New Higher 2015 Paper 1

(1) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$(8 \times (-3)) + (2 \times t) + ((-1) \times (-6)) = 0$

$-24 + 2t + 6 = 0$

$2t - 18 = 0$

$2t = 18$

$t = 9$

(2)

Point	Gradient	Equation
$y = 2(-2)^3 + 3$	$\frac{dy}{dx} = 6x^2$	$y + 13 = 24(x+2)$
$= -16 + 3$	$m = 6 \times (-2)^2$	$y + 13 = 24x + 48$
$= -13$	$= 24$	$y = 24x + 35$
$(-2, -13)$		

(3)

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$(x+3)(x^2 - 6x + 8) = 0$

$(x+3)(x-4)(x-2) = 0$

(4) $y = 3\cos 4x + 1$

(5) (a) $g(x) = 6 - 2x$

$6 - 2x = g(x)$

$-2x = g(x) - 6$

$2x = -g(x) + 6$

$x = -\frac{1}{2}g(x) + 3$

$g^{-1}(x) = -\frac{1}{2}x + 3$

(b) $g(g^{-1}(x)) = 6 - 2(-\frac{1}{2}x + 3)$

$= 6 + x - 6$

$= x$

⑥ $\log_6 12 + \frac{1}{3}\log_6 27$

$= \log_6 12 + \log_6\left(27^{\frac{1}{3}}\right)$

$= \log_6 12 + \log_6\left(\sqrt[3]{27}\right)$

$= \log_6 12 + \log_6 3$

$= \log_6 36$

$= 2$

⑦ $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$

$= x^{\frac{1}{2}}\left(3x - \frac{2}{x \times x^{\frac{1}{2}}}\right)$

$= 3x^{\frac{3}{2}} - \frac{2x^{\frac{1}{2}}}{x \times x^{\frac{1}{2}}}$

$= 3x^{\frac{3}{2}} - \frac{2}{x}$

$= 3x^{\frac{3}{2}} - 2x^{-1}$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 2x^{-2}$

$= \frac{9}{2}\sqrt{x} + \frac{2}{x^2}$

$f'(4) = \frac{9}{2}\sqrt{4} + \frac{2}{4^2}$

$= \frac{9}{2} \times 2 + \frac{2}{16}$

$= \frac{18}{2} + \frac{1}{8}$

$= 9\frac{1}{8}$

⑧ $A = l \times b$

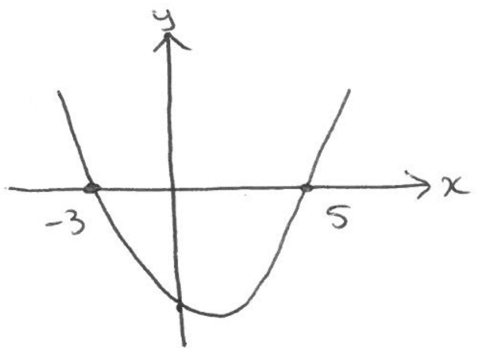
$= x(x-2)$

$x(x-2) < 15$

$x^2 - 2x - 15 < 0$

$(x-5)(x+3) < 0$

$x = 5 \qquad x = -3$

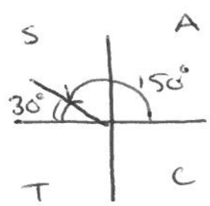


$-3 < x < 5$

However, x must be > 2 or the breadth $(x-2)$ would be negative, therefore: $\underline{\underline{2 < x < 5}}$

⑨ $y = -\sqrt{3}\,x$

$m_{AB} = -\sqrt{3}$

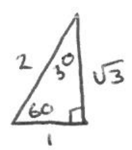


$m = \tan 150°$

$= -\tan 30$

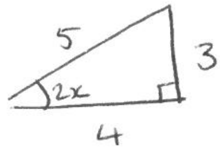
$= -\frac{1}{\sqrt{3}}$

$m_{BC} = -\frac{1}{\sqrt{3}}$



The points A, B and C are not collinear since $m_{AB} \neq m_{BC}$.

