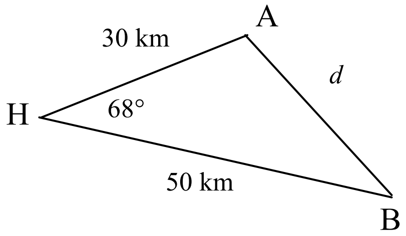
Solutions

9 Trigonometry – Sine, Cosine Rule

1. Draw a diagram, and mark in given bearings which show that ∠AHB = 68



Look at diagram - SAS - Cosine Rule

$$d^2 = 30^2 + 50^2 - 2\times 30\times 50\times \cos 68°$$

d^2 = 3400 – 1123.819… = 2276.181…

d = 47.70933…..

yachts are 47.7 km apart when they stopped.

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2. Area of triangle =1/2 a b sin C

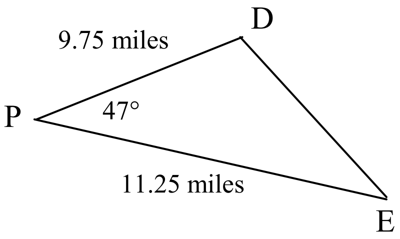
Transpose letters.

38 = ½ × 9 × 14 × sin B   38 = 63 sin B

Re-arrange: $\sin B = \dfrac{38}{63}$ B = $\sin^{-1}$ (38 ÷ 63)

Hence B = 37.096….   B = 37°

3.



PD = 13 × 0.75 = 9.75 miles

PE = 15 × 0.75 = 11.25 miles

∠DPE = 104°- 57° = 47°

Use cosine rule

$$DE^2 = 9.75^2 + 11.25^2 - 2\times 9.75\times 11.25\times \cos 47°$$

DE = 8.485…   Boat D will have to travel 8 miles

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4. Area = ½ a b sin C

So, 36 = ½ × 6 × 16 × sin R

Hence $\sin R = \dfrac{36}{48} = \dfrac{3}{4}$

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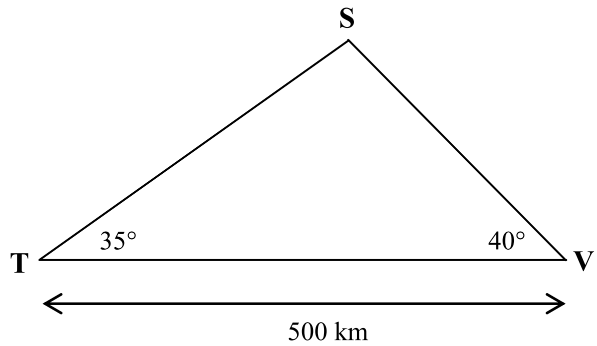
5. Use cosine Rule

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2\times 4\times 5} = \frac{5}{40} = \frac{1}{8}$$

6.

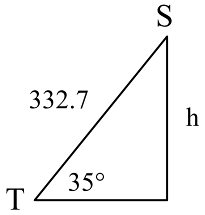


ASA - use Sine Rule to find either side ST or SV
The use SOH-CAH-TOA to find perpendicular height.

First find angle at S = 180° – (35° + 40°)   S is 105°

$$\frac{ST}{\sin 40} = \frac{500}{\sin 105}$$

$$ST = \frac{500\sin 40}{\sin 105} \Rightarrow ST = 332.731...$$



$$\sin 35 = \frac{h}{332.7}$$

$$h = 332.7\times \sin 35 = 190.828...$$

height of satellite = 190 km

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7. Basically same as previous question

∠PRQ=95°   Find RQ using sine rule

$\dfrac{RQ}{\sin 50} = \dfrac{80}{\sin 95}$ RQ = 61.5 metres

Now use SOH-CAH-TOA to find distance

Let distance between river and path be *d* metres.

