

# SINE RULE, COSINE RULE & AREA OF A TRIANGLE PAST PAPER QUESTIONS

## ANSWERS

### Calculator Questions

1.  $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$   
 $\cos P = \frac{14 \cdot 2^2 + 9 \cdot 3^2 - 7 \cdot 8^2}{2 \times 14 \cdot 2 \times 9 \cdot 3}$   
 $\cos P = \frac{227 \cdot 29}{264 \cdot 12}$   
 $\cos P = 0 \cdot 860555808$   
 $P = 30 \cdot 6^\circ$
2.  $\cos E = \frac{d^2 + f^2 - e^2}{2df}$   
 $\cos E = \frac{19 \cdot 6^2 + 10 \cdot 4^2 - 13 \cdot 2^2}{2 \times 19 \cdot 6 \times 10 \cdot 4}$   
 $\cos E = \frac{318 \cdot 08}{407 \cdot 68}$   
 $\cos E = 0 \cdot 7802197802$   
 $E = 38 \cdot 7^\circ$
3.  $\cos P = \frac{b^2 + m^2 - p^2}{2bm}$   
 $\cos P = \frac{950^2 + 1000^2 - 1100^2}{2 \times 950 \times 1000}$   
 $\cos P = \frac{692500}{1900000}$   
 $\cos P = 0 \cdot 3644736842$   
 $P = 68 \cdot 6^\circ$
4.  $A = \frac{1}{2}ab \sin C$   
 $A = \frac{1}{2} \times 120 \times 100 \times \sin 65^\circ$   
 $A = 5437 \cdot 8 \text{ m}^2$
5.  $d^2 = a^2 + c^2 - 2ac \cos D$   
 $d^2 = 9^2 + 14^2 - 2 \times 9 \times 14 \times \cos 131^\circ$   
 $d^2 = 81 + 196 - (-165 \cdot 327)$   
 $d^2 = 442 \cdot 327$   
 $d = 21 \cdot 03 \text{ cm}$
6.  $A = \frac{1}{2}ab \sin C$   
 $A = \frac{1}{2} \times 21 \times 19 \times \sin 110^\circ$   
 $A = 187 \cdot 5 \text{ cm}^2$
7. a)  $a^2 = b^2 + d^2 - 2bd \cos A$   
 $a^2 = 7 \cdot 8^2 + 11 \cdot 1^2 - 2 \times 7 \cdot 8 \times 11 \cdot 1 \times \cos 110^\circ$   
 $a^2 = 60 \cdot 84 + 123 \cdot 21 - (-59 \cdot 2242)$   
 $a^2 = 243 \cdot 2742$   
 $a = 15 \cdot 6 \text{ m}$

$$\begin{aligned} \text{b)} \quad A &= \frac{1}{2}ab \sin C \\ A &= \frac{1}{2} \times 11 \cdot 1 \times 7 \cdot 8 \times \sin 110^\circ \\ A &= 40 \cdot 7 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}ab \sin C \\ A &= \frac{1}{2} \times 15 \cdot 6 \times 9 \cdot 3 \times \sin 78^\circ \\ A &= 71 \cdot 0 \text{ m}^2 \end{aligned}$$

$$\text{Total Area} = 40 \cdot 7 + 71 \cdot 0 = 111 \cdot 7 \text{ m}^2$$

$$\begin{aligned} 8. \quad \text{a)} \quad \cos Q &= \frac{p^2 + r^2 - q^2}{2pr} \\ \cos Q &= \frac{11 \cdot 2^2 + 8 \cdot 4^2 - 12 \cdot 6^2}{2 \times 11 \cdot 2 \times 8 \cdot 4} \\ \cos Q &= \frac{37 \cdot 24}{188 \cdot 16} \\ \cos Q &= 0 \cdot 1979166667 \\ Q &= 78 \cdot 6^\circ \end{aligned}$$

