

6. Pythagoras

NB There is some overlap between these questions and those on the Circle, Similar Triangles and Trigonometry.

1. A sheep shelter is part of a cylinder as shown in Figure 1.
 It is 6 metres wide and 2 metres high.
 The cross-section of the shelter is a segment of a circle with centre O, as shown in Figure 2.
 OB is the radius of the circle.

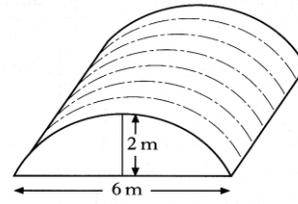


Figure 1

Calculate the length of OB.

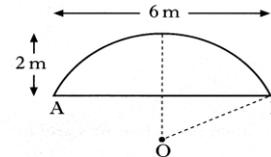
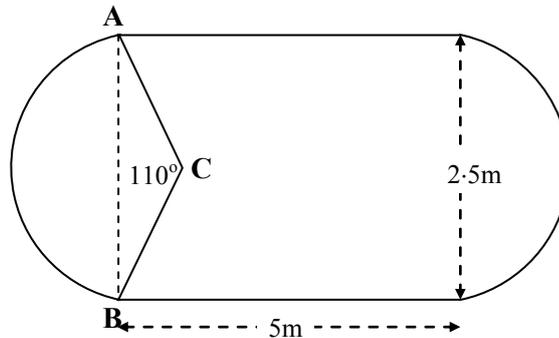


Figure 2

4 RE

2. A large shop display table is in the shape of a rectangle with a circle segment at both ends, as shown in the diagram below.

The rectangle at the centre measures 5 metres by 2.5 metres.
 AC and BC are radii of the circle and angle ACB is 110° .



- (a) Show that AC, the radius of the segment, is 1.53 m correct to 3 significant figures.

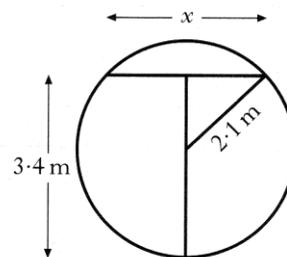
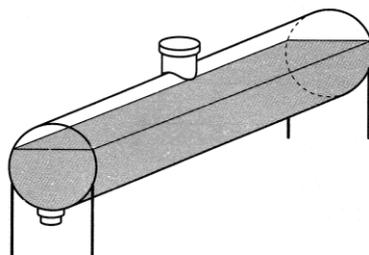
3 RE

- (b) To stand comfortably around this table it is estimated that an average person requires 75 cm of table edge.

How many people can stand comfortably at the table described above?

4 RE

3. An oil tank has a circular cross section of radius 2.1 metres.
 It is filled to a depth of 3.4 metres.



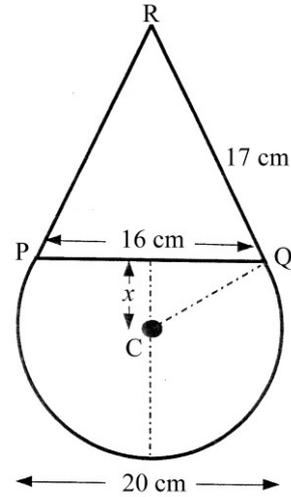
- a) Calculate x , the width in metres of the oil surface.
 b) What other depth of oil would give the same surface width.

3 KU
 1 RE

4. A clown's face consists of an isosceles triangle PQR on top of a sector of a circle.



The diameter of the circle is 20 centimetres.
The base of the triangle is 16 centimetres and its sloping sides are 17 centimetres long.

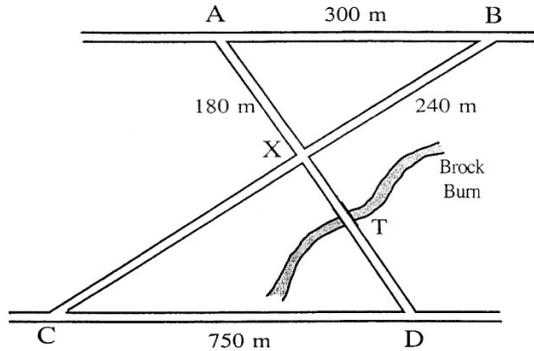


- a) Calculate x , the distance in centimeters from the centre of the circle to the base of the triangle.
b) Calculate the total height of the figure.

3 KU
3 RE

5. The road joining A to B is parallel to the road joining C to D in the diagram.

$AB = 300$ metres,
 $AX = 180$ metres,
 $BX = 240$ metres
and $CD = 750$ metres.

