

Quadratics - Past Paper Questions - ANSWERS

1)
$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9+8}}{4}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

$$x = \frac{-3 \pm 4.123}{4}$$

$$x = \frac{-3+4.123}{4} \text{ or } x = \frac{-3-4.123}{4}$$

$$x = \frac{1.123}{4} \text{ or } x = \frac{-7.123}{4}$$

$$x = 0.28075 \text{ or } x = -1.78075$$

$$x = 0.3 \text{ or } x = -1.8$$

2)
$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 4 \times 1}}{2 \times 4}$$

$$x = \frac{7 \pm \sqrt{49-16}}{8}$$

$$x = \frac{7 \pm \sqrt{33}}{8}$$

$$x = \frac{7 \pm 5.74456}{8}$$

$$x = \frac{7+5.74456}{8} \text{ or } x = \frac{7-5.74456}{8}$$

$$x = \frac{12.74456}{8} \text{ or } x = \frac{1.25544}{8}$$

$$x = 1.59307 \text{ or } x = 0.15693$$

$$x = 1.6 \text{ or } x = 0.2$$

3)
$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times (-9)}}{2 \times 2}$$

$$x = \frac{-4 \pm \sqrt{16+72}}{4}$$

$$x = \frac{-4 \pm \sqrt{88}}{4}$$

$$x = \frac{-4 \pm 9.3808}{4}$$

$$x = \frac{-4+9.3808}{4} \text{ or } x = \frac{-4-9.3808}{4}$$

$$x = \frac{5.3808}{4} \text{ or } x = \frac{-13.3808}{4}$$

$$x = 1.3452 \text{ or } x = -3.3452$$

$$x = 1.3 \text{ or } x = -3.3$$

4)
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$x = \frac{6 \pm \sqrt{36+40}}{4}$$

$$x = \frac{6 \pm \sqrt{76}}{4}$$

$$x = \frac{6 \pm 8.717798}{4}$$

$$x = \frac{6+8.717798}{4} \text{ or } x = \frac{6-8.717798}{4}$$

$$x = \frac{14.717798}{4} \text{ or } x = \frac{-2.717798}{4}$$

$$x = 3.67945 \text{ or } x = -0.67945$$

$$x = 3.7 \text{ or } x = -0.7$$

5)
$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-7)}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9+56}}{4}$$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$

$$x = \frac{-3 \pm 8.0623}{4}$$

$$x = \frac{-3+8.0623}{4} \text{ or } x = \frac{-3-8.0623}{4}$$

$$x = \frac{5.0623}{4} \text{ or } x = \frac{-11.0623}{4}$$

$$x = 1.2656 \text{ or } x = -2.7656$$

$$x = 1.3 \text{ or } x = -2.8$$

6)
$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$x = \frac{-5 \pm \sqrt{25+84}}{6}$$

$$x = \frac{-5 \pm \sqrt{109}}{6}$$

$$x = \frac{-5 \pm 10.4403}{6}$$

$$x = \frac{-5+10.4403}{6} \text{ or } x = \frac{-5-10.4403}{6}$$

$$x = \frac{5.4403}{6} \text{ or } x = \frac{-15.4403}{6}$$

$$x = 0.9067 \text{ or } x = -2.5734$$

$$x = 0.9 \text{ or } x = -2.6$$

7)
$$x^2 + 2x - 9 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

$$x = \frac{-2 \pm \sqrt{4+36}}{2}$$

$$x = \frac{-2 \pm \sqrt{40}}{2}$$

$$x = \frac{-2 \pm 6.3246}{2}$$

$$x = \frac{-2+6.3246}{2} \text{ or } x = \frac{-2-6.3246}{2}$$

$$x = \frac{4.3246}{2} \text{ or } x = \frac{-8.3246}{2}$$

$$x = 2.1623 \text{ or } x = -4.1623$$

$$x = 2.2 \text{ or } x = -4.2$$

8)
$$(2x - 1)(x + 5) = 0$$

$$2x - 1 = 0 \text{ or } x + 5 = 0$$

$$2x = 1 \text{ or } x = -5$$

$$x = \frac{1}{2}$$

$$9) \quad x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$x = \frac{-7 \pm \sqrt{49 + 24}}{4}$$

