Optimization – Questions

Q1) A storage container is in the shape of an open-topped, square-based cuboid and has a volume of $500 \ cm^3$. The length of the container is $x \ cm$ and the height is $h \ cm$.

   a) Prove that the surface area of the container is given by the formula

   $$S = x^2 + \frac{2000}{x}$$

   b) Find the dimensions of the container which minimise this surface area.

Q2) A company is contracted to provide a service. The profit made by the company can be modelled by the following formula $P(x) = 192x - x^3 + 400$, where x is the number of jobs completed each day.

   a) Determine the number of jobs that have to be completed daily to maximize Profit.

   b) Calculate the maximum profit made in one day.

Q3) David has a piece of wire, 60 cm long, which he bends into the shape of a rectangle. The length of the rectangle is $x \ cm$.

   a) Write down an expression which represents the breadth of the rectangle.

   b) Show that the area, $A \ cm^2$, of the rectangle is given by $A(x) = 30x - x^2$.

   c) What is the maximum area that the rectangle could possibly enclose?

Optimization – Solutions

Q1) a) Area of each side is given by xh. There are four sides, giving $4xh$

Area of the base is given by x^2

So, total surface area is given by $x^2 + 4xh$ (1)

Volume is given by x^2h, but the volume is $500\ cm^3$, so we have

$x^2h = 500$

$h = \frac{500}{x^2}$

Substituting this in (1) gives

$x^2 + 4x(\frac{500}{x^2}) = x^2 + \frac{2000}{x}$

b) $x = 10, h = 5$

Q2) a) 8   b) £1,424

Q3) a) $b = 30 - x$ b) Area $= length\ x\ breadth$

$= x \cdot (30 - x)$

$= 30x - x^2$

c) The maximum area enclosed by the rectangle is $225\ cm^2$.