapter 8A) • Identify the y-intercept from a graph (point where the graph crosses the y-axis • Substitute into the equation $y = mx + c$			
12B	Use the formula $y - b = m(x - a)$ to find the equation of a straight line	Identify the gradient (m) and one point on a straight line (a, b) and substitute into the formula $y - b = m(x - a)$ e.g. Straight line passes through $(3, -5)$ and $(-1, 7)$ $m = \frac{7-(-5)}{-1-3} = \frac{12}{-4} = -3$ $y - (-5) = -3(x - 3)$ $y + 5 = -3x + 9$ $y = -3x + 4$			
12C	Use and apply function notation f(x)	A function applies a rule to a set of numbers to produce a new set of numbers e.g. given that $f(x) = x^2 + 2x$, $f(-3) = (-3)^2 + 2\times(-3) = 9 + (-6) = 3$ If $g(x) = 2x - 3$ and $g(t) = 4$: $g(t) = 2t - 3 = 4$ $2t - 3 = 4$ $2t = 7$ $t = \frac{7}{2} (= 3.5)$			

Chapter	Topic	Skills	🟩	🟨	🟥
13 Working with linear equations and inequations					
13A	Solving linear equations containing brackets	Expand brackets, then solve equation e.g. $2(3x-8)-3x=17$ $6x-16-3x=17$ $3x-16=17$ $3x=33$ $x=11$			
13B	Solving linear equations containing fractions	Multiply each term in the equation by the lowest common multiple (LCM) of the denominators, then solve the equation e.g. $\frac{x}{5}+4=\frac{x-1}{3} \quad (\times 15)$ $3x+60=5(x-1)$ $3x+60=5x-5$ $65=2x$ $x=\frac{65}{2}(=32.5)$			
13C	Solving more complex linear inequations	Solve linear inequations in the same way as equations. However, multiplying or dividing each term by a negative number changes the direction of the inequality symbol e.g. $2(3-2x)+6<2x$ $6-4x+6<2x$ $12-4x<2x$ $-6x<-12$ $x>2$			

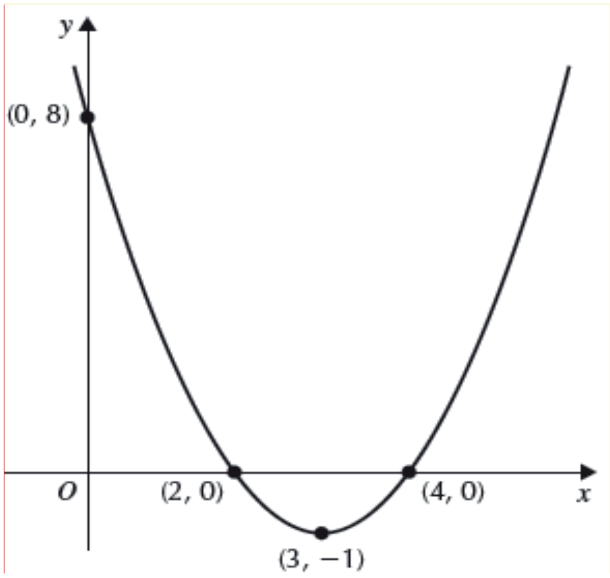
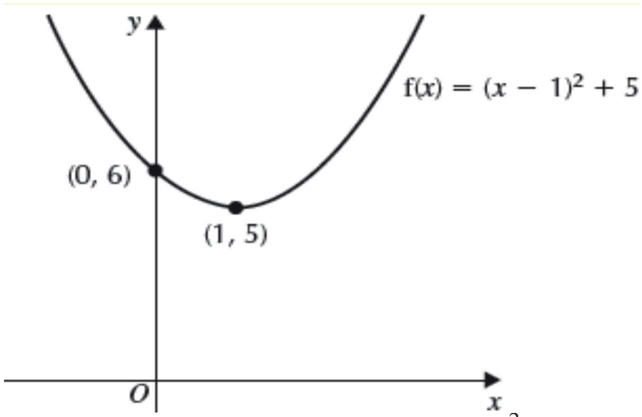
Chapter	Topic	Skills	🟩	🟨	🟥
14 Working with simultaneous equations					
14A	Solving a pair of simultaneous equations graphically	Draw each line on the same graph: • Find where the line intersects the y-axis (by choosing $x=0$) • Find where the line intersects the x-axis (by choosing $y=0$) • Draw the line Then find the point of intersection of the two lines			

Chapter	Topic	Skills			
14B	Solving a pair of simultaneous equations by substitution	At the point of intersection, the x-value and the y-value are the same for both lines. e.g. Solve $y = 4x - 1$ $y = 2x - 9$ Substitute the first equation into the second equation since y values are the same $4x - 1 = 2x - 9$ $2x = -8$ $x = -4$ $y = 4 \times (-4) - 1 = -17$ The solution is $x = -4, y = -17$			
14C	Solving a pair of simultaneous equations by elimination	Add or subtract equations to eliminate one term. Solve for the other term, then substitute to complete e.g. $x + y = 10$ $2x - y = 8$ Add together $3x = 18$ $x = 6$ Substitute $6 + y = 10 \rightarrow y = 4$ The solution is $x = 6, y = 4$			
14D	Systems which require preparation for elimination	Choose a variable to eliminate then multiply one or both equations to make elimination possible e.g. $2x + 3y = 2 \quad (\times 4)$ $3x + 4y = 4 \quad (\times 3)$ $8x + 12y = 8$ $9x + 12y = 12$ Subtract $-x = -4$ $x = 4$ Substitute $2 \times 4 + 3y = 2$ $8 + 3y = 2$ $3y = -6$ $y = -2$ The solution is $x = 4, y = -2$			
14E	Create and solve pairs of simultaneous equations from text	Use the information in the text of the question to set up two simultaneous equations, then solve using elimination, substitution or graphically.			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
15 Changing the subject of a formula					
15A	Changing the subject of a simple formula	Re-arrange the given formula to isolate a different letter on one side. e.g. Change the subject to x: $y = 2x - 5$ $2x - 5 = y$ $2x - 5 + 5 = y + 5$ $2x = y + 5$ $\frac{2x}{2} = \frac{y+5}{2}$ $x = \frac{y+5}{2}$			
15B/15C	Changing the subject of a formula containing fractions	Similar to equations: multiply each term in the equation by the lowest common multiple (LCM) of the denominators, then rearrange as before e.g. Change the subject to x: $y = \frac{x}{3} + 7 \quad (\times 3)$ $3y = x + 21$ $x + 21 = 3y$ $x = 3y - 21$			
15D	Changing the subject of a formula containing brackets, roots or powers	When making x the subject of the formula: • If x is inside a pair of brackets, multiply out the brackets first • Isolate any powers or roots on the LHS of the equation • Remove any powers or roots • Complete if required e.g. change the subject to x: $b = \sqrt{\frac{3x}{h}}$ $(b)^2 = \left(\sqrt{\frac{3x}{h}}\right)^2$ $\frac{3x}{h} = b^2$ $3x = b^2h$ $x = \frac{b^2h}{3}$			