$$\begin{aligned} \text{b)} \quad A &= \frac{1}{2}ab \sin C \\ A &= \frac{1}{2} \times 11 \cdot 2 \times 8 \cdot 4 \times \sin 78 \cdot 6^\circ \\ A &= 46 \cdot 1 \text{ cm}^2 \\ \text{Area of the parallelogram} &= 2 \times 46 \cdot 1 = 92 \cdot 2 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 9. \quad \text{a)} \quad b^2 &= a^2 + c^2 \\ b^2 &= 6 \cdot 2^2 + 4^2 \\ b^2 &= 54 \cdot 44 \\ b &= 7 \cdot 4 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \cos D &= \frac{a^2 + c^2 - d^2}{2ac} \\ \cos D &= \frac{7^2 + 5^2 - 7 \cdot 4^2}{2 \times 7 \times 5} \\ \cos D &= \frac{19 \cdot 24}{70} \\ \cos D &= 0 \cdot 2748571429 \\ D &= 74 \cdot 0^\circ \end{aligned}$$

$$\begin{aligned} 10. \quad A &= \frac{1}{2}ab \sin C \\ A &= \frac{1}{2} \times 40 \times 40 \times \sin 110^\circ \\ A &= 751 \cdot 75 \text{ cm}^2 \\ \text{Area of the paving stone} &= 2 \times 751 \cdot 75 = 1503 \cdot 5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 11. \quad A &= \frac{1}{2}ab \sin C \\ 9 &= \frac{1}{2} \times x \times x \times \sin 30^\circ \\ 18 &= x \times x \times \sin 30^\circ \\ 18 &= x \times x \times 0 \cdot 5 \\ 36 &= x^2 \\ x &= 6 \text{ cm} \end{aligned}$$

12. a)  $108^\circ$   
 b)  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $b^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 108^\circ$   
 $b^2 = 1 + 1 - (-0.618)$   
 $b^2 = 2.618$   
 $b = 1.6 \text{ cm}$
13.  $A = \frac{1}{2}ab \sin C$   
 $15 = \frac{1}{2} \times PQ \times 6 \times \sin 30^\circ$   
 $30 = PQ \times 6 \times \sin 30^\circ$   
 $30 = PQ \times 3$   
 $PQ = 10 \text{ cm}$
14.  $e^2 = d^2 + f^2 - 2df \cos E$   
 $e^2 = 26 \cdot 2^2 + 46 \cdot 4^2 - 2 \times 26 \cdot 2 \times 46 \cdot 4 \times \cos 64^\circ$   
 $e^2 = 686 \cdot 44 + 2152 \cdot 96 - 1065 \cdot 84$   
 $e^2 = 1773 \cdot 56$   
 $e = 42 \cdot 1 \text{ m}$   
 $1 \text{ lap} = 42 \cdot 1 + 46 \cdot 4 + 26 \cdot 2 = 114 \cdot 7 \text{ m}$   
 $1000 \div 114 \cdot 7 = 8 \cdot 718$   
 9 laps must be complete to cover at least 1000m
15.  $A = \frac{1}{2}ab \sin C$   
 $12 = \frac{1}{2} \times 5 \times 6 \times \sin P^\circ$   
 $24 = 5 \times 6 \times \sin P^\circ$   
 $24 = 30 \times \sin P^\circ$   
 $0 \cdot 8 = \sin P^\circ$   
 $P = 53 \cdot 1 \text{ or } 180 - 53 \cdot 1$   
 $P = 126 \cdot 9^\circ$
16.  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{h}{\sin H}$   
 $\frac{a}{\sin 50} = \frac{1500}{\sin 75}$   
 $a = \frac{1500}{\sin 75} \times \sin 50$   
 $a = 1189 \cdot 6 \text{ m}$
17.  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $b^2 = 2 \cdot 9^2 + 3 \cdot 3^2 - 2 \times 2 \cdot 9 \times 3 \cdot 3 \times \cos 130^\circ$   
 $b^2 = 8 \cdot 41 + 10 \cdot 89 - (-12 \cdot 30)$   
 $b^2 = 31 \cdot 6$   
 $b = 5 \cdot 6 \text{ m}$

