Higher 2021 Paper 1

1. $b^2 - 4ac = 0$

$3^2 - 4 \times k \times (4) = 0$

$9 + 16k = 0$

$16k = -9$

$k = \frac{-9}{16}$

2. $f'(x) = 5(x^2+1)^4 \times 2x$ ← derivative of bracket

$= 10x(x^2+1)^4$

$f'(1) = 10 \times 1 (1^2+1)^4$

$= 10 \times 16$

$= 160$

3. $f(x) = \frac{x+3}{2}$

$y = \frac{x+3}{2}$

$2y = x+3$

$2y - 3 = x$

$2x - 3 = y$

$f^{-1}(x) = 2x - 3$

4 $m = \frac{-7-2}{2--4} = \frac{-9}{6} = \frac{-3}{2}$

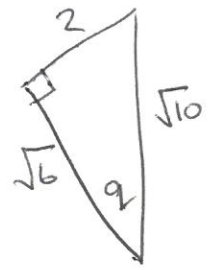
$3y = 2x + 9$

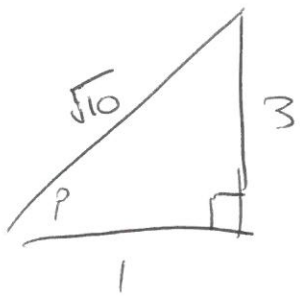
$y = \frac{2}{3}x + \frac{9}{3}$ ← gradient

$-\frac{3}{2} \times \frac{2}{3} = \frac{-6}{6} = -1$

$\therefore$ lines are perpendicular

5a)





i) $\sin p = \frac{3}{\sqrt{10}}$

$\cos q = \frac{\sqrt{6}}{\sqrt{10}}$

b) $\cos(p+q) = \cos p \cos q - \sin p \sin q$

$= \frac{1}{\sqrt{10}} \times \frac{\sqrt{6}}{\sqrt{10}} - \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{10}}$

$= \frac{\sqrt{6}}{10} - \frac{6}{10}$

$= \frac{\sqrt{6}-6}{10}$

6a) $2(x^2-2x)+5 = 2x^2-4x+5$

b) $(2x+5)^2 - 2(2x+5) = 4x^2+20x+25-4x-10 = 4x^2+16x+15$

c) $4x^2+16x+15-(2x^2-4x+5$

$4x^2+16x+15-2x^2+4x-5$

$2x^2+20x+10$

$2(x^2+10x)+10$

$2(x+5)^2+10-50$

$2(x+5)^2-40$

7. $\frac{6}{3}\sin\left(3x+\frac{\pi}{4}\right)+C$

$= 2\sin\left(3x+\frac{\pi}{4}\right)+C$

8. $m = \tan\frac{2\pi}{3} = -\sqrt{3}$

$y-0 = -\sqrt{3}(x-4)$

$y = -\sqrt{3}x+4\sqrt{3}$

$\tan 120 = -\tan 60$

$= -\sqrt{3}$

9. a) $x^3 - 7x^2 + 12x + 3 = x^3 - x^2 - 6x + 3$

$-6x^2 + 18x = 0$

$-6x(x - 3) = 0$

$-6x = 0 \qquad x - 3 = 0$

$x \neq 0 \qquad x = 3$

b) $\int_0^3 -6x^2 + 18x \, dx$

$= \left[\frac{-6x^3}{3} + \frac{18x^2}{2}\right]_0^3$

$= \left[-2x^3 + 9x^2\right]_0^3 = \left(-2(3)^3 + 9(3)^2\right) - 0$

$= 27 - 0$

$= 27 \text{ units}^2$

10. $2 \mid 6 \quad -13 \quad 0 \quad 4$

$\qquad\quad 12 \quad -2 \quad -4$

$\qquad 6 \quad -1 \quad -2 \quad 0$

$(x - 2)(6x^2 - x - 2)$

$(x - 2)(3x - 2)(2x + 1)$

11. a) $2 \times 8 - 9 = 7$

b) i) $8 + 5 = 13$

ii) $6 + 4 = 10$

Part B

15. $\text{mid}_{AD} = \left(\frac{12}{2}, \frac{2}{2}\right) = (6,1)$

$(6,1)$ $(10,1)$

$\text{mid} = \left(\frac{16}{2}, \frac{2}{2}\right) = (8,1)$

$(2,1)$
$(10,1)$

$\text{distance} = \sqrt{8^2 + 0^2}$

$= 8$

$8 \div 4 = 2 \Rightarrow$ radius

From $(8,1)$ go up 3 radii (6)

