Polynomials Questions

1. The polynomials f(x) and g(x) are defined by

$$f(x) = x^3 + px^2 - x + 5$$
$$g(x) = x^3 - x^2 + px + 1$$

where p is a constant. When f(x) and g(x) are divided by $x - 2$, the remainder is R in each case. Find the values of p and R.

2. The polynomial p(x) = $x^3 - 4x^2 + 2x + k$ has ($x - 2$) as a factor.

    (a)   Find the value of k.

    (b)   Calculate the remainder when p(x) is divided by ($x + 3$).

    (c)   Find the exact values of the roots of the equation p(x) = 0.

3. The polynomial p(x) is given by p(x) = ($x + 3$)($x - 2$)($x - 4$)

    Find the remainder when p(x) is divided by ($x + 1$).

4. The polynomial p(x) is given by p(x) = $x^3 - 3x + 2$.

    (a)   Find the remainder when p(x)is divided by $x + 1$.

    (b)   Given that $x + 2$ is a factor of p(x), express p(x) as a product of linear factors.

    (c)   Simplify the following algebraic fraction as far as possible:
    $$\frac{x^3 - 3x + 2}{x^2 - 1}$$

5. The polynomial p(x) is given by p(x) = $x^3 - 4x^2 + 3x$.

    (a)   Show that $x - 3$ is a factor of p(x).

    (b)   Express p(x) as the product of three linear factors.

    (c)   (i)   Find the remainder, r, when p(x) is divided by $x - 2$.

    (ii)   Express p(x) in the form
    $$(x - 2)(x^2 + ax + b) + r$$
    where a, b and r are constants.

Solutions

1. $p = -3, R = -1$

2.

(a) $k = 4$

(b) -65

(c) $x = 1 \pm \sqrt{3}$

3.   30

4.

(a)   4

(b) $(x + 2)(x - 1)^2$

(c) $\dfrac{(x + 2)(x - 1)}{(x + 1)}$

5.

(a) $p(3) = 0 \Rightarrow x - 3$ is a factor

(b) $p(x) = x(x - 1)(x - 3)$

(c)   (i) -2

                (ii) $a = -2$
                          $b = -1$
                          $r = -2$