

## Solutions

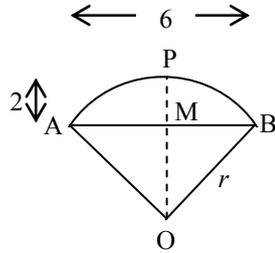
### 6 Pythagoras

1. Let mid-point of AB be M and radius OB be  $r$

$$\begin{aligned} MB &= 3\text{m} \left( \frac{1}{2} \text{ of width} \right) \\ OP &= r \text{ (radius)} \\ OM &= r - 2 \end{aligned}$$

By Pythagoras:  $r^2 = (r-2)^2 + 3^2$

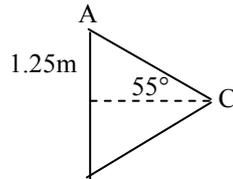
$$\begin{aligned} \text{So, } r^2 &= r^2 - 4r + 4 + 9 \\ 4r &= 13, \quad r = 3.25 \text{ metres.} \end{aligned}$$



2. a) Draw triangle as shown

$$\sin 55^\circ = \frac{1.25}{AC}$$

$$AC = \frac{1.25}{\sin 55^\circ} = 1.5259\dots \text{ hence } AC = 1.53\text{m (3 sf)}$$



- b) Find perimeter of table  
radius of arc = 1.53m (diameter = 3.06m)  
length of curved end =  $\frac{110}{360} \times \pi \times 3.06 = 2.94\text{m}$   
length of straight section = 5m  
Perimeter = 2.94 + 2.94 + 5 + 5 = 15.88m  
= 1588 cm  
No. of people = 1588 ÷ 75 = 21.17.. So **21 people**

3. a) Find MB and then double it.

$$OA = 2.1\text{m (radius)}$$

$$\begin{aligned} OM &= 3.4 - 2.1\text{ m} \\ &= 1.3\text{ m} \end{aligned}$$

By Pythagoras

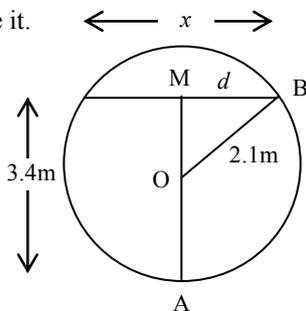
$$2.1^2 = 1.3^2 + d^2$$

Hence  $2.1^2 - 1.3^2 = d^2$  so  $d = 1.649\dots$

Hence  $x = d \times 2 = 3.298\dots = 3.3\text{ m (2 s.f.)}$

- b) By symmetry, the oil width will be the same, when it is below the centre by a distance OM.

i.e.  $2.1 - 1.3 = 0.8$  metres.



4. a) CQ = radius = 10 cm

By Pythagoras:  $10^2 = x^2 + 8^2$  hence  $10^2 - 8^2 = x^2$   
and so,  $x = 6$  cm

- b) Height of figure = Ht. of Triangle +  $x$  + 10  
Let height of triangle be  $h$ , using symmetry:  
 $h^2 + 8^2 = 17^2$   
So  $h = 15$  cm  
Height of figure = 15 + 6 + 10 cm = **31 cms**

5. a) Use converse of Pythagoras

$$AB^2 = 90\,000$$

$$AX^2 + BX^2 = 180^2 + 240^2 = 90\,000$$

Since  $AB^2 = AX^2 + BX^2$  then AXB is  $90^\circ$

(converse of Pythagoras)

So AX and BX are at right angles to each other.

- b) Shortest route is AX, XC, CD

Need to find XC

Triangles ABX and CXD are similar

(using alternate angles on the parallel lines AB, CD)

$$\frac{XC}{240} = \frac{750}{300} \text{ hence } XC = 600\text{m}$$

$$\begin{aligned} \text{Length of shortest route} \\ &= 180 + 600 + 750 = 1530 \text{ metres.} \end{aligned}$$

6. Use converse of Pythagoras

$$d^2 = 37.3^2 = 1391.29$$

$$22.5^2 + 30^2 = 506.25 + 900 = 1406.25$$

Since  $d^2 \neq 22.5^2 + 30^2$  then AXB is NOT  $90^\circ$

(converse of Pythagoras)

So No, the frame is NOT rectangular.

7. Find length of BD: OD = radius = 60 cm

BOD is a right angled triangle

Use Pythagoras:  $BD^2 = 60^2 + 60^2$

$$BD = 84.852\dots \text{ cm} = 84.9 \text{ cm (1 dp)}$$

$$\begin{aligned} \text{Perimeter of circular part of table} &= \frac{270}{360} \times \pi \times 120 \\ &= 282.7433\dots \text{ cm} = 282.7 \text{ cm (1 dp)} \end{aligned}$$

$$\text{Perimeter of table} = 84.9 + 282.7 = 367.6 \text{ cm}$$

8. Area of sector =  $\frac{280}{360} \times \pi \times 50 = 122.17\dots = 122.2 \text{ cm}^2$

To find  $l$ , we need to find OP and then add to the radius of 25 cm

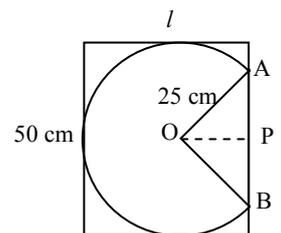
Angle AOB =  $360 - 280 = 80^\circ$

Angle AOP =  $40^\circ$

Using SOH-CAH-TOA

$$\cos 40^\circ = \frac{OP}{25} \text{ hence } OP = 19.151\dots = 19.2 \text{ cm}$$

$$\text{so min. length } l \text{ reqd.} = 19.2 + 25 = \mathbf{44.2 \text{ cms.}}$$



9. If they are to meet then this will be the angle in a semi-circle which should be  $90^\circ$

Use converse of Pythagoras

$$4.1^2 = 16.81$$

$$2.6^2 + 3.1^2 = 16.37$$

Since  $4.1^2 \neq 2.6^2 + 3.1^2$  then angle at top of bridge

is NOT  $90^\circ$  (converse of Pythagoras).