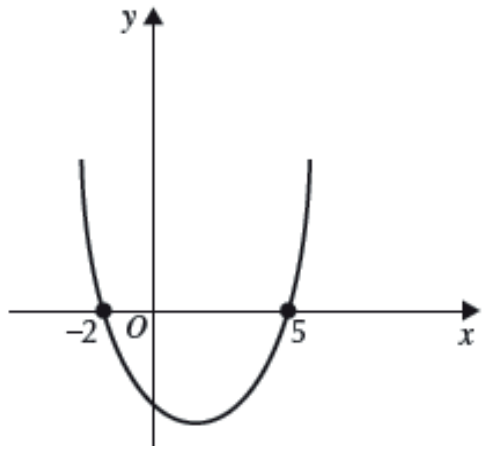
Chapter	Topic	Skills			
16 Recognise and determine the equation of a quadratic function from its graph					
16A	Recognise and determine the equation of a quadratic function of the form $y=kx^2$	• Choose any point on the graph • Substitute the values of the coordinate into $y=kx^2$ • Solve the equation to find k			
16B	Recognise and determine the equation of a quadratic function of the form $y=(x+p)^2+q$	Key features of the graph of $y=(x+p)^2+q$: • It has a minimum turning point of $(-p,q)$ • Its axis of symmetry has equation $x=-p$  e.g. The function has equation $f(x)=(x-2)^2+5$ The equation of the axis of symmetry is $x=2$ The graph of $y=-(x+p)^2+q$ has a **maximum** turning point of $(-p,q)$ and its axis of symmetry has equation $x=-p$			

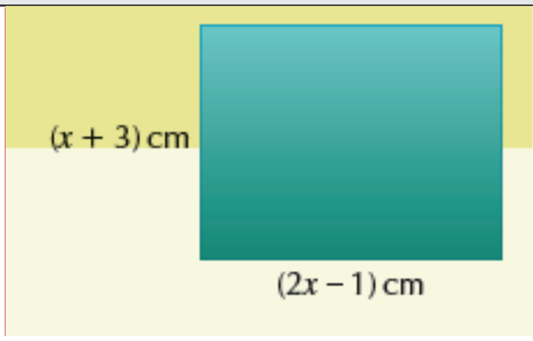
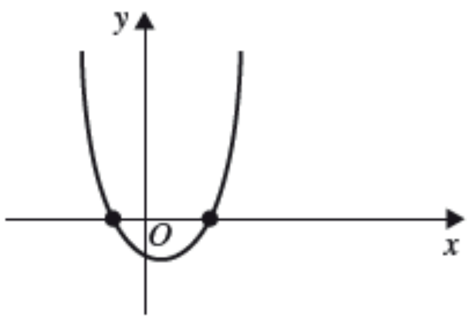
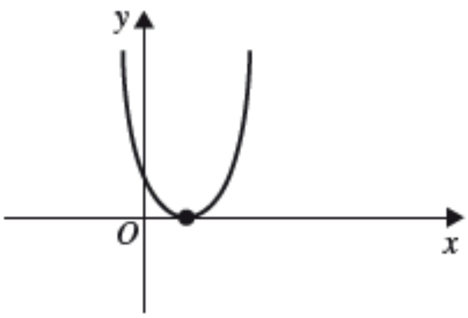
Chapter	Topic	Skills			
17 Sketching a quadratic function					
17A	Sketch the quadratic function of the form $y=(x-m)(x-n)$	• Find the roots by setting y=0 • Find the y-intercept by setting x=0 • Find the turning point • Make a neat sketch e.g. Sketch the graph $y=(x-2)(x-4)$ Roots: $(x-2)(x-4)=0$ $x-2=0,\ x-4=0$ $x=2,\ x=4$ y-intercept $y=(0-2)(0-4)=(-2)\times(-4)=8$ Turning point: x-coordinate is halfway between roots: 2 3 4 so $x=3$ $y=(3-2)(3-4)=1\times(-1)=-1$ $(3,-1)$			

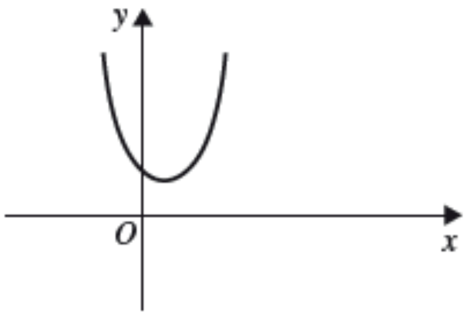
Chapter	Topic	Skills			
					
17B/C	Sketch the quadratic function of the form $y=(x+p)^2+q$ or $y=-(x+p)^2+q$ or $y=k(x+p)^2+q$	• Find the y-intercept by setting x=0 • Find the turning point • Make a neat sketch e.g. sketch the graph of $f(x)=(x-1)^2+5$ y-intercept $f(x)=(0-1)^2+5$ $=1+5=6$ Turning point: $(1,5)$  For graphs of the form $y=k(x+p)^2+q$: • If $k>0$ the graph has a minimum turning point • If $k<0$ the graph has a maximum turning point			
17D	Sketch the quadratic function of the form $y=ax^2+bx+c$	Calculate the discriminant (b^2-4ac). If $b^2-4ac>0$, the graph has two distinct real roots. Factorise and apply process from Exercise 17A If $b^2-4ac=0$ (two real and equal roots) or if $b^2-4ac<0$ (no real roots), complete the square of the function and apply process from Exercise 17B/C. The discriminant is covered in chapter 19.			

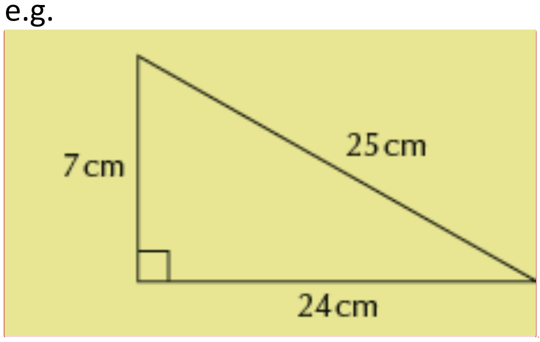
Chapter	Topic	Skills			
18 Identifying features of a quadratic function					
18A	Identify the nature and coordinates of the turning point and the equation of the axis of symmetry of the functions $y=\pm(x+p)^2+q$	Identify key features from graphs in chapter 16 e.g. A graph has the equation $y=-(x+5)-2$ It has a maximum turning point of $(-5,-2)$ Its equation of the axis of symmetry is $x=-5$			
18B	Apply knowledge of the different forms of a quadratic function to solve related problems	Use knowledge of roots, turning point and axis of symmetry of a graph to solve questions in context			

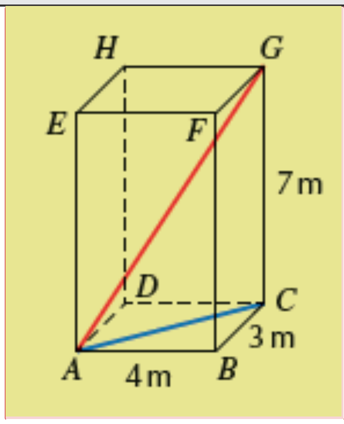
Chapter	Topic	Skills			
19 Working with quadratic equations					
19A	Solving an equation in standard form algebraically	• Factorise the quadratic expression • Set each factor equal to zero • Solve for each new equation e.g. $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x-3=0,\ x+1=0$ $x=3,\ x=-1$			
19B	Dealing with equations which are not in standard form	Rearrange into standard form $ax^2+bx+c=0$ then solve e.g. $x^2=3x+10$ $x^2-3x-10=0$ $(x-5)(x+2)=0$ $x-5=0,\ x+2=0$ $x=5,\ x=-2$			