(10) (a) $\tan 2x = \frac{3}{4}$



$\cos 2x = \frac{4}{5}$

(b) $\cos 2x = 2\cos^2 x - 1 = \frac{4}{5}$

$2\cos^2 x = \frac{9}{5}$

$\cos^2 x = \frac{9}{5} \div 2$

$\cos^2 x = \frac{9}{10}$

$\cos x = \sqrt{\frac{9}{10}}$

$\cos x = \frac{3}{\sqrt{10}}$

(11) (a) Centre $(-8, -2)$

$m_{radius} = \frac{-2-(-5)}{-8-(-2)}$

$= \frac{3}{-6}$

$= -\frac{1}{2}$

$m_{tangent} = 2$

$y-(-5) = 2(x-(-2))$

$y + \quad = 2(x+2)$

$y + 5 = 2x + 4$

$y = 2x - 1$

(b) $2x - 1 = -2x^2 + px + 1 - p$

$2x^2 + 2x - px + p - 2 = 0$

$2x^2 + x(2-p) + p - 2 = 0$

For tangency $b^2 - 4ac = 0$

$(2-p)^2 - 4 \times 2 \times (p-2) = 0$

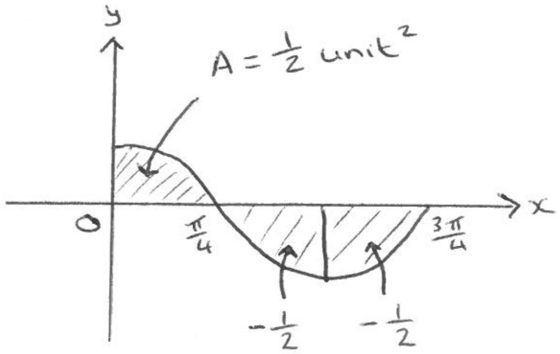
$4 - 4p + p^2 - 8p + 16 = 0$

$p^2 - 12p + 20 = 0$

$(p-10)(p-2) = 0$

$p = 10 \quad \cancel{p = 2}$

(12)



Total $A = -1$ unit^2

(Area negative since below x-axis)

$$\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$$

$$= -1 + \frac{1}{2}$$

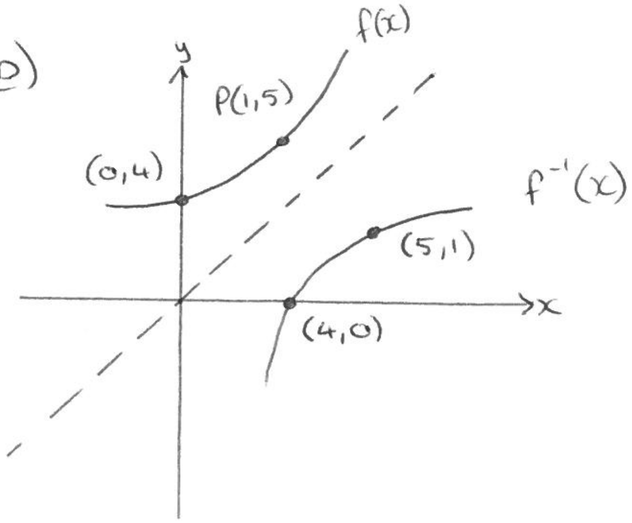
$$= -\frac{1}{2} \text{ unit}^2$$

(13) (a) $f(x) = 2^x + 3$ $\quad (1, b)$

$b = 2^1 + 3$

$b = 5$

(b)



y-intercept ($x=0$)

$y = 2^0 + 3$

$= 1 + 3$

$= 4$

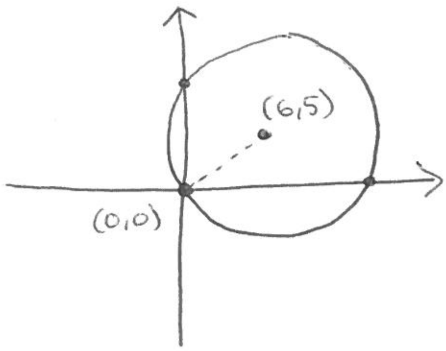
(c) $y = 4 - f(x+1)$

$= -f(x+1) + 4$

reflect in x axis | move left by 1 | move up by 4

$R(3,11) \rightarrow (3,-11) \rightarrow (2,-11) \rightarrow (2,-7)$

(14)