$$x = \frac{-7 \pm \sqrt{73}}{4}$$

$$x = \frac{-7 \pm 8.544}{4}$$

$$x = \frac{-7 + 8.544}{4} \text{ or } x = \frac{-7 - 8.544}{4}$$

$$x = \frac{1.544}{4} \text{ or } x = \frac{-15.544}{4}$$

$$x = 0.386 \text{ or } x = -3.886$$

$$x = 0.4 \text{ or } x = -3.9$$

$$10) \quad x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{2}$$

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

$$x = \frac{-5 \pm 3.60555}{2}$$

$$x = \frac{-5 + 3.60555}{2} \text{ or } x = \frac{-5 - 3.60555}{2}$$

$$x = \frac{-1.39445}{2} \text{ or } x = \frac{-8.60555}{2}$$

$$x = -0.697225 \text{ or } x = -4.302775$$

$$x = -0.7 \text{ or } x = -4.3$$

$$11) \quad x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$x = \frac{-7 \pm \sqrt{49 + 60}}{6}$$

$$x = \frac{-7 \pm \sqrt{109}}{6}$$

$$x = \frac{-7 \pm 10.4403}{6}$$

$$x = \frac{-7 + 10.4403}{6} \text{ or } x = \frac{-7 - 10.4403}{6}$$

$$x = \frac{3.4403}{6} \text{ or } x = \frac{-17.4403}{6}$$

$$x = 0.5733 \text{ or } x = -2.9067$$

$$x = 0.6 \text{ or } x = -2.9$$

$$12) \quad x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{2}$$

$$x = \frac{4 \pm \sqrt{40}}{2}$$

$$x = \frac{4 \pm 6.3246}{2}$$

$$x = \frac{4 + 6.3246}{2} \text{ or } x = \frac{4 - 6.3246}{2}$$

$$x = \frac{10.3246}{2} \text{ or } x = \frac{-2.3246}{2}$$

$$x = 5.1623 \text{ or } x = -1.1623$$

$$x = 5.2 \text{ or } x = -1.2$$

$$13) \quad x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times (-2)}}{2 \times 1}$$

$$x = \frac{5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{5 \pm \sqrt{33}}{2}$$

$$x = \frac{5 \pm 5.74456}{2}$$

$$x = \frac{5 + 5.74456}{2} \text{ or } x = \frac{5 - 5.74456}{2}$$

$$x = \frac{10.74456}{2} \text{ or } x = \frac{-0.74456}{2}$$

$$x = 5.37228 \text{ or } x = -0.37228$$

$$x = 5.4 \text{ or } x = -0.4$$

$$14) \quad x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 4 \times 1}}{2 \times 4}$$

$$x = \frac{7 \pm \sqrt{49 - 16}}{8}$$

$$x = \frac{7 \pm \sqrt{33}}{8}$$

$$x = \frac{7 \pm 5.74456}{8}$$

$$x = \frac{7 + 5.74456}{8} \text{ or } x = \frac{7 - 5.74456}{8}$$

$$x = \frac{12.74456}{8} \text{ or } x = \frac{1.25544}{8}$$

$$x = 1.59307 \text{ or } x = 0.15693$$

$$x = 1.2 \text{ or } x = 0.2$$

$$15) \quad x = \frac{-4 \pm \sqrt{4^2 - 4 \times 5 \times (-2)}}{2 \times 5}$$

$$x = \frac{-4 \pm \sqrt{16 + 40}}{10}$$

$$x = \frac{-4 \pm \sqrt{56}}{10}$$

$$x = \frac{-4 \pm 7.4833}{10}$$

$$x = \frac{-4 + 7.4833}{10} \text{ or } x = \frac{-4 - 7.4833}{10}$$

$$x = \frac{3.4833}{10} \text{ or } x = \frac{-11.4833}{10}$$

$$x = 0.34833 \text{ or } x = -1.14833$$

$$x = 0.35 \text{ or } x = -1.15$$

$$16) \quad x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$$x = \frac{7 \pm \sqrt{49 - 8}}{4}$$

