2018 Paper 1 Higher

(1) $M_{PQ} = (1 \quad , \quad 2)$

$$m_{RM} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

$$y - 6 = 2(x-3)$$
$$y - 6 = 2x - 6$$
$$y = 2x$$

(2)

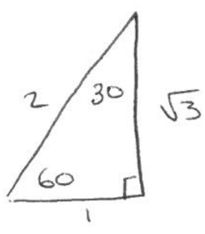
$$\frac{1}{5}x - 4 = g(x)$$
$$\frac{1}{5}x = g(x) + 4$$
$$x = 5g(x) + 20$$

$$g^{-1}(x) = 5x + 20$$

(3)

$$h'(x) = -3\sin 2x \times 2$$
$$= -6\sin 2x$$

$$h'\left(\frac{\pi}{6}\right) = -6\sin 2\left(\frac{\pi}{6}\right)$$
$$= -6\sin\left(\frac{\pi}{3}\right)$$
$$= -6 \times \frac{\sqrt{3}}{2}$$
$$= \frac{-6\sqrt{3}}{2}$$
$$= -3\sqrt{3}$$



(4) $m_{radius} = \dfrac{-5-3}{8-6} = \dfrac{-8}{2} = -4$

$m_{tangent} = \frac{1}{4}$

$y + 5 = \frac{1}{4}(x - 8)$

$y + 5 = \frac{1}{4}x - 2$

$y = \frac{1}{4}x - 7$

(5) (a)

$\vec{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$= \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ t-4 \\ 12 \end{pmatrix}$

$\vec{BC} = \underset{\sim}{c} - \underset{\sim}{b}$

$= \begin{pmatrix} 7 \\ 9 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 9-t \\ 3 \end{pmatrix}$

B divides AC in the ratio 4:1

(b)

$t - 4 = 4(9 - t)$

$t - 4 = 36 - 4t$

$5t - 40 = 0$

$5t = 40$

$t = 8$

⑥ $\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{1/3}$

$= \log_5 250 - \log_5 \sqrt[3]{8}$

$= \log_5 250 - \log_5 2$

$= \log_5 125$

$= 3$

⑦ (a) when $x=0$,

$y = 0^3 - 3(0)^2 + 2(0) + 5$

$= 5$ $(0,5)$

(b)

- $\frac{dy}{dx} = 3x^2 - 6x + 2$

- $m = 3(0)^2 - 6(0) + 2$
  $= 2$

- $y - 5 = 2(x - 0)$
  $y - 5 = 2x$
  $y = 2x + 5$

(c)

$$x^3 - 3x^2 + 2x + 5 = 2x + 5$$

$$x^3 - 3x^2 = 0$$

$$x^2(x-3) = 0$$

$x^2 = 0$ $\quad$ $x - 3 = 0$

$x = 0$ $\quad$ $x = 3$

P $\quad$ Q

$$y = 2 \times 3 + 5$$
$$= 6 + 5$$
$$= 11$$

$Q(3, 11)$

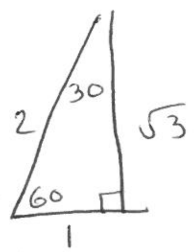
(8) $y = \sqrt{3}x - 5$

$\tan\theta = m$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = 60^\circ$



(9) (a) $\vec{BC} = -\underset{\sim}{t} + \underset{\sim}{u}$

(b) $\vec{MD}$

$= \vec{MC} + \vec{CA} + \vec{AD}$

$= \frac{1}{2}\vec{BC} + (-\underset{\sim}{u}) + \underset{\sim}{v}$

$= \frac{1}{2}(-\underset{\sim}{t} + \underset{\sim}{u}) - \underset{\sim}{u} + \underset{\sim}{v}$

$= -\frac{1}{2}\underset{\sim}{t} - \frac{1}{2}\underset{\sim}{u} + \underset{\sim}{v}$