$$18. \quad A = \frac{1}{2}ab \sin C$$

$$38 = \frac{1}{2} \times 9 \times 14 \times \sin B^\circ$$

$$76 = 9 \times 14 \times \sin B^\circ$$

$$76 = 126 \times \sin B^\circ$$

$$0.6031746 = \sin B^\circ$$

$$B = 37.1^\circ$$

$$19. \quad c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 1 \cdot 35^2 + 1 \cdot 2^2 - 2 \times 1 \cdot 35 \times 1 \cdot 2 \times \cos 35^\circ$$

$$c^2 = 1 \cdot 8225 + 1 \cdot 44 - 2 \cdot 654$$

$$c^2 = 0.6085$$

$$c = 0.78 \text{ km}$$

$$20. \quad A = \frac{1}{2}ab \sin C$$

$$A = \frac{1}{2} \times 20 \times 20 \times \sin 60^\circ$$

$$A = 173.2 \text{ cm}^2$$

$$\text{Area of the table top} = 6 \times 173.2 = 1039.2 \text{ cm}^2$$

$$21. \quad \text{a) } A = \frac{1}{2}ab \sin C$$

$$A = \frac{1}{2} \times 15 \times 18 \times \sin 70^\circ$$

$$A = 126.9 \text{ m}^2$$

$$\text{b) } 90^\circ$$

$$22. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{b}{\sin 46} = \frac{30}{\sin 96}$$

$$\cos 52 = \frac{h}{21.7}$$

$$b = \frac{30}{\sin 96} \times \sin 46$$

$$h = 13.4 \text{ m}$$

$$b = 21.7 \text{ m}$$

$$23. \quad \frac{g}{\sin G} = \frac{h}{\sin H} = \frac{t}{\sin T}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{h}{\sin 32} = \frac{46}{\sin 25}$$

$$\sin 57 = \frac{TB}{57.7}$$

$$h = \frac{46}{\sin 25} \times \sin 32$$

$$TB = 48.4 \text{ m}$$

$$h = 57.7 \text{ m}$$

$$24. \quad \tan x = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{20}{10}$$

$$x = 63.4^\circ$$

Spotlight must swing another  $63.4 - 45 = 18.4^\circ$

$$25. \quad \frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 27} = \frac{350}{\sin 111}$$

$$p = \frac{350}{\sin 111} \times \sin 27$$

$$p = 170 \cdot 2 \text{ m}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 42 = \frac{QS}{170 \cdot 2}$$

$$QS = 126 \cdot 5 \text{ m}$$

$$26. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{t}{\sin T}$$

$$\frac{a}{\sin 33} = \frac{80}{\sin 122}$$

$$a = \frac{80}{\sin 122} \times \sin 33$$

$$a = 51 \cdot 4 \text{ m}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 25 = \frac{h}{51 \cdot 4}$$

$$h = 21 \cdot 7 \text{ m}$$

Flagpole is  $21 \cdot 7 + 1 \cdot 6 = 23 \cdot 3$  m high.

$$27. \quad \frac{s}{\sin S} = \frac{t}{\sin T} = \frac{v}{\sin V}$$

$$\frac{500}{\sin 105} = \frac{t}{\sin 35}$$

$$t = \frac{500}{\sin 105} \times \sin 35$$

$$t = 296 \cdot 9 \text{ m}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 40 = \frac{h}{296 \cdot 9}$$

$$h = 190 \cdot 8 \text{ m}$$

$$28. \quad \frac{c}{\sin C} = \frac{s}{\sin S} = \frac{t}{\sin T}$$

$$\frac{4}{\sin 12} = \frac{t}{\sin 40}$$

$$t = \frac{4}{\sin 12} \times \sin 40$$

$$t = 12 \cdot 4 \text{ m}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 38 = \frac{h}{12 \cdot 4}$$

