Higher 2017 Paper 1

① (a) $g(0) = 2\cos 0$

$= 2 \times 1$

$= 2$

$f(g(0)) = f(2)$

$= 5 \times 2$

$= 10$

(b) $g(f(x))$

$= g(5x)$

$= 2\cos 5x$

② Centre $(4, 3)$

$$m_{radius} = \frac{1-3}{-2-4} = \frac{-2}{-6} = \frac{1}{3}$$

$m_{tangent} = -3$

$y - 1 = -3(x+2)$

$y - 1 = -3x - 6$

$y = -3x - 5$

③ $$\frac{dy}{dx} = 12(4x-1)^{11} \times 4$$

$$= 48(4x-1)^{11}$$

(4) For equal roots,

$$b^2 - 4ac = 0$$

$$4^2 - 4 \times 1 \times (k-5) = 0$$

$$16 - 4k + 20 = 0$$

$$-4k = -36$$

$$k = 9$$

(5) (a)

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = (5 \times 3) + (1 \times (-8)) + (-1 \times 6)$$

$$= 15 - 8 - 6$$

$$= 1$$

(b) $|\underset{\sim}{u}| = \sqrt{5^2 + 1^2 + (-1)^2}$

$$= \sqrt{27}$$

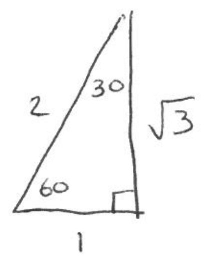
$$\underset{\sim}{u} \cdot \underset{\sim}{w} = |\underset{\sim}{u}|\,|\underset{\sim}{w}| \cos\theta$$

$$= \sqrt{27} \times \sqrt{3} \times \cos\frac{\pi}{3}$$

$$= \sqrt{81} \times \frac{1}{2}$$

$$= 9 \times \frac{1}{2}$$

$$= \frac{9}{2}$$



(6)

$$x^3 + 7 = h(x)$$

$$x^3 = h(x) - 7$$

$$x = \sqrt[3]{h(x) - 7}$$

$$h^{-1}(x) = \sqrt[3]{x - 7}$$

(7)

$$M_{AB} = \left(\frac{-3+7}{2}, \frac{5+9}{2}\right) = (2, 7)$$

$$m_{MC} = \frac{11-7}{2-2} = \frac{4}{0} = \text{undefined} \quad \text{(vertical)}$$

$$x = 2$$

(8)

$$d(t) = \frac{1}{2}t^{-1}$$

$$d'(t) = -\frac{1}{2}t^{-2}$$

$$= -\frac{1}{2t^2}$$

$$d'(5) = -\frac{1}{2\times5^2}$$

$$= -\frac{1}{50}$$

(9) (a)

$$U_2 = mU_1 + 6$$

$$13 = m\times28 + 6$$

$$7 = m\times28$$

$$\frac{7}{28} = m$$

$$\frac{1}{4} = m$$

(b) (i) A limit exists since $-1 < \frac{1}{4} < 1$

(ii)

$$L = \frac{6}{1-\frac{1}{4}}$$

$$= \frac{6}{3/4}$$

$$= \frac{24}{3}$$

$$= 8$$

(10) (a) $$\int_0^2 \left(x^3 - 4x^2 + 3x + 1 - (x^2 - 3x + 1)\right) dx$$

$$= \int_0^2 \left(x^3 - 5x^2 + 6x\right) dx$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{6x^2}{2}\right]_0^2$$

$$= \left(\frac{2^4}{4} - \frac{5(2)^3}{3} + 3(2)^2\right) - \left(\frac{0^4}{4} - \frac{5(0)^3}{3} + 3(0)^2\right)$$

$$= \left(\frac{16}{4} - \frac{40}{3} + 12\right) - 0$$

$$= 4 - 13\tfrac{1}{3} + 12$$

$$= 16 - 13\tfrac{1}{3}$$

$$= 2\tfrac{2}{3}$$

(10) (b)

$$\int_0^2 \left(1 - x - (x^2 - 3x + 1)\right) dx$$

$$= \int_0^2 (2x - x^2)\, dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^2$$

$$= \left(2^2 - \frac{2^3}{3}\right) - \left(0^2 - \frac{0^3}{3}\right)$$

$$= \left(4 - \frac{8}{3}\right) - 0$$

$$= 4 - 2\tfrac{2}{3}$$

$$= 1\tfrac{1}{3}$$

$\therefore$ Half the area is below the line, $\frac{1}{2}$.