(10)

$$y = \int (6x^2 - 3x + 4)\,dx$$

$$y = \frac{{}^{2}\cancel{6}x^3}{{}_{1}\cancel{3}} - \frac{3x^2}{2} + 4x + c$$

$$14 = 2(2)^3 - \frac{3(2)^2}{2} + 4(2) + c$$

$$14 = 16 - 6 + 8 + c$$

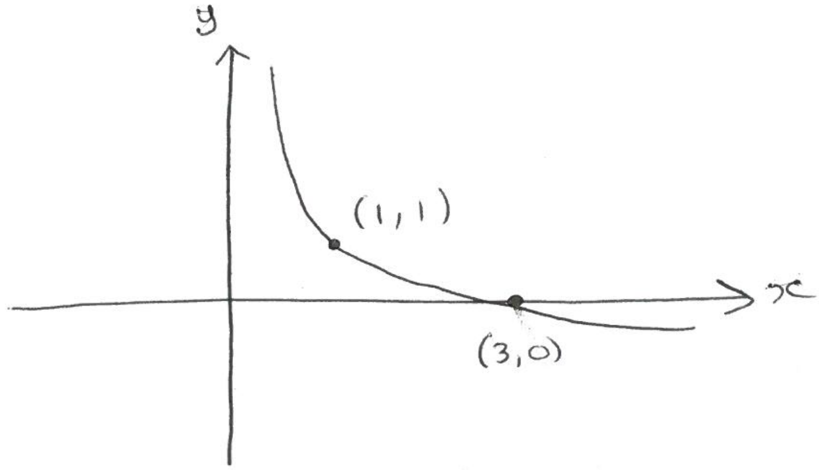
$$14 = 18 + c$$

$$-4 = c$$

$$y = 2x^3 - \frac{3x^2}{2} + 4x - 4$$

(11) (a) $y = 1 - \log_3 x$

move up 1 reflect over x-axis



(b)

$$\log_3 x = 1 - \log_3 x$$

$$2\log_3 x = 1$$

$$\log_3 x = \frac{1}{2}$$

$$x = 3^{1/2}$$

$$x = \sqrt{3}$$

(12) (a) $2\underline{a} + \underline{b}$

$$= 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$$

(b) $|2\underset{\sim}{a} + \underset{\sim}{b}| = 7$

$$\sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$$

$$36 + 9 + (4+p)(4+p) = 49$$

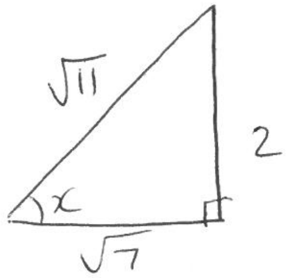
$$45 + 16 + 8p + p^2 = 49$$

$$p^2 + 8p + 12 = 0$$

$$(p+6)(p+2) = 0$$

$$p = -6 \quad , \quad p = -2$$

(13) (a) (i)



$$x^2 = (\sqrt{11})^2 - 2^2$$
$$= 11 - 4$$
$$x = \sqrt{7}$$

$$\sin 2x = 2\sin x \cos x$$
$$= 2 \times \frac{2}{\sqrt{11}} \times \frac{\sqrt{7}}{\sqrt{11}}$$
$$= \frac{4\sqrt{7}}{11}$$

(ii)

$$\cos 2x = 2\cos^2 x - 1$$
$$= 2\left(\frac{\sqrt{7}}{\sqrt{11}}\right)^2 - 1$$
$$= 2 \times \frac{7}{11} - 1$$
$$= \frac{14}{11} - 1$$
$$= \frac{3}{11}$$

(b)

$$\sin 3x = \sin(2x + x)$$
$$= \sin 2x \cos x + \cos 2x \sin x$$
$$= \left(\frac{4\sqrt{7}}{11} \times \frac{\sqrt{7}}{\sqrt{11}}\right) + \left(\frac{3}{11} \times \frac{2}{\sqrt{11}}\right)$$
$$= \frac{28}{11\sqrt{11}} + \frac{6}{11\sqrt{11}}$$
$$= \frac{34}{11\sqrt{11}}$$