$$x = \frac{7 \pm \sqrt{41}}{4}$$

$$x = \frac{7 \pm 6.4031}{4}$$

$$x = \frac{7 + 6.4031}{4} \text{ or } x = \frac{7 - 6.4031}{4}$$

$$x = \frac{13.4031}{4} \text{ or } x = \frac{0.5969}{4}$$

$$x = 3.350775 \text{ or } x = 0.149225$$

$$x = 3.35 \text{ or } x = 0.15$$

$$17) \quad x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times (-10)}}{2 \times 3}$$

$$x = \frac{2 \pm \sqrt{4 + 120}}{6}$$

$$x = \frac{2 \pm \sqrt{124}}{6}$$

$$x = \frac{2 \pm 11.13553}{6}$$

$$x = \frac{2 + 11.13553}{6} \text{ or } x = \frac{2 - 11.13553}{6}$$

$$x = \frac{13.13553}{6} \text{ or } x = \frac{-9.13553}{6}$$

$$x = 2.189255 \text{ or } x = -1.522588$$

$$x = 2.2 \text{ or } x = -1.5$$

$$18) \quad x = \frac{(-3) \pm \sqrt{3^2 - 4 \times 2 \times (-7)}}{2 \times 2}$$

$$x = \frac{(-3) \pm \sqrt{9 + 56}}{4}$$

$$x = \frac{(-3) \pm \sqrt{41}}{4}$$

$$x = \frac{(-3) \pm 8.06226}{4}$$

$$x = \frac{(-3) + 8.06226}{4} \text{ or } x = \frac{(-3) - 8.06226}{4}$$

$$x = \frac{5.06226}{4} \text{ or } x = \frac{-11.06226}{4}$$

$$x = 1.265565 \text{ or } x = -2.765565$$

$$x = 1.3 \text{ or } x = -2.8$$

$$19) \quad a) \quad \begin{aligned} b^2 - 4ac \\ 3^2 - 4 \times 1 \times 5 \\ 9 - 20 \\ -11 \end{aligned}$$

$$b) \quad \sqrt{-11} \text{ has no real values}$$

$$20) \quad x^2 - 14x + 44 = (x - 7)^2 - 5$$

$$21) \quad a = 5 \text{ and } b = -7$$

$$22) \quad \begin{aligned} y &= kx^2 \\ 48 &= k \times 4^2 \\ 48 &= k \times 16 \\ k &= 3 \end{aligned}$$

$$23) \quad \begin{aligned} y &= ax^2 \\ 45 &= a \times (-3)^2 \\ 45 &= a \times 9 \\ a &= 5 \end{aligned}$$

$$24) \quad \begin{aligned} y &= -x^2 \\ k &= -(-3)^2 \\ k &= -9 \end{aligned}$$

$$25) \quad \begin{aligned} a) & \quad (3, -4) \\ b) & \quad x = 3 \\ c) & \quad (5, 0) \end{aligned}$$

$$26) \quad \begin{aligned} a) & \quad (1, -16) \\ b) & \quad x = 1 \\ c) & \quad \begin{aligned} y &= (x - 1)^2 - 16 \\ y &= (x - 1)(x - 1) - 16 \\ y &= x^2 - 2x + 1 - 16 \\ y &= x^2 - 2x - 15 \\ y &= (x - 5)(x + 3) \\ (x - 5)(x + 3) &= 0 \\ (x - 5) = 0 \text{ or } (x + 3) &= 0 \\ x &= 5 \text{ or } x = -3 \\ AB &= 5 + 3 = 8 \end{aligned} \end{aligned}$$