$(8,7)$

$(x-8)^2 + (y-7)^2 = 4$

16. $\log_2 6 + \log_2 12 - \log_2 3^2$

$= \log_2 72 - \log_2 9$

$= \log_2 \left(\frac{72}{9}\right)$

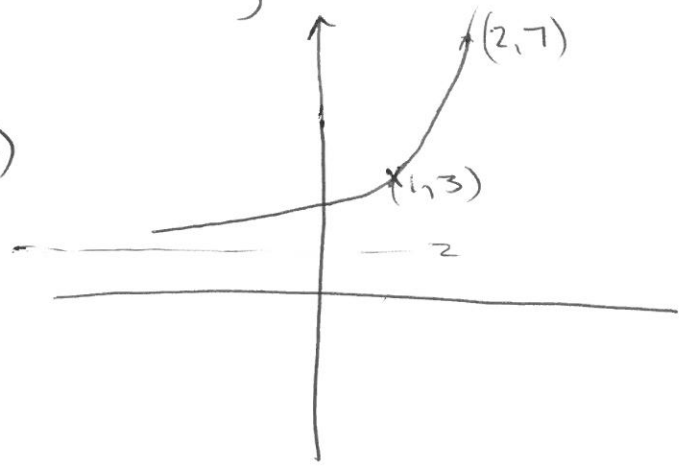
$= \log_2 8$

$= \log_2 2^3$

$= 3\log_2 2$

$= 3$

17a)



b) $\log_5(x-2) + 1 = 0$

$\log_5(x-2) = -1$

$\log_5(x-2) = -\log_5 5$

$\Rightarrow \log_5(x-2) = \log_5 5^{-1}$

$x - 2 = \frac{1}{5}$

$5x - 10 = 1$

$x = \frac{11}{5}$

$(0, \frac{11}{5})$

Higher 2021 Paper 2

1. $\frac{dy}{dx} = 6x^2 - 16x$

$m = 6(3)^2 - 16(3)$

$= 6$

$y = 2(3)^3 - 8(3)^2 + 14 = -4$

$(3, -4)$

$y + 4 = 6(x - 3)$

$y + 4 = 6x - 18$

$y = 6x - 22$

2. $\int 6(x+5)^{-3/2} dx$

$= \frac{6(x+5)^{-1/2}}{-1/2} + c$

$= \frac{12(x+5)^{-1/2}}{-1} + c$

$= \frac{-12}{\sqrt{x+5}} + c$

3. $h'(t) = 2\cos\left(2t + \frac{\pi}{6}\right)$

$h'(10) = 2\cos\left(2 \times 10 + \frac{\pi}{6}\right)$

$= 2\cos\left(20 + \frac{\pi}{6}\right)$ * put calculator into radians *

$= -0.2$

4. a) $m_{AC} = \frac{-5-1}{4--5} = \frac{-6}{9} = \frac{-2}{3}$ $\quad m_{\perp} = \frac{3}{2}$