14

$$\int_{-4}^{9} \frac{1}{(2x+9)^{2/3}} \, dx$$

$$= \int_{-4}^{9} (2x+9)^{-2/3} \, dx$$

$$= \left[\frac{(2x+9)^{1/3}}{\frac{1}{3}} \times \frac{1}{2} \right]_{-4}^{9}$$

$$= \left[\frac{\sqrt[3]{2x+9}}{2/3} \right]_{-4}^{9}$$

$$= \left[\frac{3\sqrt[3]{2x+9}}{2} \right]_{-4}^{9}$$

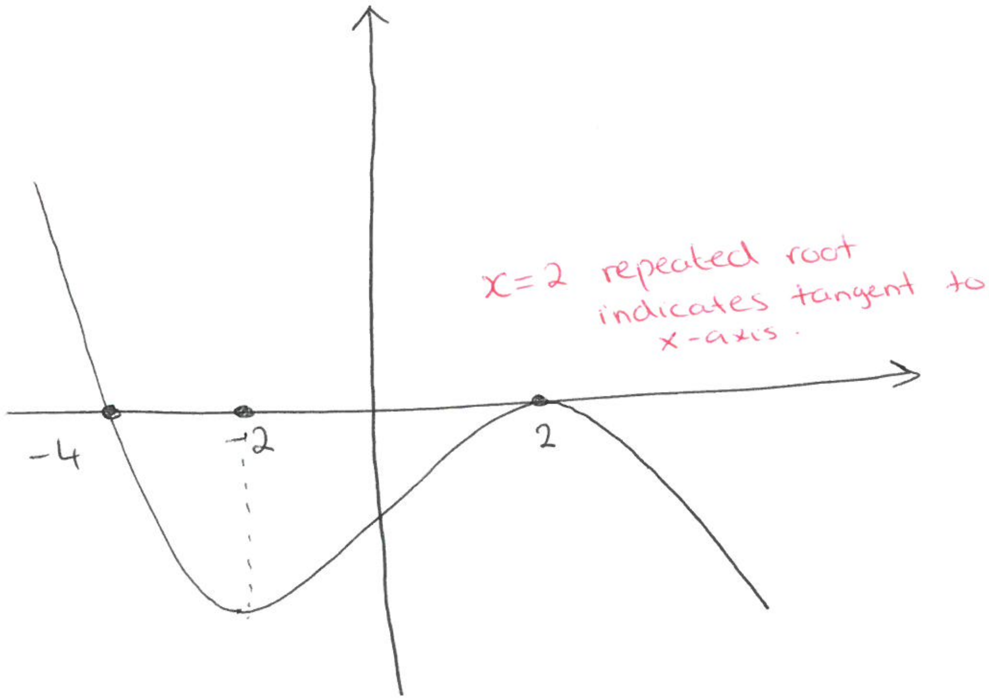
$$= \left(\frac{3\sqrt[3]{2\times 9+9}}{2} \right) - \left(\frac{3\sqrt[3]{2\times(-4)+9}}{2} \right)$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

15



$f'(-2)=0$

Stationary
when $x=-2$

2018 Paper 2 Higher

(1) $\int_{-1}^{3} (3 + 2x - x^2)\,dx$

$= \left[3x + \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3} \right]_{-1}^{3}$

$= \left(3 \times 3 + 3^2 - \frac{3^3}{3}\right) - \left(3(-1) + (-1)^2 - \frac{(-1)^3}{3}\right)$

$= (9) - \left(-3 + 1 - \frac{-1}{3}\right)$

$= 9 - \left(-1\frac{2}{3}\right)$

$= 10\frac{2}{3}$

(2) (a) $\underset{\sim}{u} \cdot \underset{\sim}{v} = (-1 \times (-7)) + (4 \times 8) + (-3 \times 5)$

$= 7 + 32 - 15$

$= 24$

(b) $|\underset{\sim}{u}| = \sqrt{(-1)^2 + 4^2 + (-3)^2} = \sqrt{26}$

$|\underset{\sim}{v}| = \sqrt{(-7)^2 + 8^2 + 5^2} = \sqrt{138}$

$\cos\theta = \frac{24}{\sqrt{26}\sqrt{138}}$

$\theta = \cos^{-1}\left(\frac{24}{\sqrt{26}\sqrt{138}}\right)$

$\theta = 66.4^\circ$

(3) $f'(x) = 3x^2 - 7$

$f'(2) = 3(2)^2 - 7 = 5$

f is increasing when $x = 2$.