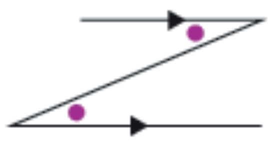
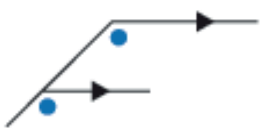
Chapter	Topic	Skills			
		$x^2 = 3x + 10$ $x^2 - 3x - 10 = 0$ $(x-5)(x+2) = 0$ $x - 5 = 0,\ x + 2 = 0$ $x = 5,\ x = -2$			
19C/D	Solving equations using the quadratic formula	The quadratic formula is used to solve a quadratic equation when quadratic cannot be factorised. For a quadratic equation written as $ax^2 + bx + c = 0$, the roots are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ e.g. Solve $3x^2 - 2x - 7 = 0$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-7)}}{2 \times 3} = \dfrac{2 \pm \sqrt{88}}{6}$ $= \dfrac{2 + \sqrt{88}}{6}$ or $\dfrac{2 - \sqrt{88}}{6}$ $= 1.9$ or -1.2 Evaluate the roots to the specified degree of accuracy			
19E	Link between the roots of a quadratic equation and the graph of the corresponding function	The roots of the quadratic equation are the points at which the graph of he corresponding quadratic function intersects the x axis. e.g. The graph of $y = x^2 - 3x - 10$ is shown here:  The graph cuts the x-axis at $x = -2$ and $x = 5$. The roots of the equation $x^2 - 3x - 10 = 0$ are therefore $x = -2$ and $x = 5$.			
19G	Problem-solving using quadratic equations	Form a quadratic equation using the information given, and use this to solve an equation e.g. The area of the rectangle is 72cm^2. Find the value of x.			

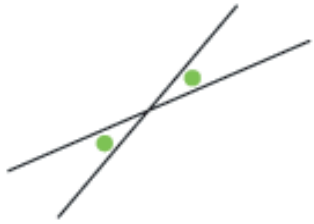
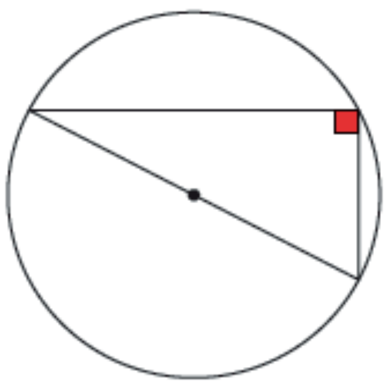
Chapter	Topic	Skills			
		 $A=(2x-1)(x+3)$ $(2x-1)(x+3)=72$ $2x^2+5x-3=72$ $2x^2+5x-75=0$ $(2x+15)(x-5)=0$ $2x+15=0,\ x-5=0$ $x=-\frac{15}{2}, x=5$ $x\neq-\frac{15}{2}$, son $x=5$			
19H	The discriminant of a quadratic equation	The discriminant (b^2-4ac) is part of the quadratic equation. Use this to determine the nature of the roots of the equation. $b^2-4ac>0$  Two real distinct roots $b^2-4ac=0$  Two real equal roots			

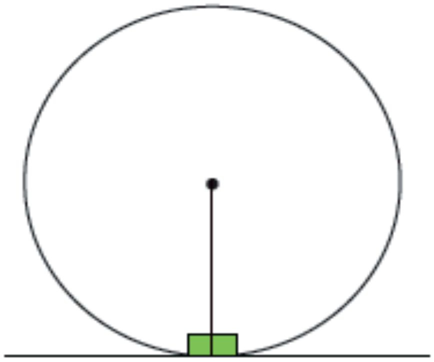
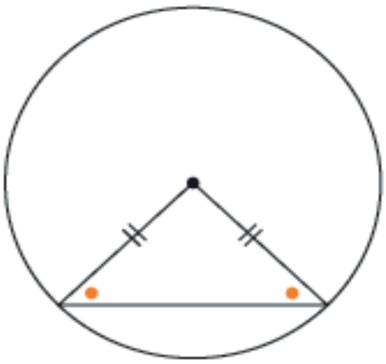
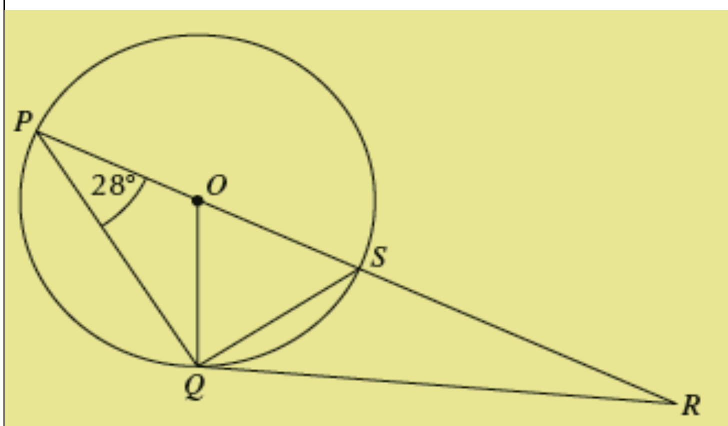
Chapter	Topic	Skills			
		$b^2 - 4ac < 0$  No real roots e.g. Determine the nature of the roots of $y = 2x^2 - 3x + 5$ $b^2 - 4ac$ $= (-3)^2 - 4 \times 2 \times 5$ $= 9 - 40 = -39 < 0$ No real roots			

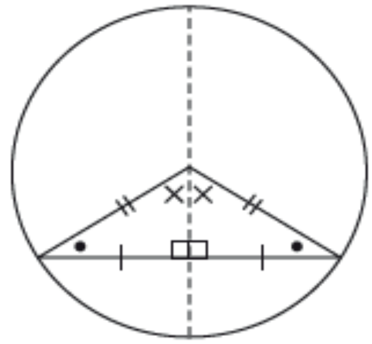
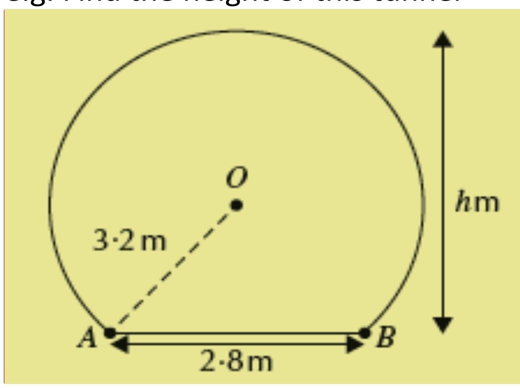
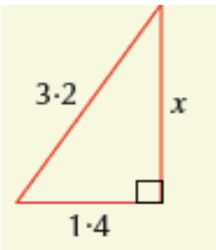
Chapter	Topic	Skills			
20A	Applying Pythagoras' theorem in complex 2D situations	Pythagoras' theorem $\left(c^2 = a^2 + b^2\right)$ can be applied twice within the same question to determine the size of a missing length of a right-angled triangle			
20B	The converse of Pythagoras' theorem	If c is the longest side of a triangle and a and b are the two shorter sides, the converse of Pythagoras' theorem states: If $c^2 = a^2 + b^2$ the triangle is right-angled. e.g.  $c^2 = 25^2 = 625$ $a^2 + b^2 = 7^2 + 24^2 = 625$ $c^2 = a^2 + b^2$ so the triangle is right-angled			
20C	Applying Pythagoras' theorem in 3D problems	A **face** diagonal lies on a face of a shape A **space** diagonal joins opposite vertices and are inside the shape. e.g. Find the length of space diagonal AG			