$\sin 35 = \dfrac{d}{61.5}$ hence, d = 35.3 metres

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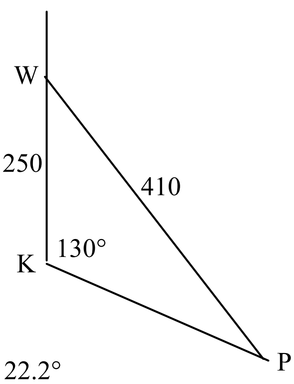
8. Draw diagram

Use sine rule to calculate angle at P.



$$\frac{\sin P}{250} = \frac{\sin 130}{410}$$

Hence sin P = 0.4671..
So, ∠ P = 27.8°

∠ KWP = 180 – (27.8 + 130) = 22.2°

Hence external angle = 157.8°

Bearing of Possum from Wallaby = 157.8°

Solutions

9 Trigonometry – Sine, Cosine Rule *(continued)*

9. Draw a larger diagram of required triangles

a) Use cosine rule: (let obtuse angle = θ)

$$\cos\theta = \frac{14^2 + 12^2 - 21^2}{2\times 14\times 12} = -\frac{101}{336}$$

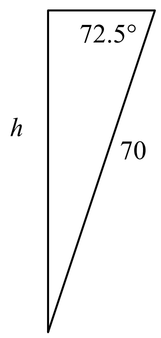
Hence acute θ = 72.5°,

so obtuse angle = 180 – 72.5 = 107.5°

b) Use SOH-CAH-TOA

Length of leg = 70 cms

Let height of table = h cms.



$$\sin 72.5 = \frac{h}{70}$$

hence h = 66.760… cms

height of table = 66.8 cms.

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10. This is exactly the same as Qu. 6

Height of B = 112.3 metres

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11. Use cosine Rule:

$$PR^2 = 101^2 + 98^2 - 2\times 101\times 98\times \cos 57°$$

PR = 94.99… = 95 cms.

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12. $$14 = \frac{1}{2}\times 6\times 7\times \sin A$$

$$\sin A = \frac{14}{21} = \frac{2}{3}$$ acute A = 41.8°

Using ASTC, the sine is positive in 2nd quadrant.

Hence there is an angle 180 – 41.8 = 138.2°

Angles are: 42° and 138°

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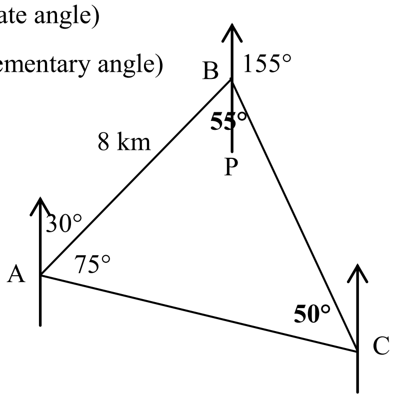
13. ∠ ABP = 30° (alternate angle)

∠ PBC = 35° (supplementary angle)

Hence,
∠ ABC = 55°

Also
∠ ACB = 50°
(angle sum triangle)



Use Sine Rule

$$\frac{BC}{\sin 75} = \frac{8}{\sin 50}$$ hence BC = 10.087…

Distance between B and C = 10.1 km (3 sf)

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14. Area of triangle = ½ a b sin C

3rd angle of triangle = 65°

Area = ½ × 7 × 11 × sin 65° = 34.9 cm^2

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15. a) ∠ RB South = 120° (alternate angles)

∠ YB South = 40° (since North B South = 180°)

Hence, ∠ RBY = 120° - 40° = 80°

b) Use cosine rule for RY

$$RY^2 = 350^2 + 170^2 - 2\times 350\times 170\times \cos 80°$$

RY = 361.6 km.

The people on the boat will be rescued first.

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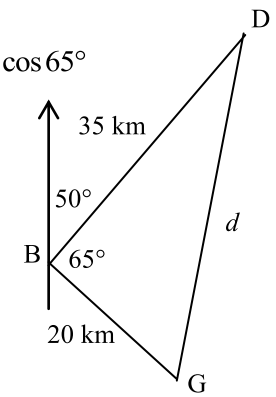
16. ∠ GBD = 125° - 50° = 65°

Use cosine rule to calculate d

$$d^2 = 35^2 + 20^2 - 2\times 35\times 20\times \cos 65°$$

Hence d = 32.145…

Distance between Delta and Gamma is 32 km.



