

Paper 2

① Equal roots $\therefore b^2 - 4ac = 0$
 $k^2 - 4(1)(k+3) = 0$
 $k^2 - 4k - 12 = 0$
 $(k-6)(k+2) = 0$
 $\downarrow \quad \downarrow$
 $k = 6$ $k = -2$

3

② $|u| = \sqrt{3^2 + 2^2 + 6^2} = 7$ $|v| = \sqrt{(-5)^2 + (-2)^2 + 4^2} = 3\sqrt{5}$ $\cos \theta = \frac{5}{7 \cdot 3\sqrt{5}}$
 $\theta = \cos^{-1}\left(\frac{5}{21\sqrt{5}}\right)$
 $\theta = 83.887...$
 $\theta \approx 83.9^\circ$

5

③ $y = \frac{1}{5}x + 6$
 $5y = x + 30$
 $x = 5y - 30$
 $g^{-1}(x) = 5x - 30$

3

④ $r = \sqrt{(3 - (-2))^2 + (7 - 5)^2}$ $(x+2)^2 + (y-5)^2 = 29$
 $= \sqrt{5^2 + 2^2}$
 $= \sqrt{29}$

2

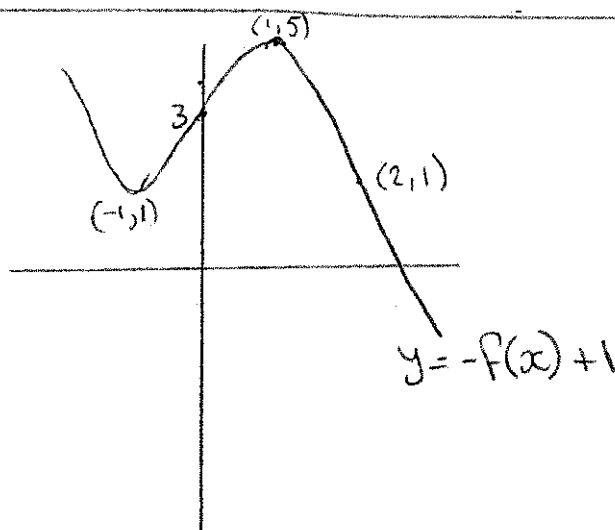
⑤ (a) $u_1 = 1.125(2) + 5 = \underline{7.25}$ (b) No limit since $1.125 > 1$.

(c) $u_1 = 7.25$
 $u_2 = 13.156...$
 $u_3 = 19.800...$
 $u_4 = 27.275...$
 $u_5 = 35.685...$ $n = 5$

4

⑥

$f(x)$	$-f(x)$	$-f(x) + 1$
$\cap (-1, 0)$	$\cup (-1, 0)$	$(-1, 1)$
$(0, -2)$	$(0, 2)$	$(0, 3)$
$\cup (1, -4)$	$\cap (1, 4)$	$(1, 5)$
$(2, 0)$	$(2, 0)$	$(2, 1)$



3

$$\textcircled{7} \int_{-1}^2 (x^2 - x - 2) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^2$$

$$= \left[\frac{2^3}{3} - \frac{2^2}{2} - 2(2) \right] - \left[\frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2(-1) \right]$$

$$= \left[\frac{8}{3} - 2 - 4 \right] - \left[-\frac{1}{3} - \frac{1}{2} + 2 \right]$$

$$= \frac{8}{3} - 6 + \frac{1}{3} + \frac{1}{2} - 2$$

$$= 3 - 8 + \frac{1}{2}$$

$$= -4\frac{1}{2}$$

$$= -\frac{9}{2} \quad \text{Area} = \underline{\underline{\frac{9}{2} \text{ units}^2}}$$

$$\textcircled{8} \text{ (a) Mid pt } AB = \left(\frac{-2+10}{2}, \frac{11+(-13)}{2} \right) \quad m_{AB} = \frac{11-(-13)}{-2-10}$$