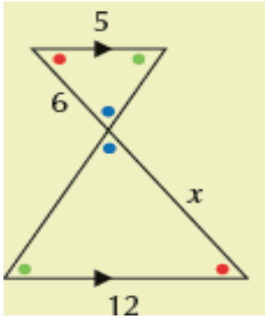
Chapter	Topic	Skills	☐	☐	☐
		 Find the length of face diagonal AC $AC^2 = 4^2 + 3^2 = 25$ $AC = \sqrt{25} = 5$ Use this to find the space diagonal AG $AG^2 = 5^2 + 7^2 = 74$ $AG = \sqrt{74} = 8.60\text{m}$			

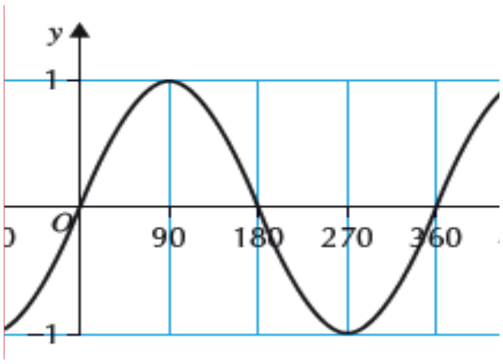
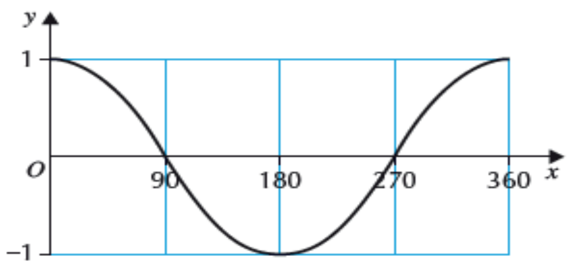
Chapter	Topic	Skills	☐	☐	☐
21 Applying the properties of shapes to determine an angle involving at least two steps					
21A	Finding angles using the angle properties of triangles and quadrilaterals	Use angle properties of lines and 2D shapes to determine the size of angles in geometric problems **Alternate angles**  Alternate angles are equal. **Corresponding angles**  Corresponding angles are equal.			

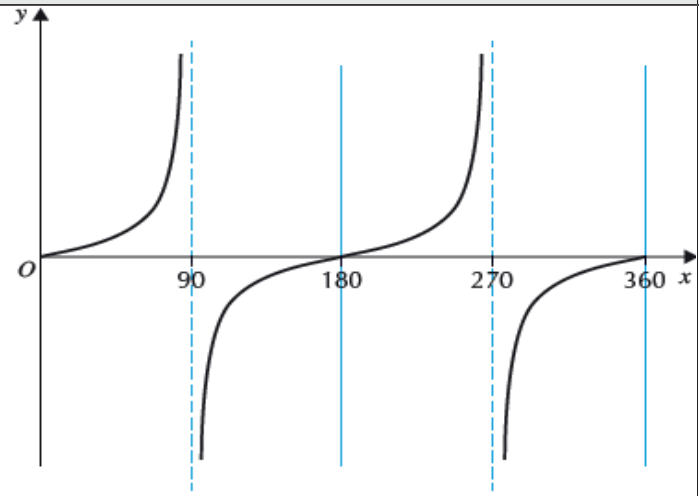
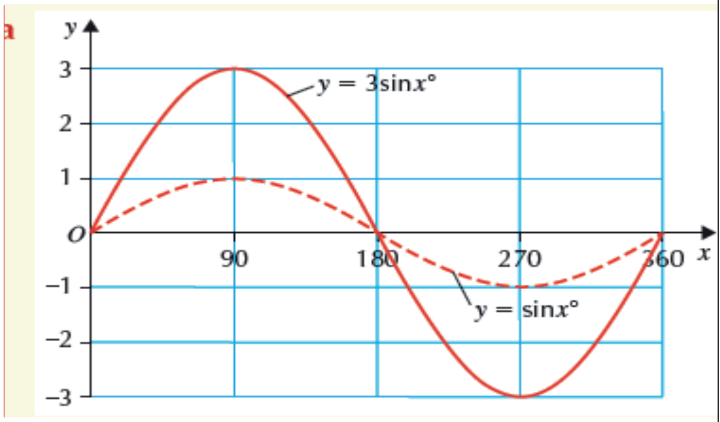
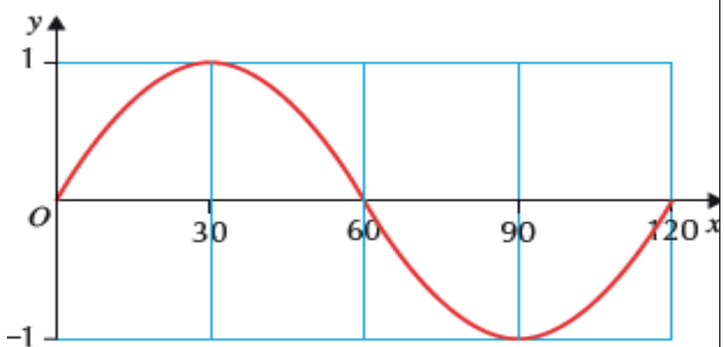
Chapter	Topic	Skills			
		Vertically opposite angles  Vertically opposite angles are equal. **Angles in a triangle**  Angles in any triangle add up to 180º. **Angles in a quadrilateral**  Angles in any quadrilateral add up to 360°.			
21B	Finding angles using angle properties of circles	Angle properties within circles are shown below: **Angles in a semi-circle**  A triangle formed by the diameter of a circle and any point on the circle has a right angle opposite the centre of the circle.			

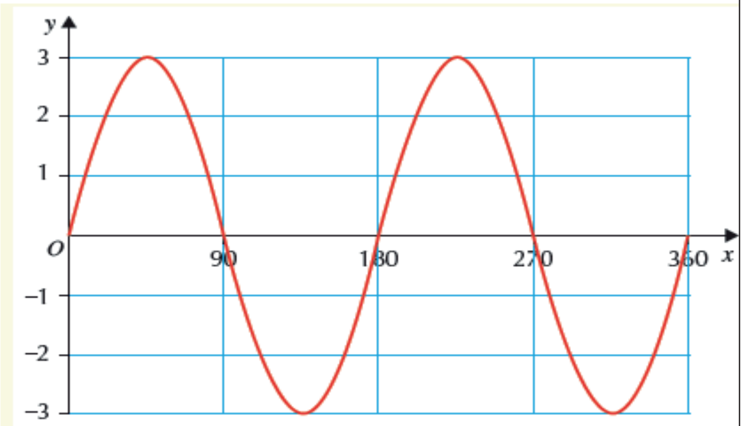
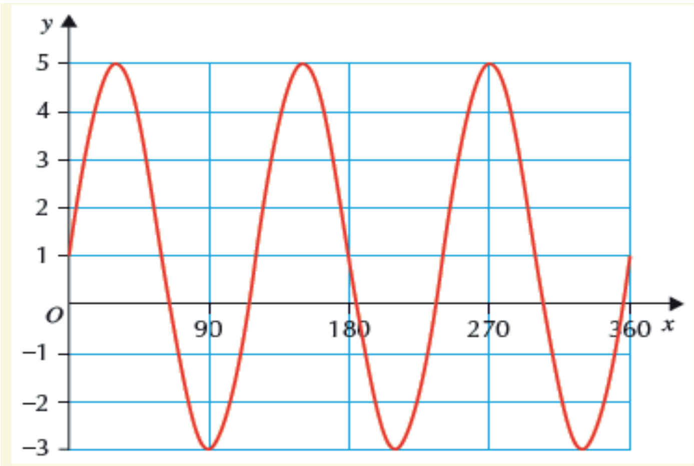
Chapter	Topic	Skills	🟩	🟨	🟥
		Angle of a tangent  A tangent is perpendicular to the radius drawn to the point of contact. **Triangle within a circle**  A triangle formed by a chord and two radii is an isosceles triangle. Use these three properties along with the properties of lines and shapes applied in Exercise 21B to determine the size of unknown angles in a diagram. e.g.  PS is a diameter and QR is a tangent. Find ∠SRQ. ∠PQO= 28° (isosceles)			