Centre $(6,5)$

$r = \sqrt{6^2 + 5^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

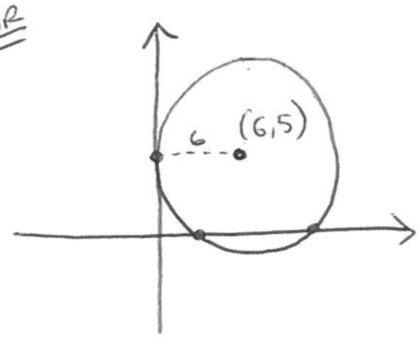
$\sqrt{(-6)^2 + (5)^2 - k} = \sqrt{61}$

$36 + 25 - k = 61$

$-k = 0$

$k = 0$

OR



$r = 6$

$\sqrt{(-6)^2 + (-5)^2 - k} = 6$

$36 + 25 - k = 36$

$-k = -25$

$k = 25$

(15)

$T = \int \frac{1}{25}t - k$

$T = \dfrac{\frac{1}{25}t^2}{2} - kt + C$

$T = \frac{1}{50}t^2 - kt + C$ when $t = 0$, $T = 100°C$

$100 = \frac{1}{50}(0)^2 - k(0) + C$

$100 = C$

$T = \frac{1}{50}t^2 - kt + 100$ when $t = 10$, $T = 82°C$

$82 = \frac{1}{50}(10)^2 - k(10) + 100$

$82 = 2 - 10k + 100$

$10k = 20$

$k = 2$

$\underline{\underline{T = \frac{1}{50}t^2 - 2t + 100}}$

New Higher 2015 Paper 2

1 (a) $m_{AB} = \frac{-5-7}{-1-(-5)} = \frac{-12}{4} = -3$ $\qquad m_{\perp} = \frac{1}{3}$

$y - 3 = \frac{1}{3}(x - 13)$

$3y - 9 = 1(x - 13)$

$3y - 9 = x - 13$

$3y - x = -4$

$x - 3y = 4$

(b) $M_{AC} = \left(\frac{-5+13}{2}, \frac{7+3}{2}\right)$ $\qquad m_{BM} = \frac{-5-5}{-1-4}$

$= (4, 5)$ $\qquad = \frac{-10}{-5}$

$= 2$

$y - 5 = 2(x - 4)$

$y - 5 = 2x - 8$

$y - 2x = -3$

(c)

$3y - x = -4$

$y - 2x = -3$ $\quad (\times(-3))$

$3y - x = -4$

$-3y + 6x = 9$

$5x = 5$

$x = 1$

$y - 2(1) = -3$

$y - 2 = -3$

$y = -1$ $\qquad (1, -1)$

② (a) $f(g(x)) = 10 + ((1+x)(3-x)+2)$

$= 10 + (3 - x + 3x - x^2 + 2)$

$= -x^2 + 2x + 15$

(b) $f(g(x)) = -1\left[x^2 - 2x\right] + 15$

$= -1\left[(x-1)^2 - 1\right] + 15$

$= -(x-1)^2 + 1 + 15$

$= -(x-1)^2 + 16$

(c) $h(x) = \dfrac{1}{-(x-1)^2+16}$ denominator cannot $=0$,

$-(x-1)^2 + 16 = 0$

$-(x-1)^2 = -16$

$(x-1)^2 = 16$

$x-1 = 4$ or -4

$x = 5$ or -3

$x \neq 5, -3$

③ (a) $t_2 = \frac{3}{4}(13) + 13 = 22.75$

(b) Frogs

$L = \dfrac{32}{1-\frac{1}{3}}$

$= 48$

Toads

$L = \dfrac{13}{1-\frac{3}{4}}$

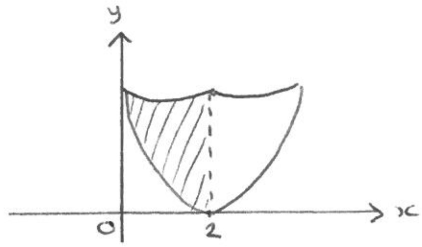
$= 52$

The toad will escape as it will reach 52 feet and $52 > 50$.

