New Higher 2016 Paper 1

① $y = -4x + 7$

$m = -4$

$y - 3 = -4(x + 2)$

$y - 3 = -4x - 8$

$y = -4x - 5$

② $y = 12x^3 + 8x^{\frac{1}{2}}$

$\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$

③ (a) $U_4 = \frac{1}{3}(6) + 10 = 2 + 10 = 12$

(b) A limit exists since $-1 < \frac{1}{3} < 1$

(c) $L = \frac{1}{3}L + 10$ $(\times 3)$

$3L = L + 30$

$2L = 30$

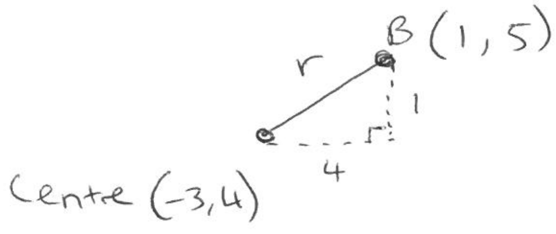
$L = 15$

④ Centre $= M_{AB} = \left(\frac{-7+1}{2}, \frac{3+5}{2}\right) = \left(\frac{-6}{2}, \frac{8}{2}\right) = (-3, 4)$

$r^2 = 4^2 + 1^2$

$= 16 + 1$

$r = \sqrt{17}$



$(x + 3)^2 + (y - 4)^2 = 17$

(5) $\int 8\cos(4x+1)\,dx$

$= \frac{1}{4} \times 8\sin(4x+1) + c$

$= 2\sin(4x+1) + c$

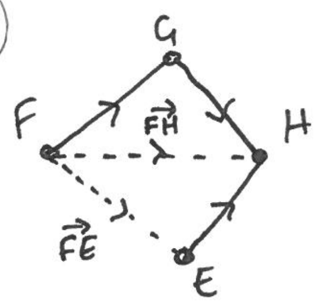
(6) (a) $f(x) = 3x+5$

$f^{-1}(x) = \dfrac{x-5}{3}$

(b) $g(2) = 7$

$g^{-1}(7) = 2$

(7)



(a) $\vec{FH} = \vec{FG} + \vec{GH}$

$= \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

$\vec{FH} = \underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$

(b) $\vec{FE} = \vec{FH} - \vec{EH}$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix}$

$\vec{FE} = -\underset{\sim}{i} - 5\underset{\sim}{k}$

(8) $x^2 + (3x-5)^2 + 2x - 4(3x-5) - 5 = 0$

$x^2 + 9x^2 - 30x + 25 + 2x - 12x + 20 - 5 = 0$

$10x^2 - 40x + 40 = 0$

$10(x^2 - 4x + 4) = 0$

$10(x-2)(x-2) = 0$

Equal roots, only 1 point of contact, the line is a tangent to the circle.

$x - 2 = 0$
$x = 2$

$y = 3(2) - 5$
$= 6 - 5$
$= 1$

$\underline{\underline{(2, 1)}}$

⑨ (a) $f(x) = x^3 + 3x^2 - 24x$

for SPs $f'(x) = 0$,

$$3x^2 + 6x - 24 = 0$$

$$3(x^2 + 2x - 8) = 0$$

$$3(x+4)(x-2) = 0$$

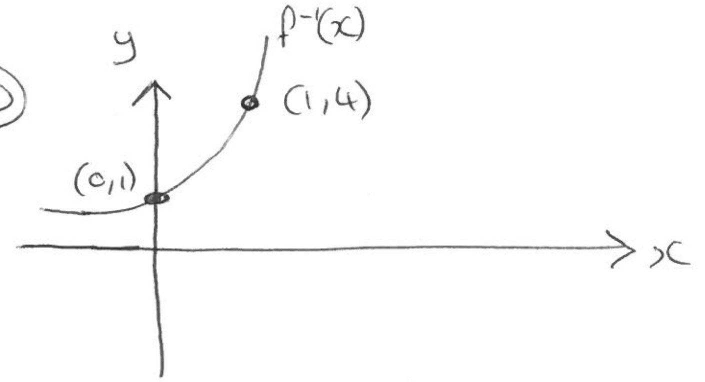
$$x = -4 \qquad x = 2$$

(b)

x	$\rightarrow$	-4	$\rightarrow$	2	$\rightarrow$
$f'(x)$	$+$	0	$-$	0	$+$
Shape	/	—	\	—	/

Increasing when $x < -4$ and $x > 2$.