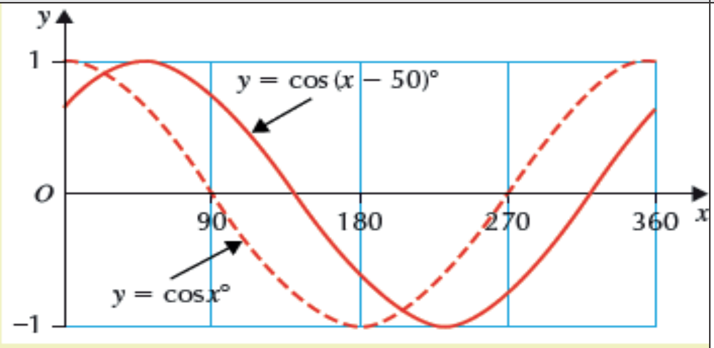
Chapter	Topic	Skills			
		∠OQS = 90 – 28 = 62° (semi-circle) ∠OSQ = ∠OQS = 62° (isosceles) ∠SQR = 90 – 62 = 28° (tangent) ∠QSR = 180 – 62 = 118° (straight line) ∠SRQ = 180 – (118+28) = 34° (triangle) Hint: demonstrate the sizes of angles within diagram given.			
21C/D	Symmetry in the circle	When the diameter meets a chord at 90°, it bisects the chord. This forms two right-angled triangles.  Pythagoras' theorem (or trigonometry) can be applied to solve problems. e.g. Find the height of this tunnel   $x^2 = 3.2^2 - 1.4^2 = 8.28$ $x = \sqrt{8.28} = 2.877\ldots$ $h = 2.877\ldots + 3.2$ $= 6.1\text{m}$			
21E	Finding angles using angle properties of polygons	For any n-sized polygon, the sum of the interior angles is given by $S = 180(n-2)$ The exterior angle = angle at the centre = $\dfrac{360°}{n}$			

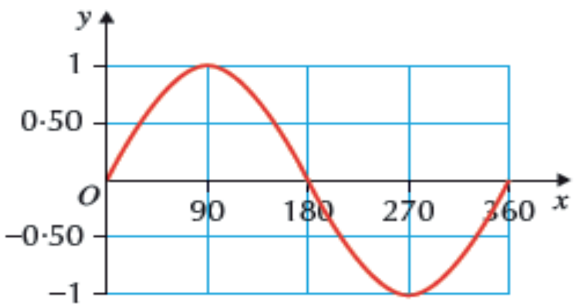
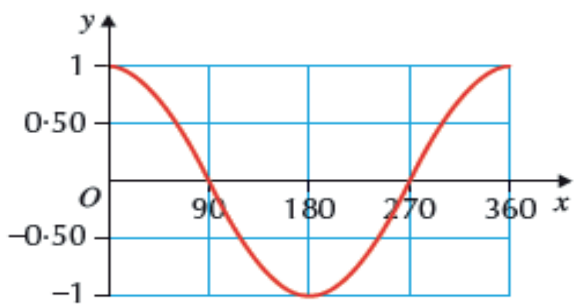
Chapter	Topic	Skills	🟩	🟨	🟥
22 Using Similarity					
22A	Similar triangles formed by parallel lines	Look for similar triangles (corresponding and alternate angles). Reduction Scale factor = $\frac{\text{small}}{\text{big}}$ Enlargement scale factor = $\frac{\text{big}}{\text{small}}$ Unknown side = scale factor × corresponding side e.g.  Enlargement scale factor = $\frac{12}{5}$ $x = \frac{12}{5} \times 6 = 14.40$ (to 2 decimal places)			
22B/C	Area and volume of similar shapes	Unknown area = (scale factor)^2 × corresponding area Unknown volume = (scale factor)^3 × corresponding volume			

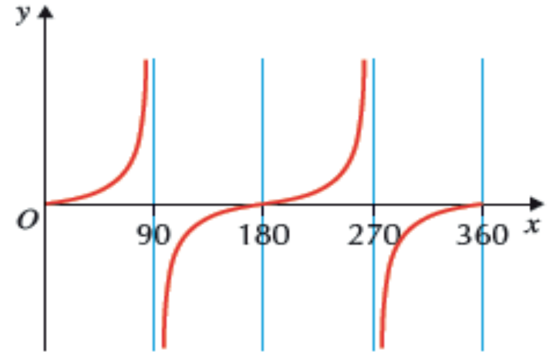
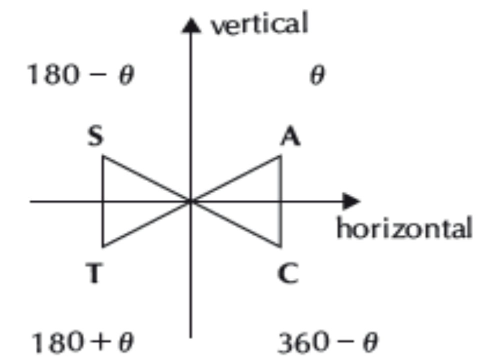
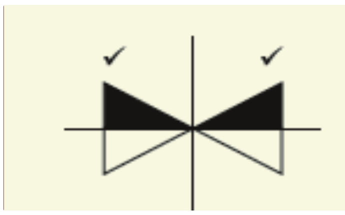
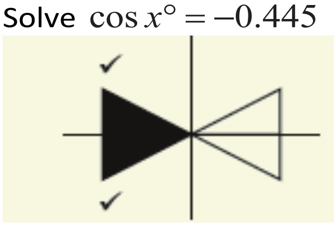
Chapter	Topic	Skills	🟩	🟨	🟥
23C Working with graphs of trigonometric functions					
23A	The graphs of the sinx and cosx functions, and the graphs of sine and cosine functions with different amplitude	The sine graph has an amplitude of 1 and a period of 360°  The cosine graph also has an amplitude of 1 and a period of 360°  The tangent graphs has an undefined amplitude and a period of 180°			

Chapter	Topic	Skills			
		 The graphs of $y = a\sin x°$ and $y = a\cos x°$ have an amplitude of $\lvert a \rvert$ (positive difference between a and zero) and a period of 360° e.g. $y = 3\sin x°$ 			
23B	The graphs of sine and cosine functions with different periods	The graphs $y = \sin bx°$ and $y = \cos bx°$ have an amplitude of 1 and a period of $\frac{360}{b}$ e.g. $y = \cos 3x°$ 			