4 (a) $f(x) = g(x)$

$$\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$$

$$x = 2$$



(b) $$\int_0^2 \left(f(x) - h(x)\right) dx$$

$$= \int_0^2 \frac{1}{4}x^2 - \frac{1}{2}x + 3 - \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right) dx$$

$$= \int_0^2 \frac{2}{8}x^2 - \frac{4}{8}x + 3 - \frac{3}{8}x^2 + \frac{18}{8}x - 3 \quad dx$$

$$= \int_0^2 -\frac{1}{8}x^2 + \frac{14}{8}x \quad dx$$

$$= \int_0^2 -\frac{1}{8}x^2 + \frac{7}{4}x \quad dx$$

$$= \left[-\frac{\frac{1}{8}x^3}{3} + \frac{\frac{7}{4}x^2}{2} \right]_0^2$$

$$= \left[-\frac{1}{24}x^3 + \frac{7}{8}x^2 \right]_0^2$$

$$= \left(-\frac{1}{24}(2)^3 + \frac{7}{8}(2)^2\right) - \left(-\frac{1}{24}(0)^3 + \frac{7}{8}(0)^2\right)$$

$$= \left(-\frac{8}{24} + \frac{28}{8}\right) - (0)$$

$$= -\frac{8}{24} + \frac{84}{24}$$

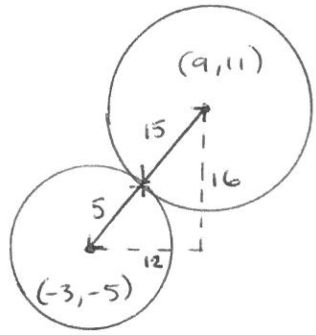
$$= \frac{76}{24}$$

$$= \frac{19}{6}$$

Total Area $= \frac{19}{6} \times 2$

$= \frac{38}{6} = \frac{19}{3}$ units^2

(5) (a) Centre C_1 $(-3, -5)$



Distance between centres C_1 & C_2

$= \sqrt{12^2 + 16^2}$

$= \sqrt{400}$

$= 20$

Radius $C_1 = \sqrt{3^2 + 5^2 - 9}$

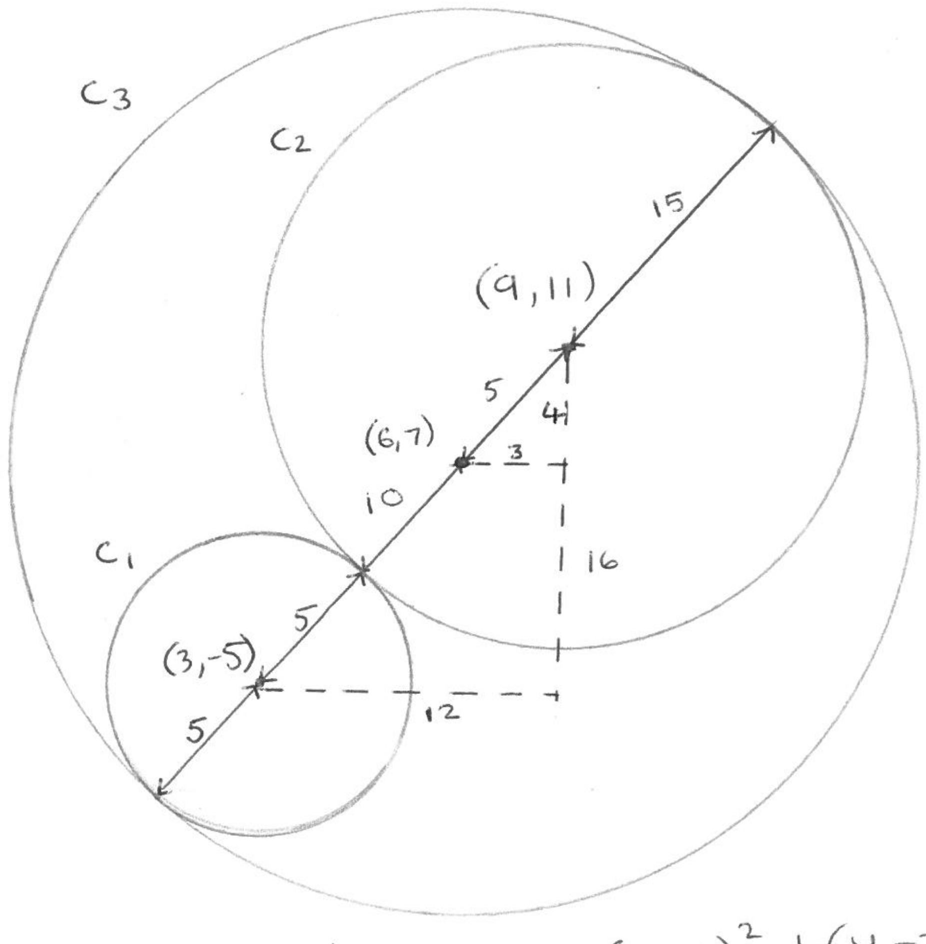
$= \sqrt{25}$

$= 5$

Radius $C_2 = 20 - 5$

$\underline{\underline{= 15}}$

(b)



Centre $C_3 = (6, 7)$

radius $C_3 = 20$

$(x-6)^2 + (y-7)^2 = 20^2$

$\underline{\underline{(x-6)^2 + (y-7)^2 = 400}}$

⑥ (a)

$$\underset{\sim}{p} \cdot (\underset{\sim}{q} + \underset{\sim}{r})$$

$$= \underset{\sim}{p} \cdot \underset{\sim}{q} + \underset{\sim}{p} \cdot \underset{\sim}{r}$$