$$h = 7 \cdot 6 \text{ m}$$

$$29. \quad \frac{c}{\sin C} = \frac{p}{\sin P} = \frac{y}{\sin Y}$$

$$\frac{89}{\sin 25} = \frac{y}{\sin 27}$$

$$y = \frac{89}{\sin 25} \times \sin 27$$

$$y = 95 \cdot 6 \text{ m}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 52 = \frac{h}{95 \cdot 6}$$

$$h = 75 \cdot 3 \text{ m}$$

$$30. \quad \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{d}{\sin D}$$

$$\frac{c}{\sin 38} = \frac{5}{\sin 17}$$

$$c = \frac{5}{\sin 17} \times \sin 38$$

$$c = 10 \cdot 5 \text{ m}$$

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 55 = \frac{h}{10 \cdot 5}$$

$$h = 8 \cdot 6 \text{ m}$$

## Bearings Questions (with a calculator)

$$1. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 80} = \frac{16.8}{\sin 70}$$

$$a = \frac{16.8}{\sin 70} \times \sin 80$$

$$a = 17.6 \text{ km}$$

Bob's average speed was  $17.6 \div 3 = 5.9$  kmhr. Bob was faster by 0.3 kmhr.

$$2. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{v}{\sin V}$$

$$\frac{5}{\sin 24} = \frac{v}{\sin 116}$$

$$v = \frac{5}{\sin 24} \times \sin 116$$

$$v = 11.0 \text{ km}$$

$$3. \quad \begin{aligned} \text{a)} \quad \cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ \cos B &= \frac{11^2 + 8^2 - 13^2}{2 \times 11 \times 8} \\ \cos B &= \frac{16}{176} \\ \cos B &= 0.09090909091 \\ B &= 84.8^\circ \end{aligned}$$

$$\begin{aligned} \text{b)} \quad 84.8 - 60 &= 24.8^\circ \\ 180 - 24.8 &= 155.2^\circ \end{aligned}$$

$$4. \quad \frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{q}{\sin 52} = \frac{25}{\sin 56}$$

$$q = \frac{25}{\sin 56} \times \sin 52$$

$$q = 23.8 \text{ km}$$

$$\begin{aligned} 5. \quad h^2 &= a^2 + b^2 - 2ab \cos H \\ h^2 &= 30^2 + 50^2 - 2 \times 30 \times 50 \times \cos 68^\circ \\ h^2 &= 900 + 2500 - 1123.8 \\ h^2 &= 2276.2 \\ h &= 47.7 \text{ km} \end{aligned}$$

$$6. \quad \frac{k}{\sin K} = \frac{p}{\sin P} = \frac{w}{\sin W}$$

$$\frac{410}{\sin 130} = \frac{250}{\sin P}$$

$$\sin P = \frac{250}{410} \times \sin 130$$

$$\sin P = 0.4671002702$$

$$P = 27.8^\circ$$

$$\text{KWP} = 180 - 130 - 27.8 = 22.2^\circ, \text{ Bearing} = 180 - 22.2 = 157.8^\circ$$

$$7. \quad \text{a)} \quad \text{BAC} = 52 - 32 = 20^\circ$$

$$\begin{aligned} \text{b)} \quad a^2 &= b^2 + c^2 - 2bc \cos A \\ a^2 &= 160^2 + 160^2 - 2 \times 160 \times 160 \times \cos 20^\circ \\ a^2 &= 25600 + 25600 - 48112 \cdot 3 \\ a^2 &= 3087.7 \end{aligned}$$

$$\begin{aligned} a &= 55.6 \text{ m} \\ \text{c)} \quad 180 + 52 + 80 &= 312^\circ \end{aligned}$$

$$8. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 65} = \frac{30}{\sin 88}$$