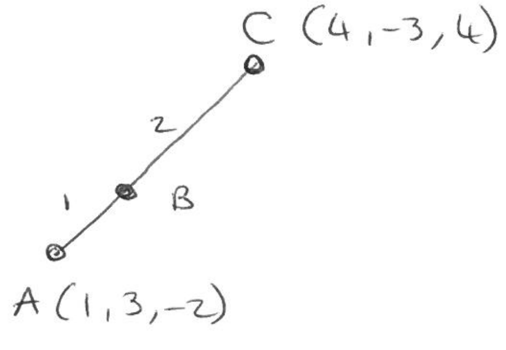
⑩



⑪

$1 : 2$

$\overrightarrow{AB} : \overrightarrow{BC}$



$$2\overrightarrow{AB} = \overrightarrow{BC}$$

$$2(\underset{\sim}{b} - \underset{\sim}{a}) = (\underset{\sim}{c} - \underset{\sim}{b})$$

$$2\underset{\sim}{b} - 2\underset{\sim}{a} = \underset{\sim}{c} - \underset{\sim}{b}$$

$$3\underset{\sim}{b} = 2\underset{\sim}{a} + c$$

$$3\underset{\sim}{b} = 2\begin{pmatrix}1\\3\\-2\end{pmatrix} + \begin{pmatrix}4\\-3\\4\end{pmatrix} = \begin{pmatrix}2\\6\\-4\end{pmatrix} + \begin{pmatrix}4\\-3\\4\end{pmatrix} = \begin{pmatrix}6\\3\\0\end{pmatrix} \qquad \underset{\sim}{b} = \begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$B = (2,1,0)$$

(11) (b) $\vec{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$$= \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$$

$$|\vec{AC}| = \sqrt{3^2 + (-6)^2 + 6^2}$$

$$= \sqrt{9 + 36 + 36}$$

$$= \sqrt{81}$$

$$= 9$$

$k = \frac{1}{9}$

(12) (a) $h(x) = f(3-x)$

$$= 2(3-x)^2 - 4(3-x) + 5$$

$$= 2(9 - 6x + x^2) - 12 + 4x + 5$$

$$= 18 - 12x + 2x^2 - 12 + 4x + 5$$

$$= 2x^2 - 8x + 11$$

(b) $h(x) = [2x^2 - 8x] + 11$

$$= 2[x^2 - 4x] + 11$$

$$= 2[(x-2)^2 - 4] + 11$$

$$= 2(x-2)^2 - 8 + 11$$

$$= 2(x-2)^2 + 3$$

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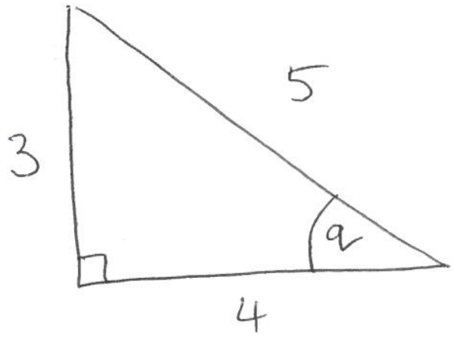
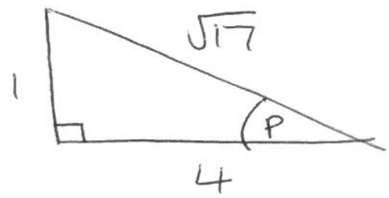




$$\cos(q-p) = \cos q \cos p + \sin q \sin p$$

$$= \left(\frac{4}{5} \times \frac{4}{\sqrt{17}}\right) + \left(\frac{3}{5} \times \frac{1}{\sqrt{17}}\right)$$

$$= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}}$$

$$= \frac{19}{5\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$$

$$= \frac{19\sqrt{17}}{5 \times 17}$$

$$= \frac{19\sqrt{17}}{85}$$

14 (a) $\log_5 25 = 2$ $\quad (5^2 = 25)$

(b) $\log_4 x + \log_4 (x-6) = \log_5 25$

$$\log_4 x(x-6) = 2$$

$$x(x-6) = 4^2$$

$$x^2 - 6x = 16$$

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

$\underline{\underline{x = 8}}$ or $\cancel{x = -2}$ since $x > 6$.