(11)

$$3y = 2x + 4$$

$$y = \frac{2}{3}x + \frac{4}{3}$$

$$\uparrow$$

$$m = \frac{2}{3}$$

$$\frac{a-2}{5+7} = \frac{2}{3}$$

$$\frac{a-2}{12} = \frac{2}{3}$$

$$3a - 6 = 24$$

$$3a = 30$$

$$a = 10$$

(12)

$$\log_a 36 - \log_a 4 = \frac{1}{2}$$

$$\log_a 9 = \frac{1}{2}$$

$$9 = a^{1/2}$$

$$9 = \sqrt{a}$$

$$81 = a$$

(13)

$$\int (5-4x)^{-1/2} \, dx$$

$$= \frac{(5-4x)^{1/2}}{\frac{1}{2}} \times \frac{1}{-4} + C$$

$$= \frac{(5-4x)^{1/2}}{-2} + C$$

$$= -\frac{(5-4x)^{1/2}}{2} + C$$

(14) (a)

$$\sqrt{3}\sin x - \cos x = k\sin(x-a)$$

$$= k\sin x\cos a \quad k\cos x\sin a$$

$k\cos x = \sqrt{3}$

$k\sin x = 1$

$$k = \sqrt{(\sqrt{3})^2 + 1^2}$$

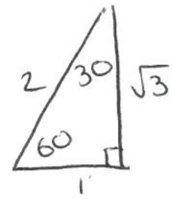
$$= \sqrt{3+1}$$

$$= \sqrt{4}$$

$$= 2$$

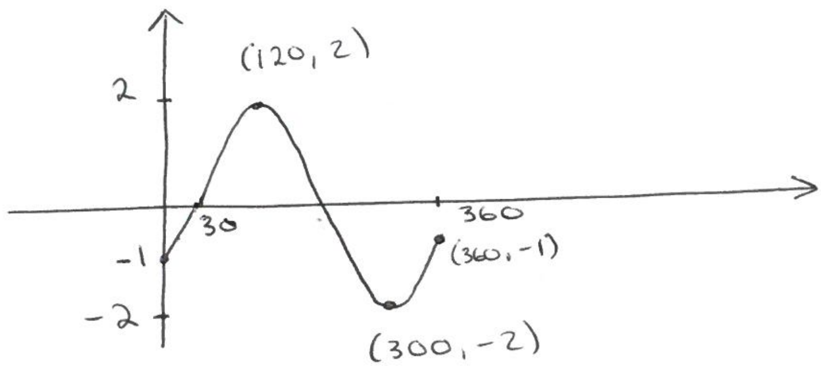
$$\tan a = \frac{1}{\sqrt{3}}$$

$$a = 30^\circ$$



$$2\sin(x-30)^\circ$$

(b)



y-Intercept:

$$y = 2\sin(0-30)^\circ$$

$$= 2\times\sin(-30)$$

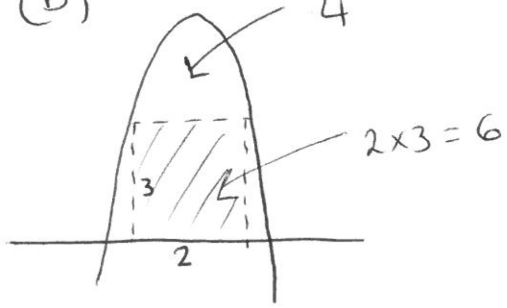
$$= 2\times\left(-\tfrac{1}{2}\right)$$

$$= -1$$

15 (a) $a = -5$ (moved right 5)

$b = 3$ (moved up 3)

(b)



$$\int_6^8 h(x)\,dx$$

$$= 10$$

(c) $f'(1) = 6$ i.e. the gradient when $x = 1$ on $f(x)$ is 6.

$h'(8) = -6$

Higher 2017 Paper 2

1 (a) $M_{BC} = (6, -1)$

$m_{BC} = \frac{-2-0}{9-3} = -\frac{2}{6} = -\frac{1}{3}$

$m_{\perp} = 3$

$y + 1 = 3(x - 6)$

$y + 1 = 3x - 18$

$y = 3x - 19$

(b) $m = \tan 45^\circ$

$= 1$

$y - 0 = 1(x - 3)$

$y = x - 3$

(c)

$y - 3x = -19$

$y - x = -3$

$-2x = -16$

$x = 8$

$y - 8 = -3$

$y = 5$

$(8, 5)$

(2) (a)

$$\begin{array}{r|rrrr} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & \boxed{0} \end{array}$$

Remainder $= 0$,
$(x-1)$ is a factor.

