

## Solutions

### 12 Functions

#### Properties of the Parabola

- $a = -1, b = 3$  (roots of the equation)
    - The point  $(0, -6)$  lies on the curve, so it will satisfy the equation of the curve.  
Hence,  $-6 = k(0+1)(0-3)$  so,  $-6 = -3k$   
 $k = 2$
    - Axis of symmetry is  $x = 1$ .  
When  $x = 1, y = 2(1+1)(1-3)$   
Hence  $y = -8$
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- Cuts  $y$  axis when  $x = 0$ , so  $y = -12$
    - B and C are roots of eqn.  $x^2 + x - 12 = 0$   
factorise  $(x+4)(x-3) = 0$  hence  $x = -4$ , or 3  
B is  $(4, 0)$  and C is  $(3, 0)$
    - Axis of symmetry is  $x = -\frac{1}{2}$ .  
When  $x = -\frac{1}{2}, y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 12$   
Hence  $y = \frac{1}{4} - \frac{1}{2} - 12 \rightarrow -12\frac{1}{4}$   
Co-ords of min t.p. are  $(-\frac{1}{2}, -12\frac{1}{4})$
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- Use the formula with  $a = 3, b = 7, c = -2$   
 $x = -0.21$  or  $x = -2.12$
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- When  $x = 0, y = -3$
    - Solve the equation by factorisation  
 $4x^2 + 4x - 3 = 0 \quad (2x-1)(2x+3) = 0$   
hence  $x = \frac{1}{2}$  or  $x = -\frac{3}{2}$
    - axis of symmetry is  $x = -\frac{1}{2}$   
when  $x = -\frac{1}{2}, y = -4$  co-ords of min t.p.  $(-\frac{1}{2}, -4)$
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#### Applications of the parabola

- Area of glass =  $(7-2x)(10-2x)$   
 $A = 70 - 14x - 20x + 4x^2$  Hence,  $A = 4x^2 - 34x + 70$
    - $28 = 4x^2 - 34x + 70$   
Re-arrange:  $4x^2 - 34x + 42 = 0$   
Divide by 2:  $2x^2 - 17x + 21 = 0$   
Factorise:  $(2x-3)(x-7) = 0$   
Hence  $x = 1\frac{1}{2}$  or  $x = 7$   
 $x$  cannot = 7, since this is width of frame,  
So  $x = 1\frac{1}{2}$  cms
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- Area of A is:  $(x+6)(x-1)$   
Area of B is:  $3(x+3)$
    - So,  $(x+6)(x-1) = 3(x+3)$   
Hence,  $x^2 + 6x - x - 6 = 3x + 9$   
simplify:  $x^2 + 2x - 15 = 0$   
factorise:  $(x+5)(x-3) = 0$   
so,  $x = 3$  or  $x = -5$ .  $x = -5$  is not possible  
Hence  $x = 3$
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- Solve the quadratic by factorisation  
 $8 + 2x - x^2 = 0$  Hence,  $(4-x)(2+x) = 0$   
 $x = 4$  or  $x = -2$ , so F is  $(4, 0)$   
The fly is 4 feet to the right of the snake.
    - axis of symmetry is when  $x = 1$   
Hence max height is  $H = 8 + 2 - 1 = 9$  feet.
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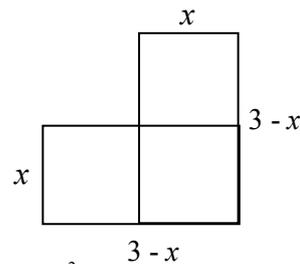
- $H(3) = 9 + 6(3) - 3(3)^2 = 0$

This indicates that the shell is now level with the cliff again.

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- Since  $BC = CD$  then  
 $2BC + 2x = 6 \rightarrow BC + x = 3$  So,  $BC = 3 - x$

- Area of rectangle  
 $= x(3-x)$   
There are 2 rectangles  
but then we have counted  
the square twice.



Hence Area =  $x(3-x) + x(3-x) - x^2$   
 $A = 3x - x^2 + 3x - x^2 - x^2$   
 $A = 6x - 3x^2$

- Find the roots of the equation  $6x - 3x^2 = 0$   
Factorise:  $3x(2-x) = 0$   
Hence  $x = 0$  or  $x = 2$   $x = 0$  is not possible, So  $x = 2$ .  
Axis of symmetry is  $x = 1$   
Max value is on axis of symmetry:  $A = 6 - 3 = 3$  m<sup>2</sup>
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- $l = w + 2$
    - Area of extension is:  $w(w+2) \rightarrow w^2 + 2w$   
This must not be more than 40% original size  
 $120 \times 0.4 = 48$  So  $w^2 + 2w = 48$  (largest extension)  
Hence  $w^2 + 2w - 48 = 0$  so  $(w-6)(w+8) = 0$   
 $w = 6$  or  $-8$  (not possible) Width = 6, Length = 8
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- $18 - 2x$  cms
    - $V = x(18 - 2x) \times 100 \quad V = 1800x - 200x^2$
    - Put  $1800x - 200x^2 = 0$  and solve equation by factorising:  $200x(9-x) = 0$   $x = 0$  or  $x = 9$   
maximum is on axis of symmetry  $x = 4\frac{1}{2}$   
dimensions of gutter are 9 cm wide  $\times$  4  $\frac{1}{2}$  cm high
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