$$= |\underset{\sim}{p}||\underset{\sim}{q}|\cos\theta + |\underset{\sim}{p}||\underset{\sim}{r}|\cos\theta$$

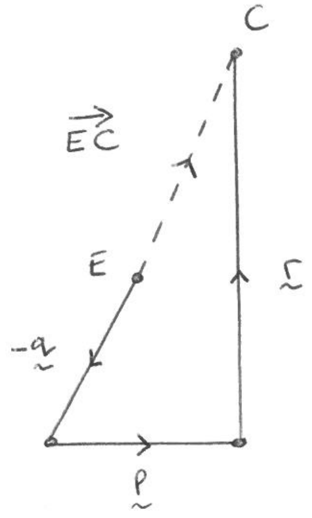
$$= 3 \times 3 \times \cos 60^\circ + 3 \times |\underset{\sim}{r}| \times \cos 90^\circ$$

$$= \frac{9}{2} + 3 \times |\underset{\sim}{r}| \times 0$$

$$= \frac{9}{2} + 0$$

$$= \frac{9}{2}$$

(b) $\overrightarrow{EC} = -\underset{\sim}{q} + \underset{\sim}{p} + \underset{\sim}{r}$



(c) $\overrightarrow{AE} \cdot \overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$

$$\underset{\sim}{q} \cdot (-\underset{\sim}{q} + \underset{\sim}{p} + \underset{\sim}{r}) = 9\sqrt{3} - \frac{9}{2}$$

$$-\underset{\sim}{q} \cdot \underset{\sim}{q} + \underset{\sim}{q} \cdot \underset{\sim}{p} + \underset{\sim}{q} \cdot \underset{\sim}{r} = 9\sqrt{3} - \frac{9}{2}$$

$$-(|\underset{\sim}{q}||\underset{\sim}{q}|\cos\theta) + (|\underset{\sim}{q}||\underset{\sim}{p}|\cos\theta) + (|\underset{\sim}{q}||\underset{\sim}{r}|\cos\theta) = 9\sqrt{3} - \frac{9}{2}$$

$$-(3 \times 3 \times \cos 0^\circ) + (3 \times 3 \times \cos 60^\circ) + (3 \times |\underset{\sim}{r}| \times \cos 30^\circ) = 9\sqrt{3} - \frac{9}{2}$$

$$-9 + \frac{9}{2} + \frac{3\sqrt{3}}{2}|\underset{\sim}{r}| = 9\sqrt{3} - \frac{9}{2}$$

$$\frac{3\sqrt{3}}{2}|\underset{\sim}{r}| = 9\sqrt{3} - \frac{9}{2} - \frac{9}{2} + 9$$

$$\frac{3\sqrt{3}}{2}|\underset{\sim}{r}| = 9\sqrt{3}$$

$$3\sqrt{3}\,|\underset{\sim}{r}| = 18\sqrt{3}$$

$$|\underset{\sim}{r}| = \frac{18\sqrt{3}}{3\sqrt{3}}$$

$$|\underset{\sim}{r}| = 6$$

(7) (a)