(b) $(x-1)(2x^2-3x-2)=0$

$(x-1)(2x+1)(x-2)=0$

$x=1$, $x=-\frac{1}{2}$, $x=2$

(3) $x^2-4x+4+y^2-2y+1=25$

$x^2+y^2-4x-2y-20=0$

$x^2+(3x)^2-4x-2(3x)-20=0$

$x^2+9x^2-4x-6x-20=0$

$10x^2-10x-20=0$

$10(x^2-x-2)=0$

$10(x-2)(x+1)=0$

$x=2$ $\quad$ $x=-1$

$y=3\times 2$ $\quad$ $y=3\times(-1)$
$=6$ $\quad$ $=-3$

$(2,6)$ $\quad$ $(-1,-3)$

(4) (a) $3x^2 + 24x + 50$

$= 3(x^2 + 8x) + 50$

$= 3\left[(x+4)^2 - 16\right] + 50$

$= 3(x+4)^2 - 48 + 50$

$= 3(x+4)^2 + 2$

(b) $f'(x) = 3x^2 + 24x + 50$

(c) $f'(x) = 3(x+4)^2 + 2$

$3(x+4)^2 + 2 \geqslant 0$ for all values of x

therefore $f(x)$ is strictly increasing for all values of x.

(5) (a) $\vec{PQ} = \vec{PR} + \vec{RQ}$

$= \begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -12 \\ -9 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$

$= -3\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$

(b) $\vec{PS} = \vec{PR} + \frac{2}{3}\vec{RQ}$

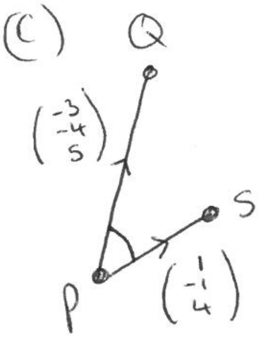
$$= \begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -12 \\ -9 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -8 \\ -6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

$$= \underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$$

(c)