$$27) \quad \begin{aligned} a) & \quad (4, 20) \\ b) & \quad x = 4 \end{aligned}$$

$$28) \quad \begin{aligned} a) & \quad \begin{aligned} 8x - x^2 &= 0 \\ x(8 - x) &= 0 \\ x = 0 \text{ or } (8 - x) &= 0 \\ x = 0 \text{ or } x &= 8 \end{aligned} \\ b) & \quad x = 4 \\ c) & \quad \begin{aligned} y &= 8x - x^2 \\ y &= 8 \times 4 - 4^2 \\ y &= 32 - 16 \\ y &= 16 \\ (4, 16) \end{aligned} \end{aligned}$$

$$29) \quad \begin{aligned} a) & \quad \begin{aligned} (7 - x)(1 + x) \\ 7 + 6x - x^2 &= 0 \\ (7 - x)(1 + x) &= 0 \\ (7 - x) = 0 \text{ or } (1 + x) &= 0 \\ x = 7 \text{ or } x &= -1 \end{aligned} \\ c) & \quad \begin{aligned} x &= 3 \\ y &= 7 + 6x - x^2 \\ y &= 7 + 6 \times 3 - 3^2 \\ y &= 16 \\ (3, 16) \end{aligned} \end{aligned}$$

$$30) \quad \begin{aligned} a) & \quad (2, 36) \\ b) & \quad x = 2 \\ c) & \quad R(-2, 20) \end{aligned}$$

$$31) \quad \begin{aligned} a) i) & \quad a = -2 \\ a) ii) & \quad b = -4 \\ b) & \quad x = 2 \end{aligned}$$

32) a) $a = -5$
 b) $Q(8,0)$
 c) $y = (x - 2)(x - 8)$
 $y = (5 - 2)(5 - 8)$
 $y = (3) \times (-3)$
 $y = -9$
 $b = -9$

33) a) $x = -3$
 b) $y = (x + 3)^2 - 4$
 c) $y = (0 + 3)^2 - 4$
 $y = 9 - 4$
 $y = 5$
 $C(0,5)$

34) a) $a = -5, b = 1$
 b) $x = 5$
 c) $y = (x - 5)^2 + 1$
 $y = (0 - 5)^2 + 1$
 $y = (5)^2 + 1$
 $y = 26$
 $P(0,26)$
 $Q(10,26)$

35) a) $(2,-5)$
 b) Minimum turning point

36) a) $x = 2, x = 4$
 b) $A(0,8), B(2,0), C(4,0)$
 c) $x = 3$

37) a) $Q(2,0), R(8,0)$
 b) $y = (x - 8)(2 - x)$
 $y = (5 - 8)(2 - 5)$
 $y = (-3) \times (-3)$
 $y = 9$
 Height = $9 - (-16) = 25$

38) a) $x = 2$
 b) $y = 5 + 4x - x^2$
 $y = 5 + 4 \times 2 - 2^2$
 $y = 9$

39) a) $(x - 7)(x + 3)$
 b) $x = 7$ or $x = -3$
 c) $x = 2$
 $y = x^2 - 4x - 21$
 $y = 2^2 - 4 \times 2 - 21$
 $y = -25$
 $(2,-25)$

40) a) $y = x^2 - 2x - 3$
 $x^2 - 2x - 3 = 0$
 $(x - 3)(x + 1) = 0$
 $(x - 3) = 0$ or $(x + 1) = 0$
 $x = 3$ or $x = -1$
 $A(-1,0), B(3,0)$
 b) $x = 1$

