

Solutions

4. Area & Volume

1. a) $V = \pi r^2 h \rightarrow V = \pi \times 5^2 \times 14 = 1099.55\dots$
 $V = 1100$ (3 s.f.) Note radius is 5 cm.

b) Cross section stays the same, height will change

$$600 = \pi \times 5^2 \times h \rightarrow h = \frac{600}{25\pi} \quad h = 7.6394\dots$$

depth of coffee = 7.6 centimetres (2 s.f.)

2. Calculate volume of cylinder.

$$V = \pi r^2 h \rightarrow V = \pi \times 0.55^2 \times 1.85 = 1.75811\dots \text{m}^3$$

This volume will have been in the top tank.

Cross section stays the same, height will change

$$1.7581 = 3 \times 9 \times h \rightarrow h = \frac{1.7581}{27} \quad h = 0.065114\dots \text{m}$$

$\times 1000$ to change to mm $\rightarrow 65.114\dots = 65$ mm

3. Volume of prism = Area cross section x length

Area of rectangle = $0.6 \times 0.25 \text{ m}^2$

Area of semi-circle = $\frac{1}{2} \pi (0.3)^2$ (NB use radius)

Area of cross section = $0.6 \times 0.25 + \frac{1}{2} \pi (0.3)^2$

Area of cross section = 0.29137...

Volume = $0.2913\dots \times 4 = 1.1654\dots = 1.2 \text{ m}^3$ (2sf)

4. Volume of prism = Area cross section x length

Area of 2 rectangles = $2 \times 4.5 \times 2 = 18 \text{ m}^2$

Area of inner semi-circle = $\frac{1}{2} \pi (2)^2$

Area of outer semi-circle = $\frac{1}{2} \pi (4)^2$

Shaded area = $\frac{1}{2} \pi (4)^2 - \frac{1}{2} \pi (2)^2 = 18.8495\dots \text{m}^2$

Area of cross section = $18 + 18.85 = 36.85 \text{ m}^2$

Volume = $36.85 \times 0.8 = 29.55 = 29.6 \text{ m}^3$ (1 dp)

*****Misprint – should ask for new diameter

5. a) $V = \pi r^2 h \rightarrow V = \pi \times 3.25^2 \times 15 = 497.746\dots$
 $V = 497.75$ (2 dp.) Note radius is 3.25 cm.

b) Volume stays the same, height is reduced

$$497.75 = \pi \times r^2 \times 12 \rightarrow r^2 = \frac{497.75}{12\pi} \quad r^2 = 13.203\dots$$

radius = 3.663... hence diameter = 7.2672..

new diameter = 7.3 cm (1 d.p.)

6. Volume of prism = Area cross section x length

Area of triangle = $\frac{1}{2} a b \sin C$

$$= \frac{1}{2} \times 8 \times 14 \times \sin 100^\circ = 55.14923\dots$$

Volume = $55.14923 \times 5 = 275.746\dots$

Volume = 276 cm^3 (2 sf)

7. Volume of space = Vol. Cylinder – Vol cuboid
Calculate volume of cylinder.

$$V = \pi r^2 h \rightarrow V = \pi \times 6^2 \times 20 = 720\pi \text{ cm}^3$$

Cross section of cuboid is a square

Diagonal = 12 cm, Area = $\frac{1}{2}$ diag x diag

$$\text{Area} = \frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$$

Or use Pythagoras

$$\text{Side of square} = \sqrt{6^2 + 6^2} = \sqrt{72} \text{ cm}$$

$$\text{Area of square} = \sqrt{72} \times \sqrt{72} = 72 \text{ cm}^2$$

Volume of cuboid = $72 \times 20 = 1440$

Hence vol of space = $720\pi - 1440$

$$= 720(\pi - 2)$$

8. a) Dimensions of packet are < 1 litre

$$\text{Vol} = 6 \times 10 \times 15 = 900 \text{ cm}^3 < 1 \text{ litre}$$

b) Volume of cylinder

$$V = \pi r^2 h \rightarrow 900 = \pi \times 6^2 \times h \rightarrow h = \frac{900}{\pi \times 6^2}$$

$h = 7.9577\dots = 8.0$ cm (1 d.p.)

9. Cross section is rectangle + trapezium
or 2 rectangles and a triangle

NB answer is in cubic metres.