$|\vec{PQ}| = \sqrt{(-3)^2 + (-4)^2 + 5^2} = \sqrt{50}$

$|\vec{PS}| = \sqrt{1^2 + (-1)^2 + 4^2} = \sqrt{18}$

$\vec{PQ} \cdot \vec{PS} = (-3 \times 1) + (-4 \times (-1)) + (5 \times 4)$

$= -3 + 4 + 20$

$= 21$

$$\cos Q\hat{P}S = \frac{21}{\sqrt{50} \times \sqrt{18}}$$

$$Q\hat{P}S = \cos^{-1}\left(\frac{21}{\sqrt{50} \times \sqrt{18}}\right)$$

$$Q\hat{P}S = 45.6^\circ$$

(6) $5\sin x - 4 = 2\cos 2x$

$-2\cos 2x + 5\sin x - 4 = 0$

$-2(1 - 2\sin^2 x) + 5\sin x - 4 = 0$

$-2 + 4\sin^2 x + 5\sin x - 4 = 0$

$4\sin^2 x + 5\sin x - 6 = 0$

$(4\sin x - 3)(\sin x + 2) = 0$

$4\sin x - 3 = 0$

$\sin x = 3/4$

$x = 0.85, 2.29$

$\pi - x$

S	A
T	C

$\sin x + 2 = 0$

$\sin x = -2$

No solutions

(7) (a) $y = 6x - 2x^{3/2}$

$\frac{dy}{dx} = 6 - 3x^{1/2}$

$= 6 - 3\sqrt{x}$

For SP's, $\frac{dy}{dx} = 0$,

$6 - 3\sqrt{x} = 0$

$-3\sqrt{x} = -6$

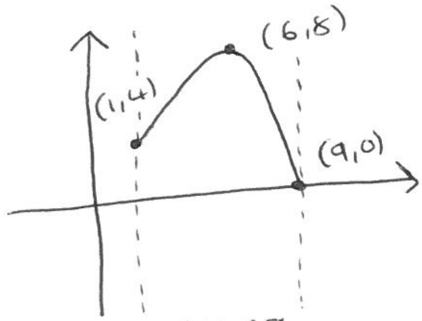
$\sqrt{x} = 2$

$x = 4$

(b) $y = 6(1) - 2\sqrt{1^3} = 4$ $(1,4)$

$y = 6(4) - 2\sqrt{4^3} = 8$ $(6,8)$

$y = 6(9) - 2\sqrt{9^3} = 0$ $(9,0)$



$1 \leq x \leq 9$

- Greatest value $= 8$
- Least value $= 0$

(8) (a) $U_0 = 5$

$U_1 = 5k - 20$

$U_2 = k(5k-20) - 20$

$= 5k^2 - 20k - 20$

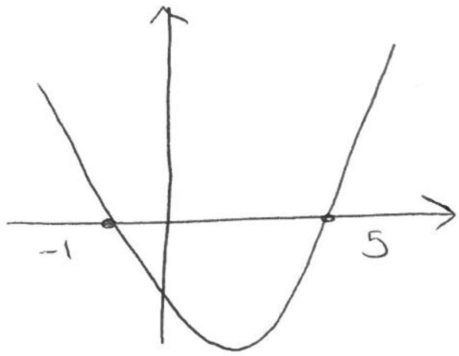
(b) $U_2 < U_0$

$5k^2 - 20k - 20 < 5$

$5k^2 - 20k - 25 < 0$

$5(k^2 - 4k - 5)$

$5(k+1)(k-5)$



$5k^2 - 20k - 25 < 0$

when: $\underline{\underline{-1 < k < 5}}$

(9) $\log_2 y = \log_2 kx^n$

$\log_2 y = \log_2 k + \log_2 x^n$

$\log_2 y = n \log_2 x + \log_2 k$

$Y = m x + c$

$n = m = \dfrac{3-0}{0-(-12)}$

$= \dfrac{3}{12}$

$= \dfrac{1}{4}$

$\log_2 k = c$

$\log_2 k = 3$

$k = 2^3$

$k = 8$

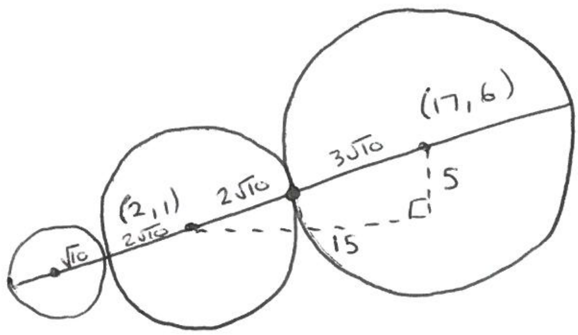
10 (a)

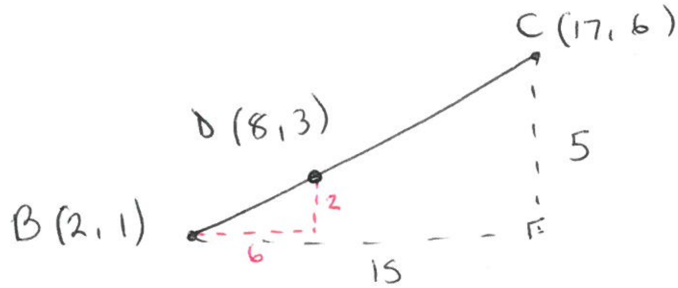
$$m_{AB} = \frac{1-(-2)}{2-(-7)} = \frac{3}{9} = \frac{1}{3}$$

$$m_{BC} = \frac{6-1}{17-2} = \frac{5}{15} = \frac{1}{3}$$

AB and BC are parallel and since B is a common point, A, B and C are collinear.

(b)





D is $^2/_5$ of the way up BC.

Centre (8, 3) radius $= 6\sqrt{10}$

$$(x-8)^2 + (y-3)^2 = (6\sqrt{10})^2$$

$$(x-8)^2 + (y-3)^2 = 360$$

11 (a) $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$

$= \dfrac{2\sin x \cos x}{2\cos x} - \sin x \cos^2 x$

$= \sin x - \sin x \cos^2 x$

$= \sin x (1 - \cos^2 x)$

$= \sin x (\sin^2 x)$

$= \sin^3 x$

(b) $\dfrac{d}{dx} \left(\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x \right)$

$= \dfrac{d}{dx} (\sin^3 x)$

$= \dfrac{d}{dx} (\sin x)^3$

$= 3\sin^2 x \times \cos x$

$= 3\sin^2 x \cos x$