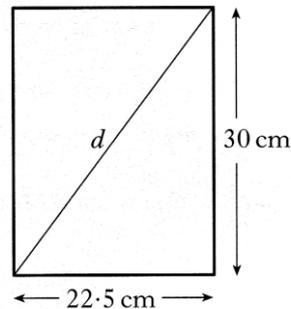


- a) Prove that the two roads AX and BX are at right angles to one another.
b) The Brock Burn burst its banks at T and the road became impassable. An alternative route had to be found in order to travel from A to D.
Calculate the length of the shortest route.

3 RE
3 RE

6. A rectangular picture frame is to be made.

It is 30 centimetres high and 22.5 centimetres wide, as shown.
To check that the frame is rectangular, the diagonal, d , is measured.
It is 37.3 centimetres long.
Is the frame rectangular?

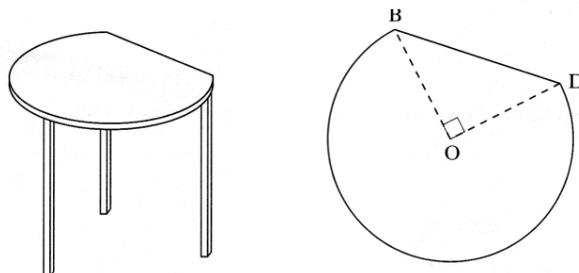


4 RE

7. The diagram shows a table whose top is in the shape of part of a circle with centre, O, and radius 60 centimetres.

BD is a straight line.
Angle BOD is 90° .

Calculate the perimeter of the table top.

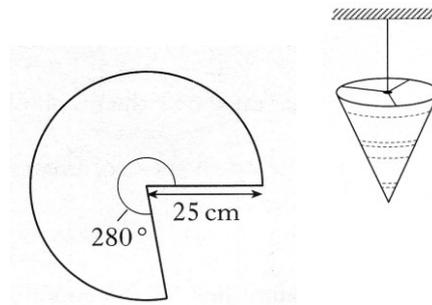


3 RE

8. A lampshade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

The circle has radius 25 centimetres and the angle of the sector is 280°

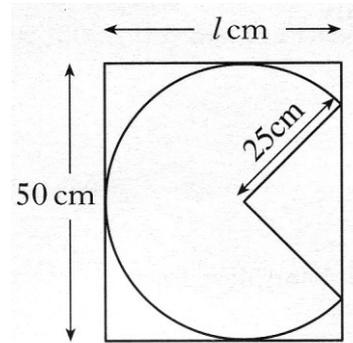


- a) Find the area of the sector of the circle.

3 KU

Each sector is cut from a rectangular piece of material, 50 centimetres wide.

- b) Find to the nearest centimetre the minimum length l , required for the piece of material.

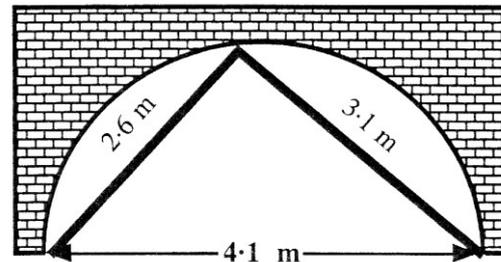


4 RE

9. The central semi-circular archway under a bridge is to be strengthened.

While the work is being carried out, 2 metal beams are to be set in place to support the archway.

For safety reasons, the beams have to just meet on the circumference of the arch.



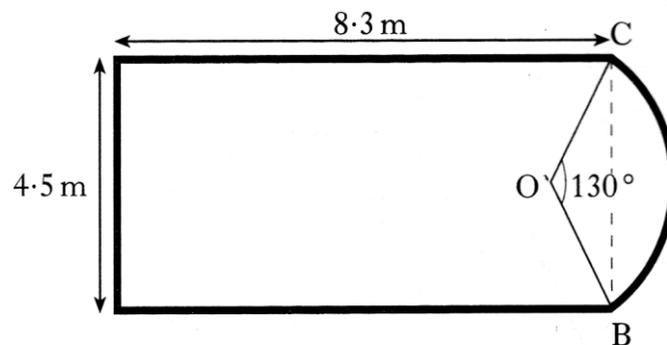
Will the beams fit this archway which is 4.1 metres wide ?

4 RE

10. The diagram shows a ceiling in the shape of a rectangle and a segment of a circle.

The rectangle measures 8.3 metres by 4.5 metres.

OB and OC are radii of the circle and angle BOC is 130° .



- a) Find the length of OB.