So work in metres

Area of cross section :

Area Trapezium

$$= \frac{1}{2} (0.8 + 0.4) \times 0.35 = 0.21 \text{ m}^2$$

Area rectangle = $0.3 \times 0.8 = 0.24 \text{ m}^2$

Cross section area = $0.21 + 0.24 = 0.45 \text{ m}^2$

Or

Area of rectangles = $0.3 \times 0.8 + 0.35 \times 0.4 = 0.38$

Area of triangle = $\frac{1}{2} \times 0.4 \times 0.35 = 0.07$

Cross section area = $0.38 + 0.07 = 0.45 \text{ m}^2$

Volume = $0.45 \times 1 = 0.45 \text{ m}^3$

10. a) $x + x + 1.5 = 3$, so $2x = 1.5$, $x = 0.75$ m

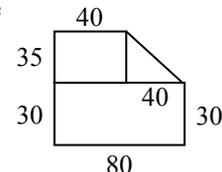
b) Cross section area: 2 triangles + 2 rectangles

Area Rect: $3 \times 0.6 + 1.5 \times 0.8 = 3 \text{ m}^2$

Area Triangles = $2 \times \frac{1}{2} \times 0.75 \times 0.8 = 0.6 \text{ m}^2$

Cross section area = $3 + 0.6 = 3.6 \text{ m}^2$

Volume = $3.6 \times 2 = 7.2$ cubic metres.



Solutions

4. Area & Volume

11. Find area of cross section

Rectangle + semi circle

$$\text{Area Rectangle} = 7 \times 5 = 35 \text{ m}^2$$

$$\text{Area semi-circle} = \frac{1}{2} \pi 3.5^2 = 19.24 \text{ m}^2$$

$$\text{Area of cross section} = 35 + 19.24 = 54.24 \text{ m}^2$$

$$\begin{aligned} \text{Volume of barn} &= 12 \times 54.24 = 650.88 \dots \text{ m}^3 \\ &= 650 \text{ m}^3 \text{ (2 sig. figs.)} \end{aligned}$$

12. Volume of prism = Area cross section x length

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \text{ base} \times \text{height (base} = 6.5 - 0.5) \\ &= \frac{1}{2} \times 6 \times 2 = 6 \text{ m}^2 \end{aligned}$$

$$\text{Area of rectangle} = 2 \times 0.5 = 1 \text{ m}^2$$

$$\text{Area of cross-section} = 6 + 1 = 7 \text{ m}^2$$

$$\text{Volume} = 7 \times 3 = 21 \text{ m}^3$$

13. **Area of existing cross section:**

$$\text{Area of Rectangle} = 14 \times 5 = 70 \text{ m}^2$$

$$\text{Area of triangles} = 2 \times \frac{1}{2} \times 10 \times 5 = 50 \text{ m}^2$$

$$\text{Area of cross section} = 120 \text{ m}^2$$

Area of new cross section:

$$\text{Area of Rectangle} = 22 \times 5 = 110 \text{ m}^2$$

$$\text{Area of triangles} = 2 \times \frac{1}{2} \times 10 \times 5 = 50 \text{ m}^2$$

$$\text{Area of cross section} = 160 \text{ m}^2$$

Area of cross section to be removed:

$$= 160 - 120 = 140 \text{ m}^2$$

Volume to be removed = $A \times l$

$$= 140 \times 10\,000 = 1\,400\,000 \text{ m}^3 \text{ (change km to m)}$$

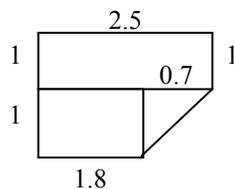
$$\text{Cost at } \pounds 4 \text{ per m}^3 = 4 \times 1\,400\,000 = \pounds 5,600,000$$

14. Area of cross section:

Area of Rectangles:

$$= 1 \times 2.5 + 1 \times 1.8$$

$$= 4.3 \text{ m}^2$$



Area of Triangle

$$= \frac{1}{2} \times 0.7 \times 1 = 0.35 \text{ m}^2$$

$$\text{Cross section area} = 4.3 + 0.35 = 4.65 \text{ m}^2$$

Volume = $A \times l$

$$= 4.65 \times 2 = 9.3 \text{ m}^3$$