(15) (a) $a = 4$, $b = -5$

$$f(x) = k(x-4)(x+5)^2$$

$$f(x) = k(x-4)(x+5)(x+5)$$ $\overset{x\ \ y}{(1, 9)}$

$$9 = k(1-4)(1+5)(1+5)$$

$$9 = k(-3)(6)(6)$$

$$9 = -108k$$

$$k = -\frac{9}{108}$$

$$k = -\frac{1}{12}$$

(b) $d > 9$

New Higher 2016 Paper 2

1 (a) (i) $M_{QR} = \left(\frac{-6+10}{2}, \frac{2+6}{2}\right) = (2, 4)$

(ii)

$$m_{PM} = \frac{4-(-4)}{2-0} = \frac{8}{2} = 4$$

$$y - 4 = 4(x-2)$$
$$y - 4 = 4x - 8$$
$$y = 4x - 4$$

(b) $$m_{PR} = \frac{6-(-4)}{10-0} = \frac{10}{10} = 1$$

$$m_{\perp} = -1$$

$$y - 4 = -1(x-2)$$
$$y - 4 = -x + 2$$
$$y = -x + 6$$

(c) $M_{PR} = \left(\frac{0+10}{2}, \frac{-4+6}{2}\right) = (\underset{x}{5}, \underset{y}{1})$

$$y = -x + 6$$
$$y = -5 + 6$$
$$y = 1$$

The point (5, 1) satisfies the equation, therefore L passes through the midpoint of PR.

(2) $b^2 - 4ac < 0$ for no real roots.

$$(-2)^2 - 4 \times 1 \times (3-p) < 0$$

$$4 - 4(3-p) < 0$$

$$4 - 12 + 4p < 0$$

$$-8 + 4p < 0$$

$$4p < 8$$

$$p < 2$$

(3) (a) (i)

$$\begin{array}{r|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & \boxed{0} \end{array}$$

$\therefore (x+1)$ is a factor.

(ii) $(x+1)(2x^2 - 11x + 14) = 0$

$(x+1)(2x-7)(x-2) = 0$

$x = -1, \ x = \frac{7}{2}, \ x = 2$

(b) (i) $A(-1, 0)$, $B(2, 0)$

(ii) $\int_{-1}^{2} (2x^3 - 9x^2 + 3x + 14)\, dx$

$$= \left[\frac{2x^4}{4} - \frac{9x^3}{3} + \frac{3x^2}{2} + 14x \right]_{-1}^{2}$$

$$= \left[\frac{x^4}{2} - 3x^3 + \frac{3x^2}{2} + 14x \right]_{-1}^{2}$$

$$= \left(\frac{2^4}{2} - 3(2)^3 + \frac{3(2)^2}{2} + 14(2) \right)$$

$$- \left(\frac{(-1)^4}{2} - 3(-1)^3 + \frac{3(-1)^2}{2} + 14(-1) \right)$$

$$= (8 - 24 + 6 + 28) - \left(\frac{1}{2} + 3 + \frac{3}{2} - 14 \right)$$

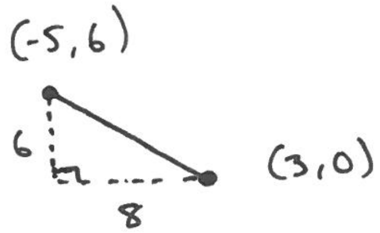
$$= (18) - (-9)$$

$$= 27$$

4 (a)

C_1	C_2
Centre $(-5, 6)$	Centre $(3, 0)$
radius $= \sqrt{9} = 3$	radius $= \sqrt{(-3)^2 + 0^2 - (-16)}$
	$= \sqrt{9 + 0 + 16}$
	$= \sqrt{25}$
	$= 5$

(b)



Distance between centres

$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

Sum of radii $= 3 + 5 = 8$

The circles do not intersect as the distance between the centres is greater than the sum of the radii.

⑤ (a) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$$= \begin{pmatrix} -10 \\ 18 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$$

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$$= \begin{pmatrix} -4 \\ -6 \\ 21 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$$

(b)

$$\cos B\hat{A}C = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|}$$

$$= -\frac{80}{18 \times 18}$$

$$B\hat{A}C = \cos^{-1}\left(-\frac{80}{324}\right)$$

$$= \underline{\underline{104.3^\circ}}$$

$\overrightarrow{AB} \cdot \overrightarrow{AC}$

$$= (-8 \times (-2)) + (16 \times (-8)) + (2 \times 16)$$

$$= 16 - 128 + 32$$

$$= -80$$

$$|\overrightarrow{AB}| = \sqrt{(-8)^2 + 16^2 + 2^2}$$

$$= \sqrt{324}$$

$$= 18$$

$$|\overrightarrow{AC}| = \sqrt{(-2)^2 + (-8)^2 + 16^2}$$

$$= \sqrt{324}$$

$$= 18$$

(6) (a) $B(t) = 200e^{0.107t}$

$B(0) = 200 \times e^{(0.107 \times 0)}$

$= 200 \times e^{0}$

$= 200 \times 1$

$= 200$

(b) $400 = 200e^{0.107t}$

$2 = e^{0.107t}$

$\ln 2 = \ln e^{0.107t}$

$\ln 2 = 0.107t \times \ln e$

$\ln 2 = 0.107t$

$t = \dfrac{\ln 2}{0.107}$

$t = 6.48$ hours.