So beams will NOT fit this archway.

## Solutions

### 6 Pythagoras (continued)

10. a) Use SOH-CAH-TOA

$$\sin 65^\circ = \frac{2.25}{OB}$$

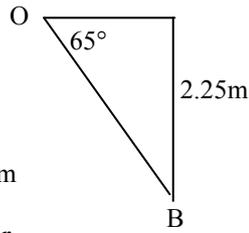
hence  $OB = 2.482\dots = 2.48$  m

- b) Length of border = perimeter.

radius of curved edge =  $2 \times 2.48 = 4.96$  m

Curved length =  $\frac{130}{360} \times \pi \times 4.96 = 5.626\dots$  m

Length of border =  $8.3 + 8.3 + 4.5 + 5.6 = 26.7$  m



11. Height of tunnel

= distance O to the floor + O to top (the radius).

Let distance O to floor be  $d$

Use Pythagoras and symmetry:

$$2.5^2 = d^2 + 1.2^2 \quad \text{hence } d = 2.193\dots$$

Hence height of tunnel =  $2.19 + 2.5 = 4.7$  m (2 sf)

6. First draw the diagram  
Mark on information given.

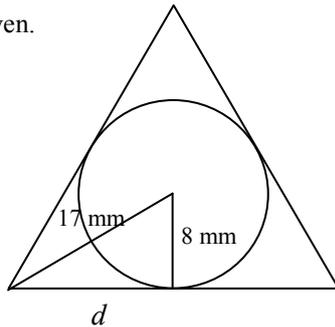
Find  $d$ .

$$17^2 = d^2 + 8^2$$

$d = 15$  cm

Hence each side  
of triangle = 30 cm

Since equilateral, perimeter =  $3 \times 30 = 90$  cms.



10. Converse of Pythagoras

$$14.5^2 = 210.25$$

$$11.6^2 + 8.7^2 = 210.25$$

Since  $14.5^2 = 11.6^2 + 8.7^2$  then

angle is a perfect right angle (converse of Pythagoras),  
so yes, it will be acceptable.

- 11.

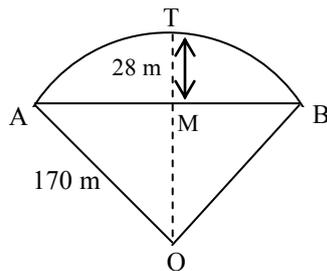
$OT = 170$  (radius)  
 $OM = 170 - 28 = 142$

Use Pythagoras

$$170^2 = AM^2 + 142^2$$

$AM = 93.466\dots$

Hence  $AB$   
=  $2 \times 93.466\dots$   
=  $187$  m (3 sf)



12. Converse of Pythagoras

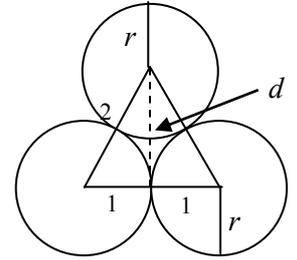
$$AC = 6 - (2 + 2.5) = 1.5$$

$$AB^2 = 2.5^2 = 6.25$$

$$AC^2 + CB^2 = 2^2 + 1.5^2 = 6.25$$

Since  $AB^2 = AC^2 + CB^2$  then angle  $ACB = 90^\circ$   
(converse of Pythagoras).

13. Height of stacked pipes =  
radius + radius  
+ distance between centres  
of two layers



Triangle is equilateral,  
by symmetry  
and sides are all 2 m

By Pythagoras:  $2^2 = 1^2 + d^2$  so  $d = 1.732\dots$  m

Hence height of pipe stack =  $1 + 1 + 1.73 = 3.73$  m

12. a) If  $AB = 2$  then  $BC = 2$  (it is a square)

By Pythagoras:  $AC^2 = 2^2 + 2^2$  so  $AC = \sqrt{8} \rightarrow 2\sqrt{2}$

- b) In any square of side  $a$ .

$$\text{Diagonal} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

Ratio of side to diagonal is:  $a : a\sqrt{2}$

which is  $1 : \sqrt{2}$

13. Since angle  $ADC = 90^\circ$

By Pythagoras

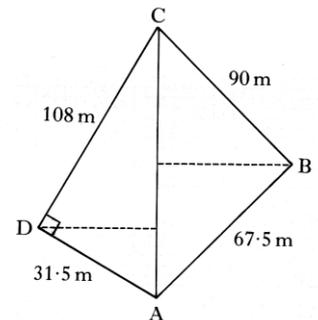
$$AC^2 = 108^2 + 31.5^2$$

$$= 12656.25$$

$$AB^2 + BC^2 = 90^2 + 67.5^2$$

$$= 12656.25$$

Hence angle  $ABC = 90^\circ$   
(converse of Pythagoras)



14. a) If  $d = 2$ , then using Pythagoras

$$R^2 = 6^2 + (R-2)^2 \quad R^2 = 36 + R^2 - 4R + 4$$

Hence  $4R = 40$  and so  $R = 10$

- b) Volume = Volume of Cap + cylinder

$$\text{Cap: } V = \frac{1}{3} \pi (2)^2 (3 \times 10 - 2) = \frac{112}{3} \pi$$

$$\text{Cylinder: } V = \pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{8}{1} = 50\pi$$

$$\text{Total volume} = \frac{150}{3} \pi + \frac{112}{3} \pi = \frac{262\pi}{3}$$