Higher Maths 2019
Paper 1

(1) $\frac{dy}{dx} = 2x^3 - 6x^2$

For S.P's $\frac{dy}{dx} = 0$,

$$2x^3 - 6x^2 = 0$$

$$2x^2(x - 3) = 0$$

$2x^2 = 0$ $\qquad$ $x - 3 = 0$

$\underline{\underline{x = 0}}$ $\qquad$ $\underline{\underline{x = 3}}$

(2) For equal roots, $b^2 - 4ac = 0$

$$(k-5)^2 - 4 \times 1 \times 1 = 0$$

$$k^2 - 10k + 25 - 4 = 0$$

$$k^2 - 10k + 21 = 0$$

$$(k-7)(k-3) = 0$$

$k - 7 = 0$ $\qquad$ $k - 3 = 0$

$\underline{\underline{k = 7}}$ $\qquad$ $\underline{\underline{k = 3}}$

(3) $\underline{C_1}$

$$r = \sqrt{(-3)^2 + (-1)^2 - (-26)}$$

$$= \sqrt{9 + 1 + 26}$$

$$= \sqrt{36}$$

$$= 6$$

$\underline{C_2}$ $(x-4)^2 + (y+2)^2 = 36$

(4) (a)

$$11 = 9m + c$$

$$9 = 6m + c$$

$$2 = 3m$$

$$\frac{2}{3} = m$$

$$9 = 6\left(\frac{2}{3}\right) + c$$

$$9 = 4 + c$$

$$5 = c$$

$m = \frac{2}{3}$, $c = 5$

(b) $U_4 = \frac{2}{3} \times 11 + 5$

$$= \frac{22}{3} + \frac{15}{3}$$

$$= \frac{37}{3}$$

(5) (a) $\vec{AB} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 8 \\ -9 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix}$

$\vec{AB}$ is parallel to $\vec{BC}$ since $\vec{AB} = \frac{3}{4}\vec{BC}$

and since B is a common point, A, B and C are collinear.

(b) 3 : 4

(6) $y = \dfrac{1}{(1-3x)^5}$

$= (1-3x)^{-5}$

$\dfrac{dy}{dx} = -5(1-3x)^{-6} \times (-3)$

$= 15(1-3x)^{-6}$

(7) $m = \tan 30^\circ$

$= \dfrac{1}{\sqrt{3}}$

$m_\perp = -\sqrt{3}$

$y + 4 = -\sqrt{3}(x - 0)$

$y + 4 = -\sqrt{3}x$

$y = -\sqrt{3}x - 4$

8 (a) $\int_{-1}^{2} \left(x^2+2x+3-(2x^2+x+1)\right) dx$

$\int_{-1}^{2} (-x^2+x+2)\, dx$

(b) $\int_{-1}^{2} (-x^2+x+2)\, dx$

$= \left[-\frac{x^3}{3}+\frac{x^2}{2}+2x\right]_{-1}^{2}$

$= \left(-\frac{2^3}{3}+\frac{2^2}{2}+2(2)\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2(-1)\right)$

$= \left(-\frac{8}{3}+2+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$

$= \left(-2\frac{2}{3}+6\right)-\left(\frac{5}{6}-2\right)$

$= 3\frac{1}{3}-\left(-1\frac{1}{6}\right)$

$= 3\frac{2}{6}+1\frac{1}{6}$

$= 4\frac{1}{2}$

9 (a) (i)

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = (p(2p+16)) + ((-2)\times(-3)) + (4\times 6)$$

$$= 2p^2 + 16p + 6 + 24$$

$$= 2p^2 + 16p + 30$$

(ii)

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$$

$$2p^2 + 16p + 30 = 0$$

$$2(p^2 + 8p + 15) = 0$$

$$2(p+3)(p+5) = 0$$

$p+3=0$ $\quad$ $p+5=0$

$p = -3$ $\quad$ $p = -5$

(b) $3\underset{\sim}{u} = 2\underset{\sim}{v}$

$$3p = 2(2p+16)$$

$$3p = 4p + 32$$

$$-p = 32$$

$$p = -32$$

(10) (a) $a = 3$ (b) $k = -2$

(11) $$\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$$

$$= \left[\sin\left(3x - \frac{\pi}{6}\right) \times \frac{1}{3} \right]_0^{\frac{\pi}{9}}$$

$$= \left(\frac{1}{3} \sin\left(3\left(\frac{\pi}{9}\right) - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(3(0) - \frac{\pi}{6}\right)\right)$$

$$= \left(\frac{1}{3} \times \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right)\right)$$