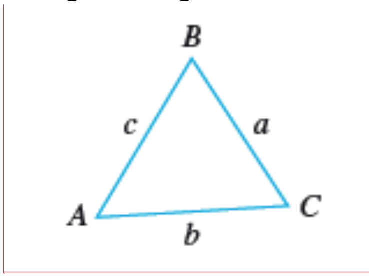
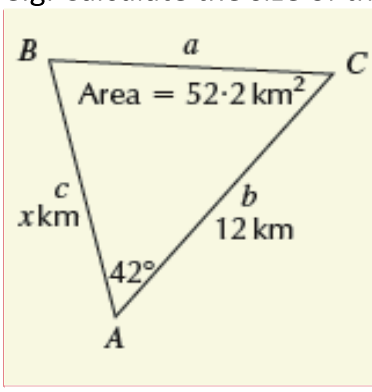
Chapter	Topic	Skills			
23C	The graphs of sine and cosine functions with different amplitudes and periods	The graphs $y = a\sin bx°$ and $y = a\cos bx°$ have an amplitude of $\lvert a \rvert$ and a period of $\frac{360}{b}$ e.g. $y = 3\sin 2x°$  			
23d	The graphs of sine and cosine functions which have undergone a vertical translation	The graphs $y = a\sin bx° + c$ and $y = a\cos bx° + c$ have an amplitude of $\lvert a \rvert$ and a period of $\frac{360}{b}$. They are shifted vertically up or down by c (up if c is positive, down if c is negative) e.g. $y = 4\sin 3x° + 1$  			
23E	Problems involving the sine and cosine graphs	Many real-life problems involving circular motion can be solved using sine and cosine graphs			
23F	The graphs of sine, cosine and tangent functions which have undergone a horizontal translation	The graphs $y = a\sin(x+d)°$, $y = a\cos(x+d)°$ and $y = a\tan(x+d)°$ are shifted horizontally by d: left if d is positive and right if d is negative e.g. $y = \cos(x-50)°$			

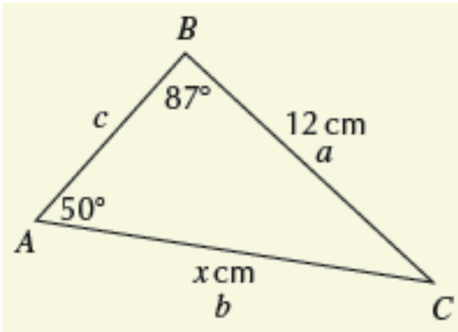
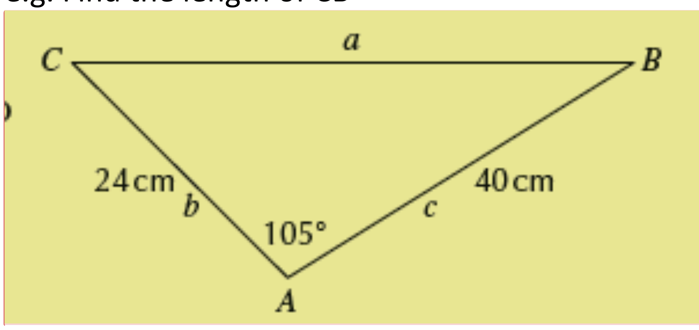
Chapter	Topic	Skills	☐	☐	☐
		 Angle d is known as the **phase angle.**			

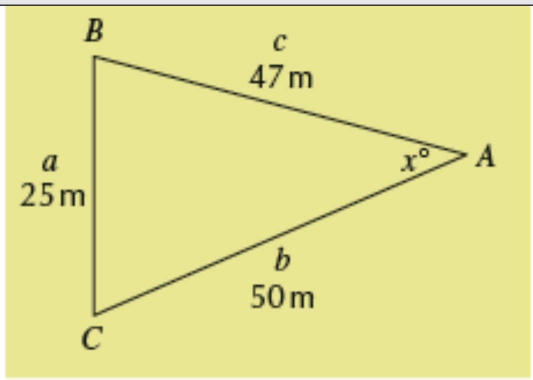
Chapter	Topic	Skills	☐	☐	☐
Working with trigonometric relationships in degrees					
24A	Key points on trigonometric graphs	**sin x°**  sin x° = 0 when x = 0, 180, 360 sin x° = 1 when x = 90 sin x° = –1 when x = 270 **cos x°**  cos x° = 0 when x = 90, 270 cos x° = 1 when x = 0, 360 cos x° = –1 when x = 180			

Chapter	Topic	Skills	🟩	🟨	🟥
		tan x°  $\tan x° = 0$ when $x = 0, 180, 360$			
24B	Working with related angles	The quadrant diagram shows where each ratio is positive: 			
24C	Solving trigonometric equations	Use knowledge of related angles and quadrant diagrams to solve trigonometric equations e.g. Solve $\sin x° = 0.835$  $x = \sin^{-1}(0.835) = 56.6°$ (first quadrant) $x = 180 - 56.6 = 123.4°$ (second quadrant) Solution: x=56.6° and x=123.4° Solve $\cos x° = -0.445$  Related acute angle = $x = \cos^{-1}(0.445) = 63.6°$ (first quadrant angle) $x = 180 - 63.6 = 116.4°$ (second quadrant) $x = 180 + 63.6 = 243.6°$ (third quadrant) Solution: $x = 116.4°$ and $x = 243.6°$			

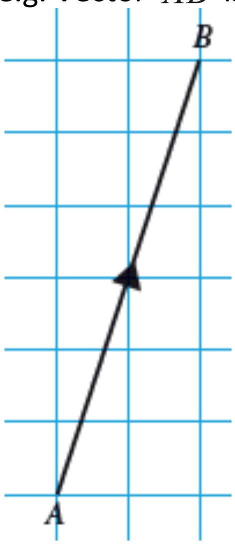
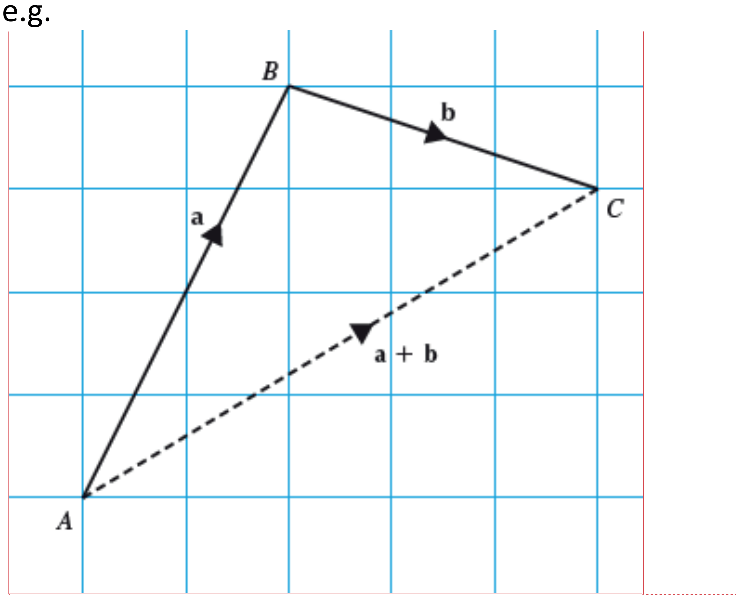
Chapter	Topic	Skills			
24D	Further equations	Rearrange to find $\sin x^\circ = \dots$ or equivalent. Then solve in same way as Exercise 24C			
24G	Trigonometric Identities	Trigonometric identities relate the three basic ratios to each other. For any angle x: • $\frac{\sin x}{\cos x} = \tan x$ • $\sin^2 x + \cos^2 x = 1$ Note: by convention $(\sin x)^2$ is written as $\sin^2 x$ e.g. Show that $\frac{1-\sin^2 x}{\cos x} = \cos x$ LHS $\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x = \text{RHS}$			