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17. Find 3rd angle in triangle = 114°

Let longer sloping edge (opp. 42°) be d metres

Use sine rule:

$$\frac{d}{\sin 42} = \frac{12.8}{\sin 114}$$ d = 9.375…

Length of longer sloping edge = 9.4 metres

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18. Use cosine Rule

$$BC^2 = 420^2 + 500^2 - 2\times 420\times 500\times \cos 52°$$

BC = 409.66….

Hence BC = 410 metres (3 sf)

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19. This is exactly the same as Qu. 6

Height of aeroplane = 16.6 metres

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20. Area ΔPQS = ½ × 62 × 87 × sin 109° = 2550 m^2

Area ΔQSR = ½ × 100 × 103 × sin 74° = 4951 m^2

Hence Area of plot of ground = 7500 m^2 (3 sf)

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21. Similar to Qu. 13. Use parallel lines etc. to find angles.

∠GAE = 52° - 36° = 16°

Use cosine Rule

$$GE^2 = 200^2 + 160^2 - 2\times 200\times 160\times \cos 16°$$

Distance between airports = 64 km (2 sf)

Solutions

9 Trigonometry – Sine, Cosine Rule *(continued)*

22. Area of triangle = ½ × 7.2 × 10.3 × sin 34°

    = 20.73 m^2

    Area of rectangle = 8.6 × 10.3 = 88.58 m^2

    Total area = 20.73 + 88.58 = 109.31 m^2

    12 litres will cover 12 x 8 = 96 m^2

    No, this is not enough paint.

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23. Use cosine Rule

    $$PR^2 = 140^2 + 120^2 - 2 \times 140 \times 120 \times \cos 132°$$

    PR = 237.66…

    Hence PR = 238 metres (3 sf)

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24. Draw diagram and fill in angles

    Use sine rule

    $$\frac{4.8}{\sin 5} = \frac{x}{\sin 64}$$

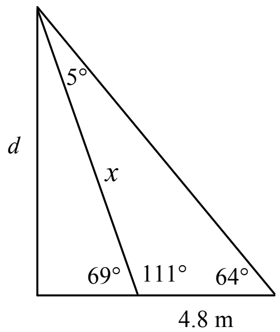
    x = 49.5 metres

    Use SOH-CAH-TOA

    $$\sin 69 = \frac{d}{49.5}$$

    Hence d = 46.21 metres (now add on height of student)

    Height of building = 46.21 + 1.5 = 47.7 metres (3 sf)



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25. Area = ½ × 300 × 340 × sin 125 = 41,776.75.. m^2

    = 41,800 m^2 (3 sf)

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26. Draw the diagram and using alternate angles find that

    ∠ PQR = 40° + 20° = 60°

    Let QR = d

    Using sine rule: $\frac{d}{\sin 85} = \frac{30}{\sin 60}$

    Hence d = 34.509…

    Distance: ship at R to lighthouse Q = 34.5 km (3 sf)

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27. Use cosine rule

    $$AB^2 = 70^2 + 100^2 - 2 \times 70 \times 100 \times \cos 65°$$

    AB = 94.7805…

    Hence AB = 95 metres (2 sf)

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28. Area = ½ × 10 × 12.6 × sin 72° = 59.9165… m^2

    = 59.9 m^2 (3 sf)

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29. ∠ABP = 40° (angle sum triangle PTB)

    Use sine rule in ΔPAB

    $\frac{5.6}{\sin 10} = \frac{AP}{\sin 40}$ hence AP = 20.73 metres

    Now use SOH-CAH-TOA in ΔPTA

    $\cos 40 = \frac{PT}{AP} = \frac{PT}{20.73}$ So, PT = 15.88 = 15.9 m (2 sf)

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30. a) Area = ½ × 6 × 7 × sin 120° = 18.186..… m^2

    = 18 m^2 (2 sf)

    b) Let angle be θ
    For maximum area, sin θ must be a maximum
    Maximum value of sine function is 1
    This occurs when angle is 90°
    Hence θ should be 90° for maximum area.

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31. Draw diagram and mark in angles – using bearings

    ∠RLT = 15° and ∠ TL West = 30°

    ∠RTL = 30° (alternate angles)

    Now use sine rule

    $\frac{10}{\sin 30} = \frac{RT}{\sin 15}$ hence RT = 5.176….

    Ship has travelled 5.2 km (2 sf) from R to T