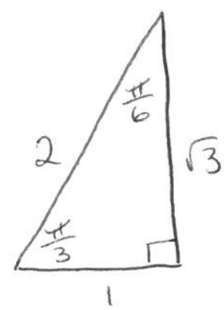
$$= \left(\frac{1}{3} \sin\left(\frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right)\right)$$

$$= \left(\frac{1}{3} \times \frac{1}{2}\right) - \left(\frac{1}{3} \times \left(-\frac{1}{2}\right)\right)$$

$$= \frac{1}{6} - \left(-\frac{1}{6}\right)$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$



(12) (a) $f(5-x)$

$$= \frac{1}{\sqrt{5-x}}$$

(b) Undefined when:

$$5 - x \leqslant 0$$

$$-x \leqslant -5$$

$$x \geqslant 5$$

(13) (a) (i) $AC^2 = (\sqrt{5})^2 - 1^2$

$$= 5 - 1$$

$$= 4$$

$$AC = 2$$

$$AD^2 = (\sqrt{10})^2 - 1^2$$

$$= 10 - 1$$

$$= 9$$

$$AD = 3$$

$$\cos p = \frac{2}{\sqrt{5}} \qquad \cos q = \frac{3}{\sqrt{10}}$$

(b) $\sin(p+q) = \sin p \cos q + \cos p \sin q$

$$= \left(\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}}\right) + \left(\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}\right)$$

$$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{25}\sqrt{2}}$$

$$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(14) (a) $\log_{10} 4 + 2\log_{10} 5$

$= \log_{10} 4 + \log_{10} 5^2$

$= \log_{10} 4 + \log_{10} 25$

$= \log_{10} 100$

$= 2$

(b) $\log_2 (7x-2) - \log_2 3 = 5$

$$\log_2 \left(\frac{7x-2}{3}\right) = 5$$

$$\frac{7x-2}{3} = 2^5$$

$$\frac{7x-2}{3} = 32$$

$$7x - 2 = 96$$

$$7x = 98$$

$$x = 14$$

(15) (a)

$$\sin 2x + 6\cos x = 0$$

$$2\sin x \cos x + 6\cos x = 0$$

$$2\cos x(\sin x + 3) = 0$$

$2\cos x = 0$

$\cos x = 0$

$x = 90^\circ, 270^\circ$

$\sin x + 3 = 0$

$\sin x = -3$

$x =$ No solutions

(b) $\sin 4x + 6\cos 2x = 0$

$$2x = \overbrace{90^\circ, 270^\circ}^{+360}, 450^\circ, 630^\circ$$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

(16) (a)

$P(4, k)$

$C(1, -2)$

$$d = \sqrt{(4-1)^2 + (k-(-2))^2}$$

$$= \sqrt{3^2 + (k+2)^2}$$

$$= \sqrt{9 + k^2 + 4k + 4}$$

$$= \sqrt{k^2 + 4k + 13}$$

(b) radius = 5

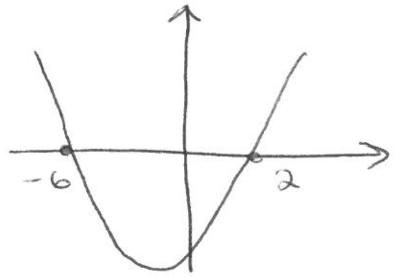
$$\sqrt{k^2+4k+13} > 5$$

$$k^2+4k+13 > 25$$

$$k^2+4k-12 > 0$$

$$(k+6)(k-2)$$

$k=-6 \quad k=2$



$$k^2+4k-12>0$$

when $\underline{\underline{k<-6\ ,\ k>2}}$

(17) (a)