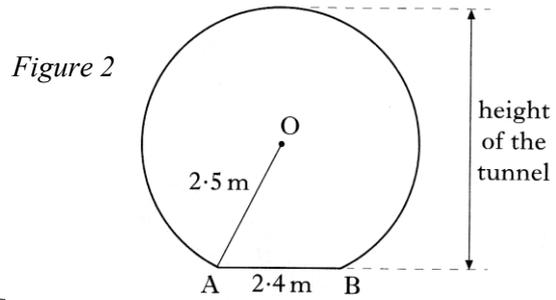
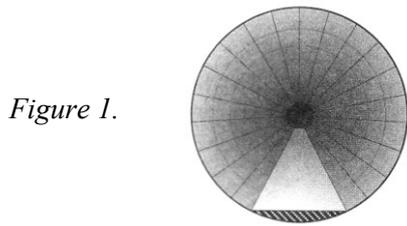
2 RE

A border has to be fitted around the perimeter of the ceiling.

- b) Find the length of border required.

4 RE

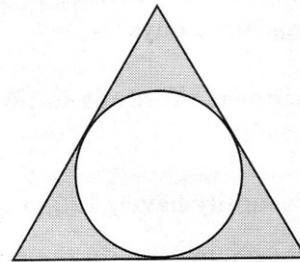
11. Figure 1 shows the circular cross section of a tunnel with a horizontal floor.



In figure 2, AB represents the floor.
 AB is 2.4 metres.
 The radius, OA, of the cross-section is 2.5 metres.
 Find the height of the tunnel.

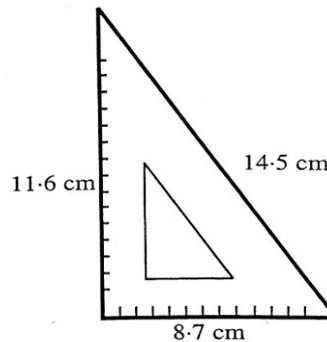
4 KU

6. The diagram shows the design of an earring.
 The earring consists of a circle inside an equilateral triangle.
 The sides of the triangle are tangents to the circle.
 The radius of the circle is 8 mm
 The distance from the centre of the circle to **each** vertex of the triangle is 17mm.
 Calculate the perimeter of the triangle.



4 RE

10. MATRIX is a company which makes mathematical instruments.
 They intend to make a new size of set square which **must have a perfect right angle** at one of its corners..
 If the set square has sides of length 8.7 cm, 11.6 cm and 14.5 cm, will it be acceptable.
 (Give reasons for your answer).



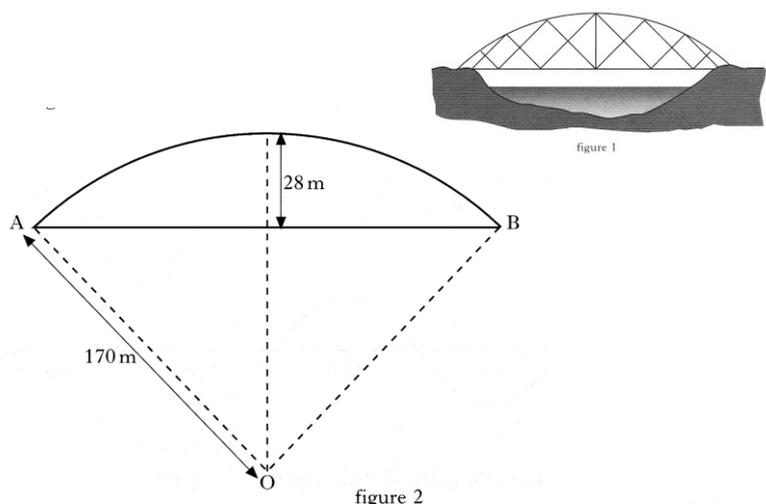
4 RE

11. Figure 1 shows a road bridge.
 The curved part of the bridge is formed from the arc of a circle, centre O, as shown in figure 2.

OA and OB are radii of length 170 metres.

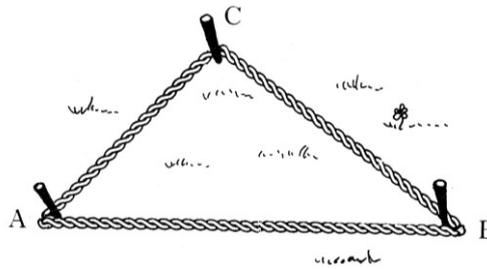
The height of the middle of the bridge above its ends is 28 metres as shown in figure 2.

Calculate the horizontal distance, AB.