(4) $-3x^2 - 6x + 7$

$= -3\left[x^2 + 2x\right] + 7$

$= -3\left[(x+1)^2 - 1\right] + 7$

$= -3(x+1)^2 + 3 + 7$

$= -3(x+1)^2 + 10$

(5) (a) $M_{PQ} = (6, 1)$

$m_{PQ} = \dfrac{-2-4}{9-3} = \dfrac{-6}{6} = -1$

$m_{\perp} = 1$

$y - 1 = 1(x - 6)$

$y - 1 = x - 6$

$y = x - 5$

(5) (b)

$$3y + x = 25$$
$$y - x = -5$$
$$4y = 20$$
$$y = 5$$

$$5 - x = -5$$
$$-x = -10$$
$$x = 10$$

$C(10, 5)$

(c)

$$\underset{(PC)}{d} = \sqrt{(3-10)^2 + (4-5)^2}$$
$$= \sqrt{(-7)^2 + (-1)^2}$$
$$= \sqrt{50}$$

$$(x-10)^2 + (y-5)^2 = 50$$

(6) (a) (i) $f(g(x)) = 3 + \cos 2x$

(ii) $g(f(x)) = 2(3 + \cos x) = 6 + 2\cos x$

(b)

$$3 + \cos 2x = 6 + 2\cos x$$

$$\cos 2x - 2\cos x - 3 = 0$$
$$2\cos^2 x - 1 - 2\cos x - 3 = 0$$
$$2\cos^2 x - 2\cos x - 4 = 0$$
$$2(\cos^2 x - \cos x - 2) = 0$$
$$2(\cos x + 1)(\cos x - 2) = 0$$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

$\cos x - 2 = 0$

$\cos x = 2$

No Solutions

7 (a) (i)

$$\begin{array}{c|cccc} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$$

$(x-2)$ is a factor since remainder $= 0$.

(ii) $(x-2)(2x^2+x-1)=0$

$(x-2)(2x-1)(x+1)=0$

(b) $U_6 = a(2a-3)-1 = 2a^2-3a-1$

$U_7 = a(2a^2-3a-1)-1 = 2a^3-3a^2-a-1$

(c) (i) $2a^3-3a^2-a-1 = 2a-3$

$2a^3-3a^2-3a+2=0$

$(a-2)(2a-1)(a+1)=0$

$a=2 \qquad 2a=1 \qquad a=-1$

$a=\frac{1}{2}$

For a limit to exist $-1<a<1$, therefore $a=\frac{1}{2}$.

(ii) $L = \dfrac{-1}{1-\frac{1}{2}} = -\dfrac{1}{\frac{1}{2}} = -2$

⑧ (a) $2\cos x - \sin x = k\cos(x-a)^\circ$

$= k\cos x\cos a + k\sin x\sin a$

$k\cos a = 2$

$k\sin a = -1$

$k = \sqrt{2^2 + (-1)^2}$

$= \sqrt{5}$

$\tan a = \dfrac{\sin a}{\cos a}$

$\tan a = \dfrac{-1}{2}$

$a = -26.6^\circ$

$a = 360 - 26.6$

$= 333.4^\circ$

$\sqrt{5}\cos(x - 333.4)^\circ$

(b) (i) min value $= -3\sqrt{5}$

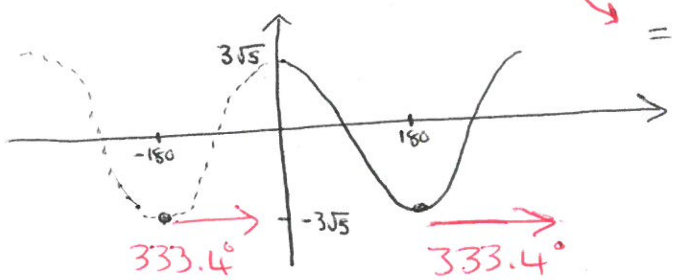
$6\cos x - 3\sin x$

$= 3(2\cos x - \sin x)$

$= 3(\sqrt{5}\cos(x - 333.4)^\circ)$

$= 3\sqrt{5}\cos(x - 333.4)^\circ$

(ii)



$x = -180 + 333.4 = \underline{\underline{153.4^\circ}}$

(9) $P = 2x + \frac{128}{x}$

$= 2x + 128x^{-1}$

For SP's, $\frac{dP}{dx} = 0$

$\frac{dP}{dx} = 2 - 128x^{-2} = 0$

$= 2 - \frac{128}{x^2} = 0$

$-\frac{128}{x^2} = -2$

$128 = 2x^2$

$64 = x^2$

$x = \pm 8$

x	-10	-8	1	8	10
$\frac{dP}{dx}$	+	0	−	0	+
shape	/	—	\	—	/

Min value occurs when $x = 8$

$P = 2(8) + \frac{128}{8} = 16 + 16 = 32$

Min value of $P = 32$.