$y - 1 = \frac{3}{2}(x - 3)$

$2y - 2 = 3(x - 3)$

$2y - 2 = 3x - 9$

$2y - 3x = -7$

b) $mid_{AB} = \left(\frac{-5+3}{2}, \frac{1+1}{2}\right) = (-1, 1)$

$m_{AB} = 0$ $\quad x = -1$

$m_{\perp} = \infty$

c) $2y - 3(-1) = -7$

$2y + 3 = -7$ $\quad (-1, -5)$

$2y = -10$

$y = -5$

5.a) $k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

$3\cos t + 5\sin t$

$k\cos a = 5$ $\quad \tan a = \frac{3}{5}$

$k\sin a = 3$ $\quad a = 31$

$k = \sqrt{5^2 + 3^2}$ $\quad \sqrt{34}\sin(t+31)$

$= \sqrt{34}$

b) i) $-\sqrt{34}$

ii) $+31$ means left $31°$

$270 - 31 = 239°$

6.a) $f'(x) = 6 - 2 \times \frac{3}{2}x^{1/2} = 0$

$= 6 - 3\sqrt{x} = 0$

$3\sqrt{x} = 6$

$\sqrt{x} = 2$

$x = 4$

6b) $\int_4^9 6x - 2x^{3/2}\,dx$

$= \left[\frac{6x^2}{2} - \frac{2x^{5/2}}{5/2}\right]_4^9$

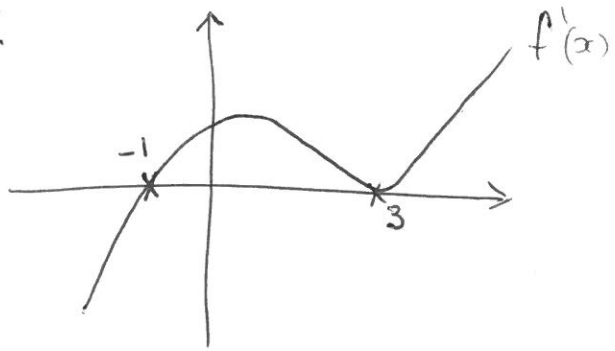
$= \left[3x^2 - \frac{4x^{5/2}}{5}\right]_4^9$

$= \left(3(9)^2 - \frac{4(9)^{5/2}}{5}\right) - \left(3(4)^2 - \frac{4(4)^{5/2}}{5}\right)$

$= \frac{243}{5} - \frac{112}{5}$

$= \frac{131}{5}$ units^2

7.



8. $2\sin(3x-60) = -1$

$\sin(3x-60) = \frac{-1}{2}$

S	A
T	C

180+ 360-

$\sin^{-1}\left(\frac{1}{2}\right) = 30$

$3x-60 = 180+30$ $\quad 3x-60 = 360-30$

$3x-60 = 210, 570,$ $\quad 3x-60 = 330, 690, -30$

$3x = 270, 630,$ $\quad 3x = 390, 750, 30$

$x = 90, 210$ $\quad x = 130, 250, 10$

$x = 90°, 130°, 10°$

9.a) $SA = 2\pi r^2 + \pi dh$

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi r \times \frac{450}{\pi r^2}$

$= 2\pi r^2 + \frac{900}{r}$ as required $= 2\pi r^2 + 900r^{-1}$

$\pi r^2 h = 450$

$h = \frac{450}{\pi r^2}$

b) $A'(r) = 4\pi r - 900r^{-2} = 0$

$4r(\pi - 225r^{-3}) = 0$

$4r = 0$ $\quad \pi - \frac{225}{r^3} = 0$

$r \neq 0$

$\frac{225}{r^3} = \pi$

$r^3\pi = 225$

$r^3 = \frac{225}{\pi}$

$r = \sqrt[3]{\frac{225}{\pi}}$

$r = 4.15cm$

x	$3 \rightarrow 4.15 \rightarrow 5$
A'	$- \quad 0 \quad +$
slope	$\backslash \quad - \quad /$

10a) $2\tan x \cos^2 x$

$= 2\frac{\sin x}{\cos x} \times \frac{\cos^2 x}{1}$

$= 2\sin x \cos x$

$= \sin 2x$

b) $6\tan x \cos^2 x = 3\sin 2x$

$\int 3\sin 2x \, dx$

$y = -\frac{3}{2}\cos 2x + c$

$3 = \frac{-3}{2}\cos(2 \times 0) + c$

$3 = \frac{-3}{2} \times 1 + c$

$c = 3/2$

$y = \frac{-3}{2}\cos 2x + 3/2$

Part B

14. Centre $(5,-1)$

$m_{radius} = \frac{-1-5}{5-3} = \frac{-6}{2} = -3$

$m_{tangent} = \frac{1}{3}$

$y-5 = \frac{1}{3}(x-3)$

$3y-15 = x-3$

$3y - x = 12$

15. $x^2 + (4-2x)^2 - 10x - 8(4-2x) + 1 = 0$

$x^2 + 16 + 4x^2 - 16x - 10x - 32 + 16x + 1 = 0$

$5x^2 - 10x - 15 = 0$

$5(x^2 - 2x - 3)$

$(x-3)(x+1)$

$x = 3 \quad x = -1$

$y = 4-2(3) \quad y = 4-2(-1)$

$= 2 \quad = 6$

$(3,2) \quad (-1,6)$

16. $y = ab^x$

$\log_8 y = \log_8 ab^x$

$= \log_8 a + \log_8 b^x$

$\log_8 y = \log_8 a + x\log_8 b$

$y = mx + c$

$\log_8 a = 2 \Rightarrow 8^2 = a$

$\log_8 b = \frac{4-2}{6-0} = \frac{2}{6} = \frac{1}{3}$

$\Rightarrow 8^{\frac{1}{3}} = b$

$a = 64 \quad b = 2$