(7) (a) $L = 9x + 8y$

$= 9x + 8\left(\frac{18}{x}\right)$

$L(x) = 9x + \dfrac{144}{x}$

$A = l \times b$

$108 = 3x \times 2y$

$2y = \dfrac{108}{3x}$

$2y = \dfrac{36}{x}$

$y = \dfrac{18}{x}$

(b)

$$L(x) = 9x + \frac{144}{x}$$

$$= 9x + 144x^{-1}$$

For SP's $L'(x) = 0$,

$$L'(x) = 9 - 144x^{-2}$$

$$9 - \frac{144}{x^2} = 0 \qquad (\times x^2)$$

$$9x^2 - 144 = 0$$

$$9(x^2 - 16) = 0$$

$$9(x+4)(x-4) = 0$$

$$x = -4, \quad x = 4$$

x	-5	-4	1	4	5
$f'(x)$	$+$	0	$-$	0	$+$
shape	/	—	\	_	/

The minimum length of fencing required is when $x = 4$.

(8) (a) $5\cos x - 2\sin x = k\cos(x + a)$

$= k(\cos x\cos a - \sin x\sin a)$

$= k\cos a\cos x - k\sin a\sin x$

$k\sin a = 2$

$k\cos a = 5$

$k = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$

$\tan a = \frac{2}{5}$

$a = 0.38$

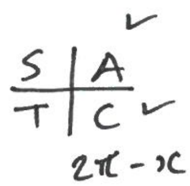
$5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.38)$

(b) $10 + 5\cos x - 2\sin x = 12$

$5\cos x - 2\sin x = 2$

$\sqrt{29}\cos(x + 0.38) = 2$

$\cos(x + 0.38) = \frac{2}{\sqrt{29}}$



$x + 0.38 = 1.19, 5.09$

$x = 0.81, 4.71$

(9) $f'(x) = \dfrac{2x+1}{\sqrt{x}} = \dfrac{2x}{x^{1/2}} + \dfrac{1}{x^{1/2}} = 2x^{1/2} + x^{-\frac{1}{2}}$

$f(x) = \int \left(2x^{1/2} + x^{-1/2}\right) dx$

$f(x) = \dfrac{2x^{3/2}}{3/2} + \dfrac{x^{1/2}}{1/2} + C$

$f(x) = \dfrac{4x^{3/2}}{3} + \dfrac{2x^{1/2}}{1} + C$

$f(x) = \dfrac{4\sqrt{x}^{3}}{3} + 2\sqrt{x} + C$

$\boxed{f(9) = 40}$

$40 = \dfrac{4\sqrt{9}^{3}}{3} + 2\sqrt{9} + C$

$40 = \dfrac{108}{3} + 6 + C$

$40 = 36 + 6 + C$

$40 = 42 + C$

$C = -2$

$f(x) = \dfrac{4\sqrt{x^{3}}}{3} + 2\sqrt{x} - 2$

10 (a) $y = (x^2+7)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+7)^{-\frac{1}{2}} \times 2x$$

$$= x(x^2+7)^{-\frac{1}{2}}$$

$$= \frac{x}{(x^2+7)^{\frac{1}{2}}}$$

$$= \frac{x}{\sqrt{x^2+7}}$$

(b) $\int \frac{4x}{\sqrt{x^2+7}} \, .dx$

$$= 4\int \frac{x}{\sqrt{x^2+7}} \, .dx$$

$$= 4(x^2+7)^{\frac{1}{2}} + C$$

11 (a) $\sin 2x \tan x$

$= 2\sin x \cos x \times \frac{\sin x}{\cos x}$

$= \frac{2\sin^2 x \cos x}{\cos x}$

$= 2\sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$= 1 - \cos 2x$

$\cos 2x + 2\sin^2 x = 1$

$2\sin^2 x = 1 - \cos 2x$

(b) $f(x) = \sin 2x \tan x$

$= 1 - \cos 2x$

$f'(x) = -(-2\sin 2x)$

$= 2\sin 2x$