$$\int (3\cos 2x + 1)\,dx$$

$$= 3\sin 2x \times \frac{1}{2} + x + C$$

$$= \frac{3}{2}\sin 2x + x + C$$

(b)

$$3\cos 2x + 1$$

$$= 3(\cos^2 x - \sin^2 x) + 1$$

$$= 3\cos^2 x - 3\sin^2 x + 1 \qquad \boxed{\sin^2 x + \cos^2 x = 1}$$

$$= 3\cos^2 x - 3\sin^2 x + \sin^2 x + co$$

$$= 4\cos^2 x - 2\sin^2 x$$

(c)

$$4\cos^2 x - 2\sin^2 x = 3\cos 2x + 1$$

$$-2\cos^2 x + \sin^2 x = -\frac{1}{2}(3\cos 2x + 1)$$

$$\sin^2 x - 2\cos^2 x = -\frac{3}{2}\cos 2x - \frac{1}{2}$$

$$\int (\sin^2 x - 2\cos^2 x)\,dx$$

$$= \int \left(-\frac{3}{2}\cos 2x - \frac{1}{2}\right) dx$$

$$= -\frac{3}{2}\sin 2x \times \frac{1}{2} - \frac{1}{2}x + C$$

$$= -\frac{3}{4}\sin 2x - \frac{1}{2}x + C$$

(8) (a) (i) If it doesn't travel on land then the distance, x, that it swims upstream is 20m.

when $x = 20$,

$$T(20) = 5\sqrt{36 + 20^2} + 4(20-20)$$
$$= 5\sqrt{436} + 4(0)$$
$$= 104 \text{ tenths of a second.}$$

(ii) If it swims the shortest distance i.e. directly to the opposite bank, then it does not swim upstream. Therefore $x = 0$.

when $x = 0$, $T(0) = 5\sqrt{36 + 0^2} + 4(20-0)$
$$= 5\sqrt{36} + 4(20)$$
$$= 30 + 80$$
$$= 110 \text{ tenths of a second.}$$

(b) $T(x) = 5(36+x^2)^{\frac{1}{2}} + 80 - 4x$

$$T'(x) = \tfrac{1}{2} \times 5(36+x^2)^{-\frac{1}{2}} \times (2x) - 4$$
$$= \frac{5}{2}(2x)\left(\frac{1}{\sqrt{36+x^2}}\right) - 4$$
$$= 5x\left(\frac{1}{\sqrt{36+x^2}}\right) - 4$$
$$= \frac{5x}{\sqrt{36+x^2}} - 4$$

For S.P's $T'(x) = 0$,

$$\frac{5x}{\sqrt{36+x^2}} - 4 = 0$$
$$\frac{5x}{\sqrt{36+x^2}} = 4$$
$$5x = 4\sqrt{36+x^2}$$
$$25x^2 = 16(36+x^2)$$
$$25x^2 = 576 + 16x^2$$
$$9x^2 = 576$$
$$x^2 = 64$$
$$x = 8 \text{ or } \cancel{-8}$$

The minimum time taken occurs when $x = 8$, $T(8) = 5\sqrt{36+8^2} + 4(20-8)$
$= 98$ tenths of a second.

(9) $36\sin(1.5t) - 15\cos(1.5t) = k\sin(1.5t - a)$

$$= k(\sin 1.5t \cos a - \cos 1.5t \sin a)$$

$$= k\cos a \sin 1.5t - k\sin a \cos 1.5t$$

$k\cos a = 36$

$k\sin a = 15$

$k = \sqrt{36^2 + 15^2}$

$= \sqrt{1521}$

$= 39$

$\tan a = \dfrac{\sin a}{\cos a}$

$= \dfrac{15}{36}$

$a = 0.395$

$$39\sin(1.5t - 0.395)$$

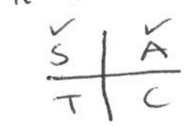
$h = 36\sin(1.5t) - 15\cos(1.5t) + 65 = 100$

$39\sin(1.5t - 0.395) + 65 = 100$

$39\sin(1.5t - 0.395) = 35$

$\sin(1.5t - 0.395) = \dfrac{35}{39}$

$\pi - t$



$1.5t - 0.395 = 1.114, 2.03$

$1.5t = 1.509, 2.425$

$t = 1.006, 1.617$