$$= \left(\frac{8}{2}, \frac{-2}{2} \right) = \left(4, -1 \right)$$

$$= \frac{24}{-12} = -2$$

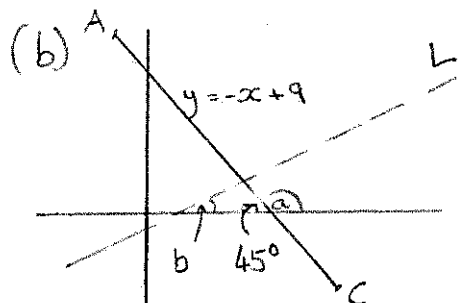
$$m_{\perp} = \frac{1}{2}$$

$$-1 = \frac{1}{2}(4) + C$$

$$-1 = 2 + C$$

$$C = -3$$

$$\underline{\underline{y = \frac{1}{2}x - 3}}$$



$$m_{AC} = -1$$

$$m = \tan a$$

$$-1 = \tan a$$

$$a = 135$$

$$m_L = \frac{1}{2}$$

$$m = \tan b$$

$$\frac{1}{2} = \tan b$$

$$b = 26.6$$



$$\underline{\underline{108.4^\circ}}$$

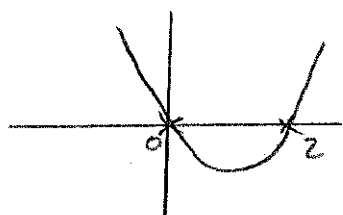
6

$$\textcircled{9} f(x) = 4x^3 - 12x^2 + 7$$

$$f'(x) = 12x^2 - 24x$$

$$12x^2 - 24x < 0$$

$$x(x-2) < 0$$



$$\underline{\underline{0 < x < 2}}$$

4

$$\begin{aligned}
 (10) \quad & 3\cos 2x + 10\cos x - 1 = 0 \\
 & 3(2\cos^2 x - 1) + 10\cos x - 1 = 0 \\
 & 6\cos^2 x - 3 + 10\cos x - 1 = 0 \\
 & 6\cos^2 x + 10\cos x - 4 = 0 \\
 & 3\cos^2 x + 5\cos x - 2 = 0 \\
 & (3\cos x - 1)(\cos x + 2) = 0 \\
 & \quad \downarrow \\
 & \cos x = \frac{1}{3} \quad \cos x = -2 \\
 & \therefore a = \cos^{-1}\left(\frac{1}{3}\right) \quad \text{No Soln} \\
 & = 70.5^\circ
 \end{aligned}$$

S/A✓
T/C✓

1st Quad: 70.5°

4th Quad: 289.5°

$x = 70.5^\circ, 289.5^\circ$

5

$$\begin{aligned}
 (11) \quad & y = ab^x \\
 & \log_4 y = \log_4 ab^x \\
 & \log_4 y = \log_4 a + \log_4 b^x \\
 & \log_4 y = x \underbrace{\log_4 b}_m + \underbrace{\log_4 a}_{y\text{-int.}}
 \end{aligned}$$

gradient

$$\begin{aligned}
 m &= \frac{9-3}{3-0} \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \log_4 b &= 2 \\
 b &= 4^2 \\
 b &= 16
 \end{aligned}$$

y-int

$$\log_4 a = 3$$

$$a = 4^3$$

$$\underline{a = 64}$$

5

$$\begin{aligned}
 (12)(a) \quad & f(x) = x^3 + 2x^2 - 4x + 1 \\
 & f'(x) = 3x^2 + 4x - 4 \\
 & \text{SPs when } 3x^2 + 4x - 4 = 0 \\
 & \quad (3x-2)(x+2) = 0 \\
 & \quad \quad \downarrow \quad \quad \downarrow \\
 & \quad \underline{x = \frac{2}{3}} \quad \underline{x = -2}
 \end{aligned}$$

$$(b) \quad f(-1) = (-1)^3 + 2(-1)^2 - 4(-1) + 1 = 6$$

$$f(1) = 1 + 2 - 4 + 1 = 0$$

$$\begin{aligned}
 f\left(\frac{2}{3}\right) &= \left(\frac{2}{3}\right)^3 + 2\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 1 \\
 &= -\frac{13}{27}
 \end{aligned}$$

$$\underline{\underline{\text{min} = -\frac{13}{27}}}$$

$$\underline{\underline{\text{max} = 6}}$$

6

$$\begin{aligned} \textcircled{13} (a) \quad 2 \sin x - 7 \cos x &= k \sin(x-a) \\ &= k \sin x \cos a - k \cos x \sin a \\ &= \underline{k \cos a \sin x} - \underline{k \sin a \cos x} \end{aligned}$$

$$\begin{aligned} k \cos a &= 2 \\ k \sin a &= 7 \end{aligned}$$

$$\tan a = \frac{7}{2}$$

$$a = 74.1^\circ$$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$$\begin{aligned} k &= \sqrt{2^2 + 7^2} \\ &= \sqrt{53} \end{aligned}$$

$$\underline{2 \sin x - 7 \cos x = \sqrt{53} \sin(x - 74.1^\circ)}$$

$$(b) \quad 2 \sin x - 7 \cos x = 3$$

$$\sqrt{53} \sin(x - 74.1^\circ) = 3$$

$$\sin(x - 74.1^\circ) = \frac{3}{\sqrt{53}}$$

$$A = \sin^{-1}\left(\frac{3}{\sqrt{53}}\right)$$

$$= 24.3^\circ$$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array} \quad \begin{array}{l} 1^{\text{st}} \text{ Quad: } 24.3^\circ, -335.7^\circ \\ 2^{\text{nd}} \text{ Quad: } 155.7^\circ, -204.3^\circ \end{array}$$

$$x - 74.1^\circ = -204.3^\circ$$

$$\underline{x = -130.2^\circ}$$

7

$$\textcircled{14} (a) \quad T = 800e^{-0.124t} + 25$$

$$\underline{\underline{= 825^\circ\text{C}}}$$

$$(b) \quad 150 = 800e^{-0.124t} + 25$$

$$125 = 800e^{-0.124t}$$

$$\frac{5}{32} = e^{-0.124t}$$

$$\ln \frac{5}{32} = -0.124t$$

$$t = \frac{\ln \frac{5}{32}}{-0.124}$$

$$\underline{\underline{t = 14.970 \approx 15.0s}}$$

5

$$\textcircled{15} \quad (3 \sin \theta + \cos \theta)(\sin \theta + 3 \cos \theta)$$

$$= 3 \sin^2 \theta + 9 \sin \theta \cos \theta + \sin \theta \cos \theta + 3 \cos^2 \theta$$

$$= 3(\sin^2 \theta + \cos^2 \theta) + 10 \sin \theta \cos \theta$$

$$= 3 + 5(2 \sin \theta \cos \theta)$$

$$\underline{\underline{= 3 + 5 \sin 2\theta}}$$

3