41) a) $y = 4 \times 0^2 + 4 \times 0 - 3 = -3$
 $A(0,-3)$
 b) $y = 4x^2 + 4x - 3$
 $4x^2 + 4x - 3 = 0$
 $(2x + 3)(2x - 1) = 0$
 $(2x + 3) = 0$ or $(2x - 1) = 0$
 $2x = -3$ or $2x = 1$
 $x = -\frac{3}{2}$ or $x = \frac{1}{2}$
 $B(-\frac{3}{2}, 0), C(\frac{1}{2}, 0)$
 c) Min Value = $4 \times (-\frac{1}{2})^2 + 4 \times (-\frac{1}{2}) - 3 = -4$

42) a) $a = -1, b = 3$
 b) $y = k(x + 1)(x - 3)$
 $-6 = k(0 + 1)(0 - 3)$
 $-6 = k \times (1) \times (-3)$
 $k = 2$
 c) $x = 1$
 $y = 2(1 + 1)(1 - 3)$
 $y = 2 \times (2) \times (-2) = -8$
 Min Turning Pt $(1,-8)$

43) a) $P(-2,-16)$
 b) $Q(6,-16)$
 c) $y = (x - 14)^2 - 16$

44) a) $h(t) = 16t - t^2$
 $60 = 16t - t^2$
 $t^2 - 16t + 60 = 0$
 $(t - 6)(t - 10) = 0$
 $(t - 6) = 0$ or $(t - 10) = 0$
 $t = 6$ or $t = 10$
 after 6 seconds

b) max height at 8 secs
 $h(8) = 16 \times 8 - 8^2$
 $h(8) = 128 - 64 = 64$
 will NOT reach a height of 70m

45) a) $h(t) = -2t(t - 14)$
 $0 = -2t(t - 14)$
 $-2t = 0$ or $(t - 14) = 0$
 $t = 0$ or $t = 14$
 in flight for 14 secs

b) max height at 7 secs
 $h(7) = -2 \times 7 \times (7 - 14)$
 $h(7) = -14 \times (-7)$
 $h(7) = 98m$
 max height = 98m

46) $h = 48 + 8t - t^2$
 $0 = 48 + 8t - t^2$
 $t^2 - 8t - 48 = 0$
 $(t - 12)(t + 4) = 0$
 $(t - 12) = 0$ or $(t + 4) = 0$
 $t = 12$ or $t = -4$
 Flare enters the sea after 12 secs

47) a) $f(x) = g(x)$
 $x^2 - 4x = 2x + 7$
 $x^2 - 6x = 7$
 $x^2 - 6x - 7 = 0$

b) $x^2 - 6x - 7 = 0$
 $(x - 7)(x + 1) = 0$
 $(x - 7) = 0$ or $(x + 1) = 0$
 $x = 7$ or $x = -1$

48) $c^2 = a^2 + b^2$
 $(x + 8)^2 = x^2 + (x + 7)^2$
 $x^2 + 16x + 64 = x^2 + x^2 + 14x + 49$
 $16x + 64 = x^2 + 14x + 49$
 $64 = x^2 - 2x + 49$
 $0 = x^2 - 2x - 15$
 $(x - 5)(x + 3) = 0$
 $x = 5$ or $x = -3$
 $x = 5$, can't be negative

49) a) $2x \times 3 = x^2 + 5$
 $6x = x^2 + 5$
 $x^2 - 6x + 5 = 0$

b) $x^2 - 6x + 5 = 0$
 $(x - 1)(x - 5) = 0$
 $x = 1$ or $x = 5$
 If QR is the shortest side then $x = 5$

50) $W = \frac{1}{4}(M^2 - 4M + 272)$
 $83 = \frac{1}{4}(M^2 - 4M + 272)$
 $332 = M^2 - 4M + 272$
 $0 = M^2 - 4M - 60$
 $(M - 10)(M + 6) = 0$
 $M = 10$ or $M = -6$
 Age 10 as it can't be negative