(10) $b^2 - 4ac > 0$

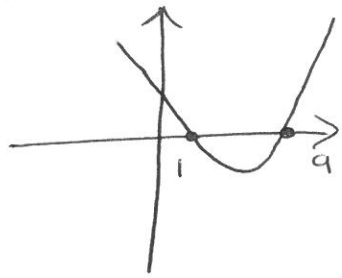
$(m-3)^2 - 4 \times 1 \times m > 0$

$m^2 - 6m + 9 - 4m > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1)$

$m = 9 \quad m = 1$



$m^2 - 10m + 9 > 0$

when:

$\underline{\underline{m < 1}}, \underline{\underline{m > 9}}$

(11) (a) $P = 100(1 - e^{kt})$

$50 = 100(1 - e^{3k})$

$0.5 = 1 - e^{3k}$

$-0.5 = -e^{3k}$

$e^{3k} = 0.5$

$\ln e^{3k} = \ln 0.5$

$3k \ln e = \ln 0.5$

$3k = \ln 0.5$

$k = \frac{\ln 0.5}{3}$

$k = -0.231$

(b) $P = 100(1 - e^{-0.231 \times 5})$

$= 68.5\ \%$ wait for <u>less</u> than 5 mins.

$100 - 68.5 = 31.5\ \%$ wait for 5 mins or longer.

(12) (a) (i) $(13, -4)$

(ii) $13^2 + (-4)^2 + 14(13) - 22(-4) + c = 0$

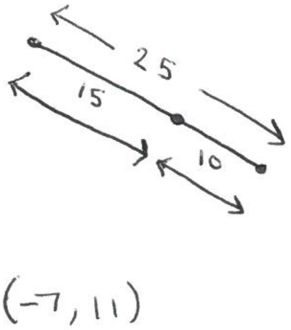
$169 + 16 + 182 + 88 + c = 0$

$455 + c = 0$

$c = -455$

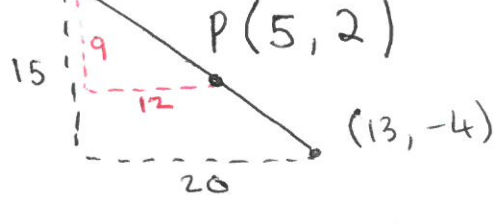
(b) (i)

Radius C_1	Radius C_2
10	$\sqrt{7^2 + (-11)^2 + 455}$
	$= \sqrt{625}$
	$= 25$

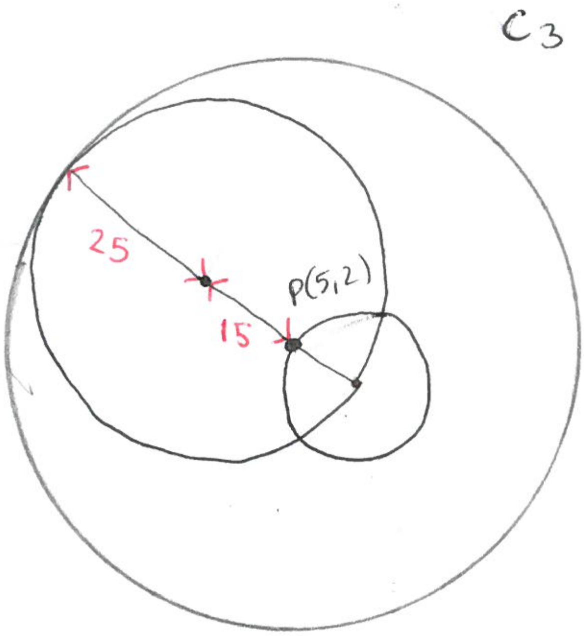


$15 : 10$

$3 : 2$

(ii) 

(12) (c)



Centre (5,2)

Radius $= 15 + 25$
$= 40$

$(x-5)^2 + (y-2)^2 = 40^2$

$(x-5)^2 + (y-2)^2 = 1600$