$$(\sin x - \cos x)^2$$

$$= \sin^2 x - 2\sin x\cos x + \cos^2 x$$

$$= \sin^2 x + \cos^2 x - 2\sin x\cos x$$

$$= 1 - \sin 2x$$

(b) $\int (\sin x - \cos x)^2 \, dx$

$$= \int (1 - \sin 2x)\, dx$$

$$= x + \tfrac{1}{2}\cos 2x + C$$

Higher Maths 2019
Paper 2

1 (a) $M_{AC} = \left(\frac{-5+(-3)}{2}, \frac{(-12)+6}{2}\right) = (-4, -3)$

$m_{BM} = \frac{-3-(-8)}{-4-11} = \frac{5}{-15} = -\frac{1}{3}$

$y + 3 = -\frac{1}{3}(x+4)$

$3y + 9 = -1(x+4)$

$3y + 9 = -x - 4$

$3y = -x - 13$

(b) $m_{BC} = \frac{6-(-8)}{-3-11} = \frac{14}{-14} = -1$

$m_{\perp} = 1$

$y + 12 = 1(x+5)$

$y = x - 7$

(c) $3y + x = -13$

$y - x = -7$

$4y = -20$

$y = -5$

$-5 - x = -7$

$-x = -2$

$x = 2$

$(2, -5)$

(2) $\int (6\sqrt{x} - 4x^{-3} + 5)\,dx$

$= \int (6x^{1/2} - 4x^{-3} + 5)\,dx$

$= \dfrac{6x^{3/2}}{3/2} - \dfrac{4x^{-2}}{-2} + 5x + C$

$= \dfrac{12x^{3/2}}{3} + 2x^{-2} + 5x + C$

$= 4x^{3/2} + 2x^{-2} + 5x + C$

$= 4\sqrt{x^3} + \dfrac{2}{x^2} + 5x + C$

(3) (a) $\overrightarrow{BE} = -\underset{\sim}{p} + \underset{\sim}{r}$ (b) $\overrightarrow{EF} = -\underset{\sim}{r} + \underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q}$

(4) (a) $a = 0.973$

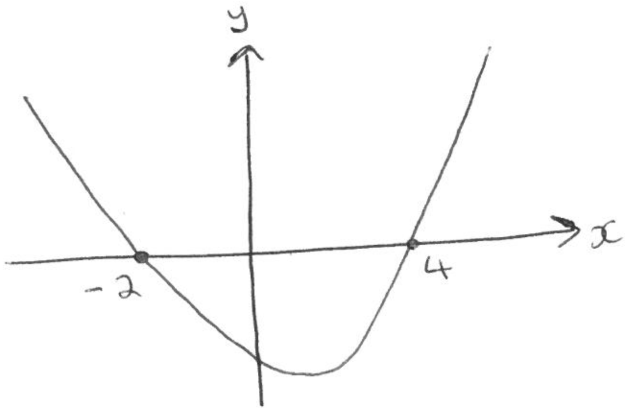
$b = 30$

(b) (i) A limit exists since $-1 < 0.973 < 1$

(ii) $L = \dfrac{30}{1 - 0.973} = 1111.11\ldots$

$= 1100$

(5)



(6) (a) $2\cos x - 3\sin x = k\cos(x+a)$

$= k\cos x\cos a - k\sin x\sin a$

$k\cos a = 2$

$k\sin a = 3$

$k = \sqrt{2^2+3^2}$

$= \sqrt{13}$

$\tan a = \frac{3}{2}$

$a = 56.3^\circ$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$\sqrt{13}\cos(x+56.3)^\circ$

(b) $\sqrt{13}\cos(x+56.3)^\circ = 3$

$\cos(x+56.3)^\circ = \frac{3}{\sqrt{13}}$

$x+56.3 = 33.7^\circ, 326.3^\circ, 393.7^\circ$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$x = \cancel{-22.6^\circ}, 270.0^\circ, 337.4^\circ$

(7) (a) $-6x^2+24x-25$

$= -6[x^2-4x]-25$

$= -6[(x-2)^2-4]-25$

$= -6(x-2)^2+24-25$

$= -6(x-2)^2-1$

(b) $f(x) = -2x^3 + 12x^2 - 25x + 9$

$f'(x) = -6x^2 + 24x - 25$

$= -6(x-2)^2 - 1$

$(x-2)^2 \geq 0$ for all values of x,

therefore $-6(x-2)^2 - 1 < 0$ for all values of x;

therefore $f(x)$ is strictly decreasing for all $x \in R$.

8 (a) $f(x) = \sqrt[3]{x} + 8$

$f(x) - 8 = \sqrt[3]{x}$

$(f(x) - 8)^3 = x$

$f^{-1}(x) = (x-8)^3$

(b) Domain of $f(x)$ is $1 \leq x \leq 1000$

Range of $f(x)$ is: $x=1$, $\sqrt[3]{1} + 8 = 9$

$x=1000$, $\sqrt[3]{1000} + 8 = 18$

$\therefore$ Domain of $f^{-1}(x)$ is $9 \leq x \leq 18$

(9) (a) $P_0 = 120e^{-0.0079\times 0}$

$= 120 \times e^0$

$= 120$

(b) $0.85 \times 120 = 102$

$102 = 120e^{-0.0079t}$

$\frac{102}{120} = e^{-0.0079t}$

$\ln 0.85 = \ln e^{-0.0079t}$

$\ln 0.85 = -0.0079t \ln e$

$t = \frac{\ln 0.85}{-0.0079}$

$t = 20.6$

(10) (a)

-3	3	10	1	-8	-6
		-9	-3	6	6
	3	1	-2	-2	0

Remainder = 0, $(x+3)$ is a factor.