51) $d = \frac{n(n-3)}{2}$
 $20 = \frac{n(n-3)}{2}$
 $40 = n(n - 3)$
 $40 = n^2 - 3n$
 $n^2 - 3n - 40 = 0$
 $(n - 8)(n + 5) = 0$
 $n = 8$ or $n = -5$
 8 sides as it can't be negative

52) a) $r = \frac{1}{2}n(n - 1)$
 $r = \frac{1}{2} \times 7 \times (7 - 1)$
 $r = \frac{1}{2} \times 7 \times 6 = 21$

b) $55 = \frac{1}{2}n(n - 1)$
 $110 = n(n - 1)$
 $110 = n^2 - n$
 $0 = n^2 - n - 110$

c) $(n - 11)(n + 10) = 0$
 $n = 11$ or $n = -10$
 11 towns as it can't be negative

53) a) $A = (x + 2) \times x = x^2 + 2x$

b) $A = (x + 1)^2 = x^2 + 2x + 1$
 Square is greater by 1cm^2

54) a) $A = (x + 7)(x + 3)$
 $A = x^2 + 3x + 7x + 21$
 $A = x^2 + 10x + 21$

b) $45 = x^2 + 10x + 21$
 $0 = x^2 + 10x - 24$
 $(x + 12)(x - 2) = 0$
 $x = -12$ or $x = 2$
 $x = 2$, can't be negative

55) $A = \frac{1}{2} \times 2x \times (2x - 5)$
 $7 = x \times (2x - 5)$
 $7 = 2x^2 - 5x$
 $0 = 2x^2 - 5x - 7$
 $(2x - 7)(x + 1) = 0$
 $x = \frac{7}{2}, x = -1$
 $x = \frac{7}{2}$, can't be negative

56) a)i) $l = x + x + 13 = 2x + 13$
a)ii) $b = x + x + 9 = 2x + 9$
 $A = (2x + 13)(2x + 9)$
 $270 = 4x^2 + 18x + 26x + 117$
 $4x^2 + 44x - 153 = 0$

b) $x = \frac{-44 \pm \sqrt{(44)^2 - 4 \times 4 \times (-153)}}{2 \times 4}$
 $x = \frac{-44 \pm \sqrt{1936 + 2448}}{8}$
 $x = \frac{-44 \pm \sqrt{4384}}{8}$
 $x = \frac{-44 \pm 66 \cdot 212}{8}$
 $x = \frac{-44 + 66 \cdot 212}{8}$ or $x = \frac{-44 - 66 \cdot 212}{8}$
 $x = \frac{22 \cdot 212}{8}$ or $x = \frac{-110 \cdot 212}{8}$
 $x = 2 \cdot 7765$ or $x = -13 \cdot 7765$
 $x = 2 \cdot 8$ or $x = -13 \cdot 8$
 $x = 2 \cdot 8$, can't be negative

57) a) A of lawn = $3x \times x = 3x^2$
 A of path = $x + 1 + 3x + 1 + x = 5x + 2$
 $3x^2 = 5x + 2$
 $3x^2 - 5x - 2 = 0$

b) $(3x + 1)(x - 2) = 0$
 $x = -\frac{1}{3}, x = 2$
 $x = 2$, can't be negative
length = $3 \times 2 = 6$ m

58) a) $length = 10 - x$
 $width = 6 - x$
 $A = (10 - x)(6 - x)$
 $A = 60 - 10x - 6x + x^2$
 $A = x^2 - 16x + 60$

b) $12 = x^2 - 16x + 60$
 $0 = x^2 - 16x + 48$
 $(x - 12)(x - 4) = 0$
 $x = 12$ or $x = 4$
 $x = 4$, can't be 12

59) a) $V = (x + 5) \times x \times 1$
 $24 = x^2 + 5x$
 $0 = x^2 + 5x - 24$

b) $x^2 + 5x - 24 = 0$
 $(x + 8)(x - 3) = 0$
 $x = -8$ or $x = 3$
breadth = 3m, can't be negative