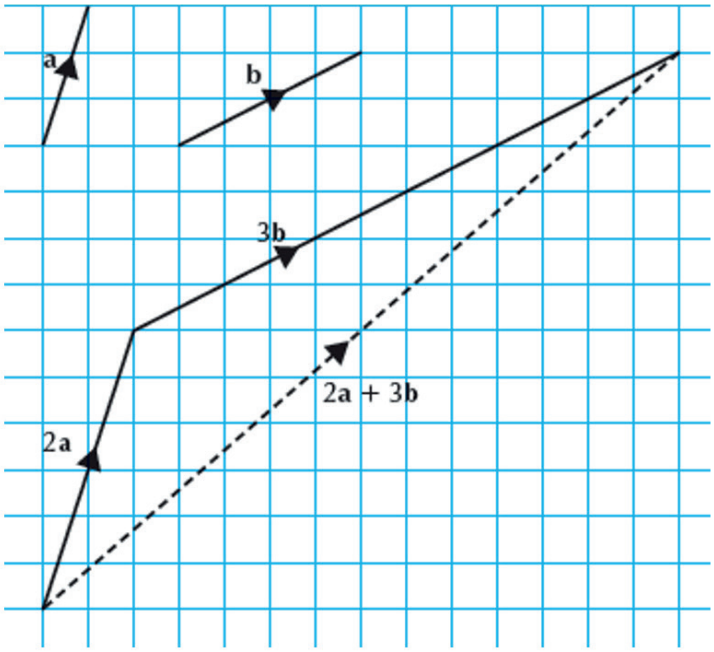
Chapter	Topic	Skills			
25 Calculating the area of a triangle using trigonometry					
25A	Calculating the area of a triangle using trigonometry	Using a triangle labelled in this way:  The area of a triangle can be calculated using the formula: $A = \frac{1}{2}ab\sin C$			
25B	Using the formula to find a missing side or angle	Use rule for calculating area, and re-arrange e.g. Calculate the size of the missing side AB  $52.2 = \frac{1}{2} \times 12 \times x \times \sin 42^\circ$ $52.2 = 4.01x$ $4.01x = 52.2$ $x = \frac{52.2}{4.01} = 13.1\text{km}$			

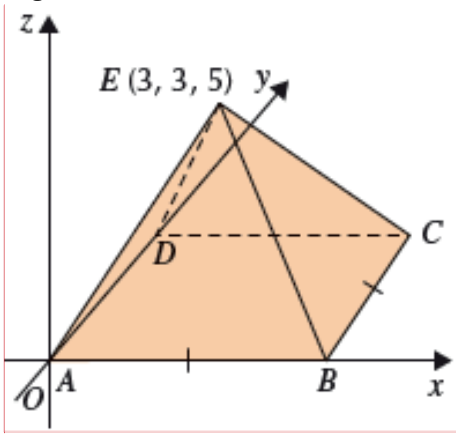
Chapter	Topic	Skills	🟩	🟨	🟥
26 Using the sine rule and cosine rules to find a side or angle					
26A	Using the sine rule	Use the following rule to find missing sides and angles in a non-right angled triangle: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$ e.g. Find the length of AC  $\dfrac{a}{\sin A}=\dfrac{B}{\sin B}$ $\dfrac{12}{\sin 50°}=\dfrac{x}{\sin 87°}$ $x=\dfrac{12\sin 87°}{\sin 50°}=15.6\,\text{cm}$			
26B	Using the cosine rule	The cosine rule relates all three lengths of sides of any triangles to the cosine of one of its angles: $a^2=b^2+c^2-2bc\cos A$ e.g. Find the length of CB  $a^2=b^2+c^2-2bc\cos A$ $=24^2+40^2-2\times24\times40\times\cos 105°$ $=2672.93$ $a=\sqrt{2672.93}=51.7\,\text{cm}$			
26C	The cosine rule for an angle with all three sides unknown	We can rearrange the cosine rule formula to calculate missing angles: $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$ e.g. Find the size of angle BAC			

Chapter	Topic	Skills	Green	Amber	Red
		 $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ $= \dfrac{50^2 + 47^2 - 25^2}{2 \times 50 \times 47} = 0.869$ $x = \cos^{-1}(0.869) = 29.7°$			
26D	Choosing the correct formula	<table><tr><td>To find</td><td>Given in question</td><td>Formula</td></tr><tr><td>side</td><td>1 side, 2 angles</td><td>$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$</td></tr><tr><td>angle</td><td>2 sides, 1 angle</td><td>$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$</td></tr><tr><td>side</td><td>2 sides, 1 angle</td><td>$a^2 = b^2 + c^2 - 2bc \cos A$</td></tr><tr><td>angle</td><td>3 sides</td><td>$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$</td></tr></table>			

Chapter	Topic	Skills	Green	Amber	Red
27 Using bearings with trigonometry					
27A/B	Bearings and trigonometry	Bearings are angles measured from a North line reading clockwise. They usually have three digits. Use the work covered in the chapter 26 to solve problems involving bearings. It is helpful to extend north lines and look for corresponding and alternate angles as well as angles in a triangle			

Chapter	Topic	Skills	🟩	🟨	🟥
28 Working with 2D vectors					
28A	Drawing and writing 2D vectors	A vector has magnitude (size) and direction. A directed line segment is a line joining any two points. The arrow indicates the direction of the line segment (or vector). Components can be used to show the difference in direction: $\begin{pmatrix} \text{horizontal} \\ \text{vertical} \end{pmatrix}$. e.g. Vector $\overrightarrow{AB}$ is shown below  It can be written as $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ in component form. Vectors can also be named by a single letter e.g. $\underline{\mathbf{v}}$			
28B	Adding and subtracting vectors	Join "nose-to-tail" to produce a resultant vector. e.g.  Using the vectors a and b, find the resultant vector **2a** +3**b** Method 1: using directed line segments			

Chapter	Topic	Skills			
		 $2\mathbf{a}+3\mathbf{b}=\begin{pmatrix}14\\12\end{pmatrix}$ Method 2: using components $2\mathbf{a}=\begin{pmatrix}2\\6\end{pmatrix}$ $3\mathbf{b}=\begin{pmatrix}12\\6\end{pmatrix}$ $2\mathbf{a}+3\mathbf{b}=\begin{pmatrix}2\\6\end{pmatrix}+\begin{pmatrix}12\\6\end{pmatrix}=\begin{pmatrix}14\\12\end{pmatrix}$			

Chapter	Topic	Skills			
29 Working with 3D Coordinates					
29A	Determining coordinates in 3 dimensions	3D coordinates work in a similar way to 2D coordinates with the addition of an extra axis (the z-axis). Coordinates are plotted as (x, y, z). e.g. Find the coordinates of C  Solution: $(6,6,0)$			

Chapter	Topic	Skills	🟩	🟨	🟥
30 Using vector components					
30A	Adding and subtracting using vector components	2D and 3D vectors can be added and subtracted using their component forms to find the **resultant** vector. e.g. $\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}5\\-2\end{pmatrix}=\begin{pmatrix}7\\1\end{pmatrix}$ $\begin{pmatrix}5\\2\\-7\end{pmatrix}-\begin{pmatrix}3\\-5\\-1\end{pmatrix}=\begin{pmatrix}2\\7\\-6\end{pmatrix}$			
30B	Calculating the magnitude of a vector	Using Pythagoras' theorem to calculate the longest side in a right-angled triangle, the magnitude (length) of a 2D vector $AB=\begin{pmatrix}x\\y\end{pmatrix}$ can be written as: $\lvert AB\rvert=\sqrt{x^2+y^2}$ e.g. Find $\lvert a\rvert$, the magnitude of $a=\begin{pmatrix}3\\-4\end{pmatrix}$ $\lvert a\rvert=\sqrt{3^2+(-4)^2}=5$ The magnitude (length) of a 3D vector $AB=\begin{pmatrix}x\\y\\z\end{pmatrix}$ can be written as: $\lvert AB\rvert=\sqrt{x^2+y^2+z^2}$ e.g. Find $\lvert b\rvert$, the magnitude of $b=\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $\lvert b\rvert=\sqrt{2^2+(-3)^2+1^2}=\sqrt{14}=3.74$			