(b) $(x+3)(3x^3+x^2-2x-2)=0$

1	3	1	-2	-2
		3	4	2
	3	4	2	0

$(x+3)(x-1)(3x^2+4x+2)$

b^2-4ac

$= 4^2-4\times3\times2$

$= -8$

No solutions.

$(x+3)(x-1)(3x^2+4x+2)$

11 (a)

$$\text{Surface Area} = \underbrace{4(3x \times h)}_{\text{Sides}} + \underbrace{2(3x \times 3x - x^2)}_{\text{Top \& Bottom}} + \underbrace{4(x \times h)}_{\text{insides}}$$

$$= 12xh + 16x^2 + 4xh$$

$$= 16xh + 16x^2$$

$$V = 2000$$

$$\underbrace{lbh}_{\text{Box}} - \underbrace{lbh}_{\text{space}} = 2000$$

$$(3x \times 3x \times h) - (x \times x \times h) = 2000$$

$$8x^2 h = 2000$$

$$\boxed{h = \frac{2000}{8x^2}}$$

$$= 16x\left(\frac{2000}{8x^2}\right) + 16x^2$$

$$= \frac{32000x}{8x^2} + 16x^2$$

$$= \frac{4000}{x} + 16x^2$$

$$A = 16x^2 + \frac{4000}{x}$$

(b) $A = 16x^2 + 4000x^{-1}$

$$32x - 4000x^{-2} = 0$$

$$32x - \frac{4000}{x^2} = 0$$

$$32x^3 - 4000 = 0$$

$$32x^3 = 4000$$

$$x^3 = 125$$

$$x = 5$$

x	$\rightarrow$	5	$\rightarrow$
$A'(x)$	$-$	0	$+$
Shape	\	_	/

Minimum value of A occurs when $x = 5$,

$$A = 16(5^2) + \frac{4000}{5}$$

$$= 1200 \text{ cm}^2$$

(12)

$$y = ab^x$$

$$\log_4 y = \log_4 ab^x$$

$$\log_4 y = \log_4 a + \log_4 b^x$$

$$\log_4 y = \underline{x \log_4 b} + \underline{\log_4 a}$$

$$Y = mx + c$$

$$\log_4 a = c$$

$$\log_4 a = -1$$

$$a = 4^{-1}$$

$$a = \frac{1}{4}$$

$$\log_4 b = m$$

$$\log_4 b = \frac{8-(-1)}{3-0}$$

$$\log_4 b = \frac{9}{3}$$

$$\log_4 b = 3$$

$$b = 4^3$$

$$b = 64$$

(13)

$$f(x) = \int (3x^2 - 16x + 11)\, dx$$

$$= x^3 - 8x^2 + 11x + c$$

x y
(7, 0)

$$0 = 7^3 - 8(7)^2 + 11(7) + c$$

$$0 = 343 - 392 + 77 + c$$

$$0 = 28 + c$$

$$-28 = c$$

$$f(x) = x^3 - 8x^2 + 11x - 28$$

(14) $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v}) = 21$

$\underset{\sim}{u} \cdot \underset{\sim}{u} + \underset{\sim}{u} \cdot \underset{\sim}{v} = 21$

$|\underset{\sim}{u}||\underset{\sim}{u}|\cos\theta + |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = 21$

$4 \times 4 \times \cos\theta + 4 \times 5 \times \cos\theta = 21$

$16 + 20\cos\theta = 21$

$20\cos\theta = 5$

$\cos\theta = \frac{5}{20}$

$\theta = 75.5°$

(15) (a) $m_{cp} = \frac{13-12}{5-8}$

$= -\frac{1}{3}$

$m_\perp = 3$

$y - 13 = 3(x-5)$

$y - 13 = 3x - 15$

$y = 3x - 2$

(b)(i) $T(0, -2)$

$y = 3 \times 0 - 2 = -2$

(ii) CT is the diameter.

- $M_{CT} = \left(\frac{8+0}{2}, \frac{12+(-2)}{2}\right) = (4, 5)$
- $d_{MC} = \sqrt{(4-8)^2 + (5-12)^2}$

$= \sqrt{16 + 49}$

$= \sqrt{65}$

$(x-4)^2 + (y-5)^2 = 65$