Higher 2022 Paper 1

1. $5x + 2y = 7$

$2y = -5x + 7$

$y = \frac{-5}{2}x + \frac{7}{2}$

$m = \frac{-5}{2}$

$m_{\perp} = \frac{2}{5}$

$(-1, 6)$

$y - b = m(x - a)$

$y - 6 = \frac{2}{5}(x + 1)$

$5y - 30 = 2(x + 1)$

$5y - 30 = 2x + 2$

$\underline{\underline{5y - 2x = 32}}$

2. $2\log_3 6 - \log_3 4$

$= \log_3 6^2 - \log_3 4$

$= \log_3 \frac{36}{4}$

$= \log_3 9$

$= \log_3 3^2$

$= 2\log_3 3$

$= \underline{\underline{2}}$

3. $h = 4 + \frac{1}{3}x$

$y = 4 + \frac{1}{3}x$

$y - 4 = \frac{1}{3}x$

$3y - 12 = x$

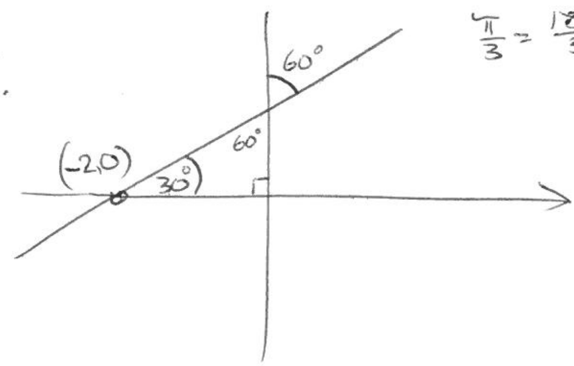
$3x - 12 = y$

$h^{-1}(x) = 3x - 12$

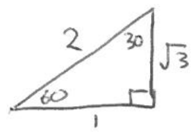
4. $y = x^{3/2} - 2x^{-1}$

$\underline{\underline{\frac{dy}{dx} = \frac{3}{2}x^{1/2} + 2x^{-2}}}$

5.



$\frac{\pi}{3} = \frac{180}{3} = 60$



$m = \tan 30 = \frac{1}{\sqrt{3}}$

$y - 0 = \frac{1}{\sqrt{3}}(x + 2)$

$\sqrt{3}y = 1(x + 2)$

$\underline{\underline{\sqrt{3}y = x + 2}}$

6. $\int_{-5}^{2} (10-3x)^{-1/2}\,dx$

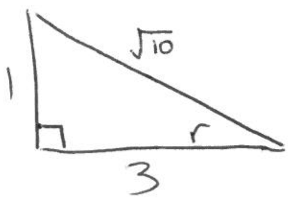
$$= \left[\frac{(10-3x)^{1/2}}{\frac{1}{2}\times -3}\right]_{-5}^{2} = \left[\frac{\sqrt{10-3x}}{-3/2}\right]_{-5}^{2} = \left[\frac{2\sqrt{10-3x}}{-3}\right]_{-5}^{2}$$

$$= \left(\frac{2\sqrt{10-3(2)}}{-3}\right) - \left(\frac{2\sqrt{10-3(-5)}}{-3}\right)$$

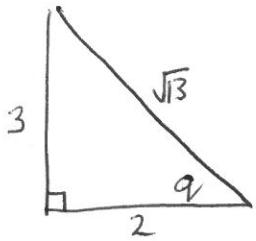
$$= \left(\frac{2\sqrt{4}}{-3}\right) - \left(\frac{2\sqrt{25}}{-3}\right)$$

$$= \frac{-4}{3} - \frac{-10}{3} = \frac{-4}{3} + \frac{10}{3} = \frac{6}{3} = \underline{\underline{2}}$$

7. a) i)