Chapter	Topic	Skills	🟩	🟨	🟥
31 Working with percentages					
31A/B	Calculating percentage increases and decreases over periods of time	• Find multiplier by adding to 100 or subtracting from 100, then dividing by 100 to give decimal fraction • New value = multiplier × original value			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
31C	Calculating compound interest	• Find multiplier in same way as Exercise 31A • New value = (multiplier)^n × original value where n is the number of years (or other period of time) e.g. David invested £6000 for four years at a rate of 2.4% per annum. How much money did he have after four years? $(1.024)^4 \times 6000 = £6597.07$ Note: a "year-by-year" approach can be calculated as an alternative strategy			
31D/E	Finding original values	• Add to 100 or subtract from 100 dependent on increase or decrease • Find 1% (or equivalent) • Find 100% e.g. A painting was bought a year ago. It has increased in value by 6% since bought and is now worth £2544. How much was it bought for a year ago? 106% = 2544 1% = $\frac{2544}{106}$ =24 100% = 24 × 100 = £2400			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
32 Working with fractions					
32A	Dividing by a fraction	Division by a fraction is the same as multiplication by its reciprocal e.g. $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{2}{1} \times \frac{2}{5} = \frac{4}{5}$			
32B	The four operations with mixed numbers	Adding and subtracting Method 1: Split into two calculations (add/subtract the whole numbers then add the fractions) e.g. $2\frac{2}{3} + 3\frac{4}{5}$ $2 + 3 = 5$, $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$ $5 + 1\frac{7}{15} = 6\frac{7}{15}$ Method 2: Convert to improper fractions $2\frac{2}{3} + 3\frac{4}{5} = \frac{8}{3} + \frac{19}{5} = \frac{40}{15} + \frac{57}{15} = \frac{97}{15} = 6\frac{7}{15}$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
		Multiplying To multiply, convert to improper fractions. Simplify, then multiply numerators and multiply denominators e.g. $3\frac{1}{5}\times2\frac{1}{4}=\frac{16}{5}\times\frac{9}{4}=\frac{4}{5}\times\frac{9}{1}=\frac{36}{5}=7\frac{1}{5}$ Dividing To divide, convert to improper fractions and multiply by the reciprocal of the 2nd fraction (use skills from Exercise 32A) e.g. $4\frac{1}{3}\div2\frac{3}{4}=\frac{13}{3}\div\frac{11}{4}=\frac{13}{3}\times\frac{4}{11}=\frac{52}{33}=1\frac{19}{33}$			
32C	Calculations involving combinations of the four operations with fractions and mixed numbers	Operations are applied in this order: • Calculations within brackets • Powers and roots • Multiplication and division • Addition and subtraction			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
33 Compare data using statistics					
33A	Standard deviation	There are two versions of the standard deviation formula given in the exam: • $s=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n-1}}$ • $s=\sqrt{\frac{\sum x^2-\frac{\left(\sum x\right)^2}{n}}{n-1}}$ Where $\sum$ means add together, x represents each value, $\bar{x}$ represents the mean and n represents the number of values of data. e.g. find the standard deviation of 7, 9, 12, 10, 14, 12, 6			

Chapter	Topic	Skills	☐	☐	☐

Method 1: $\bar{x} = \frac{70}{7} = 10$

x	$x - \bar{x}$	$(x - \bar{x})^2$
52	–23	529
60	–15	225
77	2	4
82	7	49
88	13	169
91	16	256
Total	**0**	$\sum(x - \bar{x})^2 = 1232$

$s = \sqrt{\frac{50}{6}} = 2.89$

Method 2

x	x^2
7	49
9	81
12	144
10	100
14	196
12	144
6	36
$\sum x = 70$	$\sum x^2 = 750$

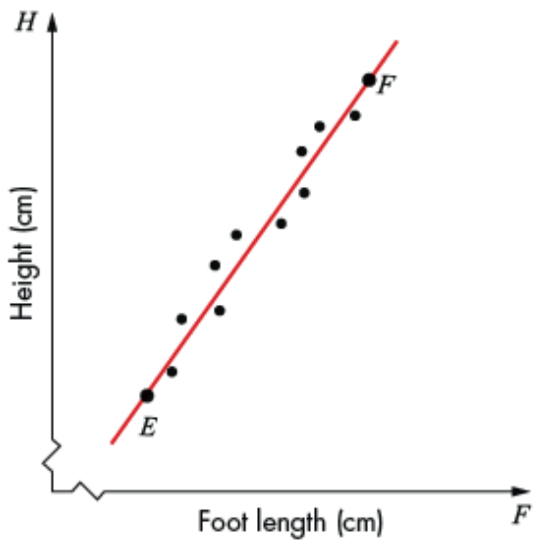
S= $\sqrt{\frac{750 - \frac{70^2}{7}}{6}} = \sqrt{\frac{50}{6}} = 2.89$

Comparing data: use the word "average" to compare means, and to compare the standard deviation, use spread of data, consistency of data or variation of data.

Chapter	Topic	Skills	☐	☐	☐
33B	Interquartile range	• Calculate the median (Q_2): the middle value • Calculate the lower quartile (Q_1): the median of the lower half of the numerical data • Calculate the lower quartile (Q_3): the median of the lower half of the numerical data Interquartile range = Q_3 — Q_1			

Chapter	Topic	Skills			
		Note: some questions ask you to calculate the semi-interquartile range, but this is no longer assessed at National 5. e.g. find the interquartile range of: 5 6 9 10 12 13 13 15 15 21 5 6 9 10 (12 13) 13 15 15 21 $Q_2 = 12{\cdot}5$ 5 6 (9) 10 12 $Q_1 = 9$ 13 13 (15) 15 21 $Q_3 = 15$ Interquartile range = 15 – 9 = 6 Comparing data: similar to Exercise 33A, use the word “average” to compare medians, and to compare the interquartile range, use spread of data, consistency of data or variation of data.			

Chapter	Topic	Skills			
34 Forming a linear model from a given set of data					
34A	Drawing a scattergraph and line of best fit	• Choose scale carefully to show all data • Plot points (first set of data on horizontal axis, second set on vertical axis) • Draw a straight line that best fits the data • Calculate the equation of the line of best fit (select two points and use skills from Exercise 12B)			
34B	Use the equation of a best-fitting straight line to estimate solutions	The equation found can be used to analyse and interpret data, and forecast outcomes e.g. Pupils in a class measured their height (H cm) and the length of their feet (F cm). Their results are recorded in the scatter graphs below.			

Chapter	Topic	Skills			
		 Point E represents a pupil with a foot length of 25cm and a height of 146cm Point F represents a pupil with a foot length of 33cm and a height of 170cm a. Find the equation of the best-fitting line $m = \frac{170-146}{33-25} = \frac{24}{8} = 3$ Using point E, $y - 146 = 3(x - 25)$ $y - 146 = 3x - 75$ $y = 3x + 71$ $H = 3F + 71$ b. Estimate the height of a pupil with a foot of length 28cm. $H = 3 \times 28 + 71 = 155\,\text{cm}$			