4 KU

12. A loop of rope is used to mark out a triangular plot, ABC.
 The loop of rope measures 6 metres.
 Pegs are positioned at A and B such that AB is 2.5 metres.
 The third peg is positioned at C such that BC is 2 metres.

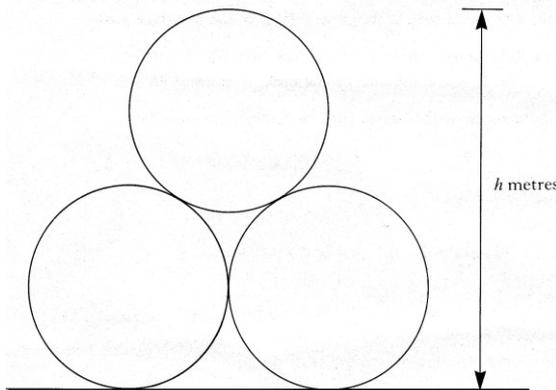


Prove that angle $ACB = 90^\circ$.

Do not use a scale drawing.

4 KU

13. Three pipes are stored on horizontal ground as shown in the diagram.
 Each pipe has a circular cross-section with radius 1 metre.

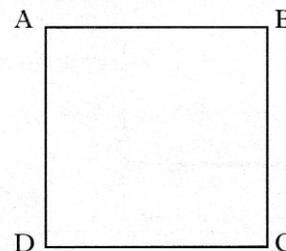


Calculate the height, h metres, of the stacked pipes. (Ignore the thickness of the pipes.)

Give your answer in metres correct, to two decimal places.

4 RE

12. a) ABCD is a square of side 2 cms
 Write down the ratio of the length AB to the length of AC.
 b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$



2 KU

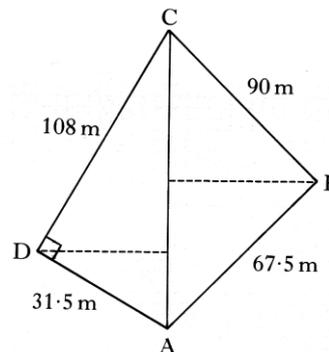
3 RE

13. A school's playing fields have recently been surveyed and the following plan produced.
 The plan is not drawn to scale.

- AB = 67.5 metres
- BC = 90 metres
- AD = 31.5 metres
- DC = 108 metres
- Angle $ADC = 90^\circ$

Without doing any further measurements, the surveyor realises that angle ABC is a right angle.

Prove that angle $ABC = 90^\circ$



4 KU

This next question is quite an involved one from 1990. It is unlikely to be set today, however, if you can do this then you have demonstrated an excellent understanding of mathematics.

14. The diagram of a rivet is shown opposite.
The body of the rivet is in the shape of a cylinder.



Figure 1

The head of the rivet is a cap of a sphere of radius R , which is obtained as shown in figure 2.

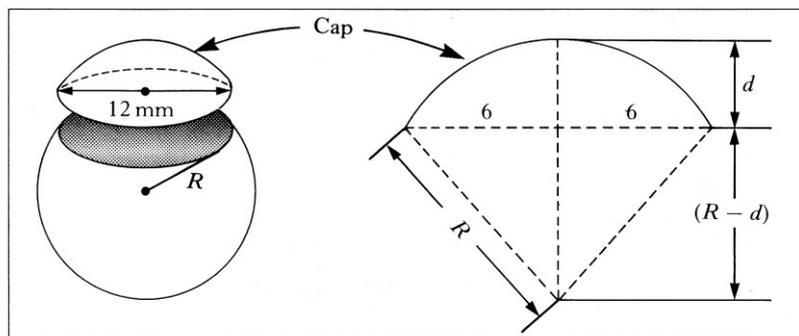


Figure 2

- a) Find the value of R for this cap of width 12mm when its height in mm is given by $d = 2.4 RE$
- b) The length of the cylindrical body of this rivet is 8mm and the diameter of the base is 5mm.

Assuming that the volume of the cap of the sphere is given by

$$V = \frac{1}{3} \pi d^2 (3R - d)$$

show that the total volume of the rivet

is $\frac{262\pi}{3} \text{ mm}^3$

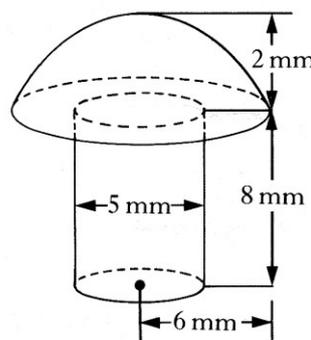


Figure 3

4 RE