$\sin r = \underline{\underline{\frac{1}{\sqrt{10}}}}$



ii) $\sin q = \underline{\underline{\frac{3}{\sqrt{13}}}}$

b) $\sin(q-r) = \sin q \cos r - \cos q \sin r$

$$= \frac{3}{\sqrt{13}} \times \frac{3}{\sqrt{10}} - \frac{2}{\sqrt{13}}\,\frac{1}{\sqrt{10}}$$

$$= \frac{9}{\sqrt{130}} - \frac{2}{\sqrt{130}}$$

$$= \underline{\underline{\frac{7}{\sqrt{130}}}}$$

8. $\log_6 x + \log_6(x+5) = 2\log_6 6$

$\log_6 x(x+5) = \log_6 6^2$

$x^2 + 5x = 36$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4)$

$x = \cancel{-9}$ $\quad x = 4$

9. $\cos 2x - 5\cos x + 3 = 0$

$2\cos^2 x - 1 \quad - 5\cos x + 3 = 0$

$2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2)$

$\cos x = \frac{1}{2}$ $\qquad$ $\cancel{\cos x = 2}$

no solutions

$x = 60$

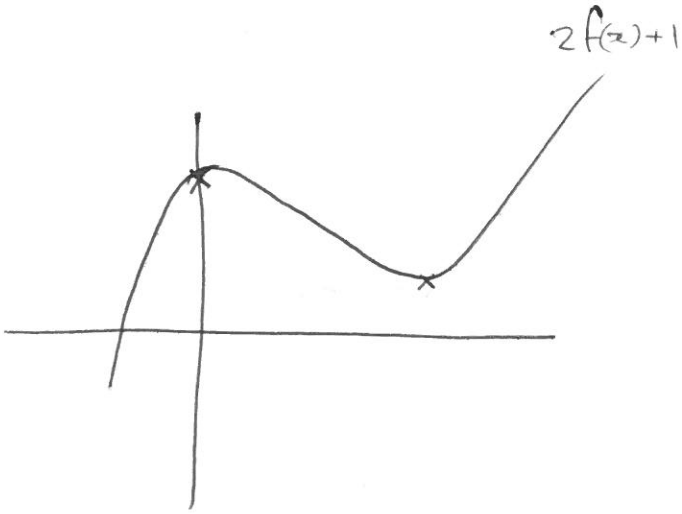
$x = 360 - 60 = 300$

$\underline{x = 60^\circ, 300^\circ}$

10. a) $(0,3)$ $(4,0)$

$(0,6)$ $(4,0)$ $\quad 2f(x)$

$(0,7)$ $(4,1)$ $\quad +1$



b) $f\left(\frac{1}{2}x\right)$ means $\div\, x$ by $\frac{1}{2}$

$\underline{(0,3)\ (8,0)}$

11. $2(x^2 + 6x) + 23$

$2(x+3)^2 + 23 - 18$

$\underline{2(x+3)^2 + 5}$

12. $f'(x) = -4\cos\left(3x - \frac{\pi}{6}\right) \times 3$

$= -12\cos\left(3x - \frac{\pi}{6}\right)$

$\boxed{\cos\frac{\pi}{3} = \cos 60 = \frac{1}{2}}$

$f'\left(\frac{\pi}{6}\right) = -12\cos\left(3 \times \frac{\pi}{6} - \frac{\pi}{6}\right)$

$= -12\cos\left(\frac{2\pi}{6}\right)$

$= -12\cos\left(\frac{\pi}{3}\right)$

$= -12 \times \frac{1}{2}$

$= \underline{-6}$

13 a) i) $-2 \left| \begin{array}{cccc} 1 & -2 & -20 & -24 \\ & -2 & 8 & 24 \\ \hline 1 & -4 & -12 & 0 \end{array} \right.$ remainder $= 0$ so $(x+2)$ is a factor

ii) $(x+2)(x^2-4x-12)$

$x=-2$ $(x-6)(x+2)$

$x=6$ $x=-2$

b) $x=-2$ is root and s.p. for $f(x)$

moves to $(1,0)$ which is $\rightarrow 3$

$k=3$

14. a) i) centre $(7,-5)$ radius $= 10$

ii) $(-2,7)$ x y

$(-2-7)^2 + (7+5)^2$

$= -9^2 + 12^2$

$= 81 + 144$

$= 225 > 100$ so P lies outside circle

b) Distance between centres $(7,-5)$ $(-2,7)$ $\sqrt{9^2+12^2} = 15$

$r_1 + r_2 = 15$

$10 + r_2 = 15$

$r = 5$