$$a = \frac{30}{\sin 88} \times \sin 65$$

$$a = 27.2 \text{ km}$$

$$9. \quad \text{a)i)} \quad \text{HKM} = 77 + 70 = 147^\circ$$

$$\begin{aligned} \text{a)ii)} \quad k^2 &= h^2 + m^2 - 2hm \cos K \\ k^2 &= 22^2 + 30^2 - 2 \times 22 \times 30 \times \cos 147^\circ \\ k^2 &= 484 + 900 - (-1107) \\ k^2 &= 2491 \\ k &= 49.9 \text{ km} \end{aligned}$$

$$\text{Total distance} = 22 + 30 + 49.9 = 101.9 \text{ km}$$

$$\begin{aligned} \text{b)} \quad A &= \frac{1}{2} ab \sin C \\ A &= \frac{1}{2} \times 22 \times 30 \times \sin 147^\circ \\ A &= 179.7 \text{ km}^2 \end{aligned}$$

$$10. \quad \text{a)} \quad 74 + 50 = 124^\circ$$

$$\begin{aligned} \text{b)} \quad b^2 &= a^2 + c^2 - 2ac \cos B \\ b^2 &= 230^2 + 110^2 - 2 \times 230 \times 110 \times \cos 124^\circ \\ b^2 &= 52900 + 12100 - (-28295 \cdot 2) \\ b^2 &= 93295 \cdot 2 \\ b &= 305.4426296 \text{ m} \\ b &= 305 \text{ m} \end{aligned}$$

## Non-Calculator Questions

1.  $A = \frac{1}{2}ab \sin C$   
 $A = \frac{1}{2} \times 16 \times 20 \times \sin P^\circ$   
 $A = \frac{1}{2} \times 16 \times 20 \times \frac{1}{4}$   
 $A = 8 \times 20 \times \frac{1}{4}$   
 $A = 160 \times \frac{1}{4}$   
 $A = 40 \text{ cm}^2$

2.  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$   
 $\cos B = \frac{6^2 + 3^2 - 5^2}{2 \times 6 \times 3}$   
 $\cos B = \frac{20}{36}$   
 $\cos B = \frac{5}{9}$

3.  $\frac{k}{\sin K} = \frac{l}{\sin L} = \frac{m}{\sin M}$

$$\frac{k}{0.4} = \frac{18}{0.9}$$

$$k = \frac{18}{0.9} \times 0.4$$

$$k = 20 \times 0.4$$

$$k = 8 \text{ cm}$$

4.  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{10}{\sin 150} = \frac{4}{\sin B}$$

$$10 \times \sin B = 4 \times \sin 150$$

$$10 \times \sin B = 4 \times \frac{1}{2}$$

$$10 \times \sin B = 2$$

$$\sin B = \frac{2}{10}$$

$$\sin B = \frac{1}{5}$$

$$5. \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$$

$$\cos A = \frac{5}{40}$$

$$\cos A = \frac{1}{8}$$

$$6. \quad A = \frac{1}{2} ab \sin C$$

$$A = \frac{1}{2} \times 12 \times 10 \times \sin B^\circ$$

$$A = \frac{1}{2} \times 12 \times 10 \times \frac{2}{3}$$

$$A = 6 \times 10 \times \frac{2}{3}$$

$$A = 60 \times \frac{2}{3}$$

$$A = 40 \text{ cm}^2$$

$$7. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{1/3} = \frac{12}{1/2}$$

$$b = \frac{12}{1/2} \times 1/3$$

$$b = 24 \times 1/3$$

$$b = 8 \text{ cm}$$

$$8. \quad A = \frac{1}{2} ab \sin C$$

$$20 = \frac{1}{2} \times 16 \times BC \times \frac{1}{4}$$

$$20 = 8 \times BC \times \frac{1}{4}$$

$$20 = 2 \times BC$$

$$BC = 10 \text{ cm}$$

$$9. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin A} = \frac{10}{\sin 30}$$

$$6 \times \sin 30 = 10 \times \sin A$$

$$6 \times 0.5 = 10 \times \sin A$$

$$3 = 10 \times \sin A$$

$$\sin A = \frac{3}{10}$$

$$\sin A = 0.3$$