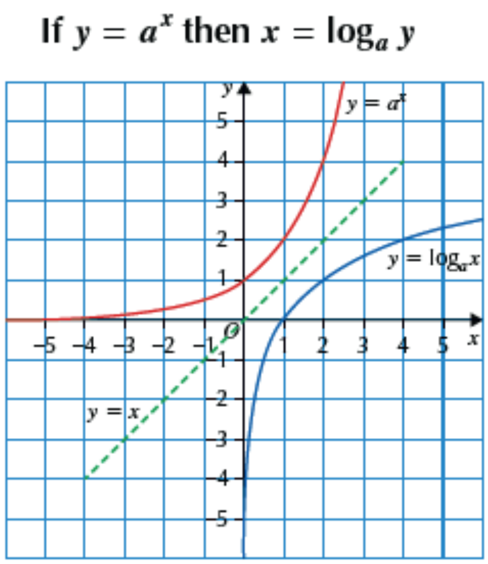
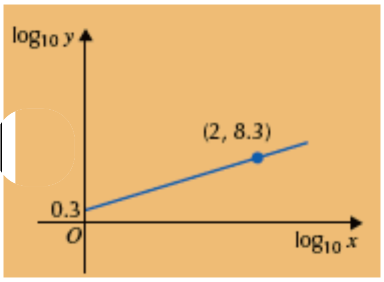
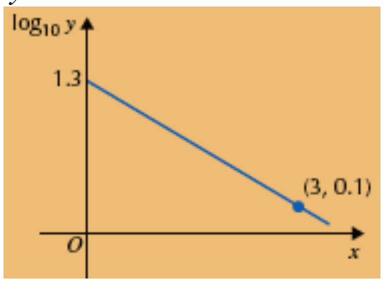
Higher Checklist

Check your readiness for Higher Maths and tick the relevant box to show how confident you feel answering questions on each topic.

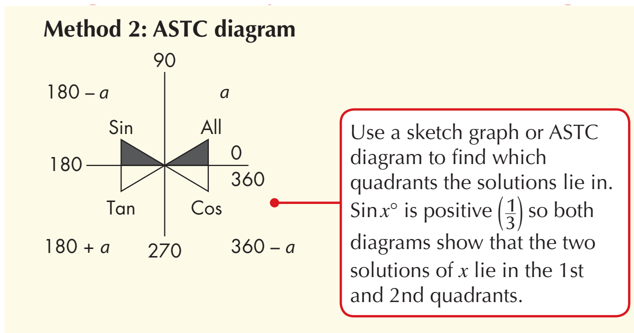
Chapter	Topic	Skills			
1 Exponential and Logarithms					
1A	Exponential population growth	$y=a^x$ is an exponential functions with base a and exponent x. It increases rapidly (exponential growth) when $a>1$ and decreases rapidly when $0<a<1$ The function $y=e^x$ features widely in real-life applications of mathematics.			
1B	Introduction to the logarithmic function	If $y=a^x$ then $x=\log_a y$ In other words, $y=\log_a x$ is a reflection of $y=a^x$ in the line $y=x$  e.g. $2^4=16$ can be expressed as $\log_2 16=4$			
1C	Rules of logarithms	Using rules of exponential functions from N5, there are rules that can be used to simplify expressions involving logs Rule 1: $\log_a 1=0$ (since $a^0=1$) Rule 2: $\log_a a=1$ (since $a^1=a$) Rule 3: $\log_a x+\log_a y=\log_a xy$ e.g. $\log_3 8+\log_3 2=\log_3(8\times 2)=\log_3 16$ Rule 4: $\log_a x-\log_a y=\log_a \frac{x}{y}$ e.g. $\log_{10} 50-\log_{10} 5=\log_3 \frac{50}{5}=\log_{10} 10=1$ Rule 5: $\log_a x^n=n\log_a x$ e.g. $\log_5 25=\log_5 5^2=2\log_5 5=2\times 1=2$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
1D	The natural logarithm	The inverse of the exponential function $y = e^x$ is called the natural logarithm $y = \log_e x$. This can also be written as $y = \ln x$.			
1E	Solving simple logarithmic and exponential functions	Laws of exponents can be used to solve simple equations $e^x = 5$ e.g. $x = \log_e 5$ $= 1.6094...... = 1.61$			
1F	Solving logarithmic and exponential functions using the laws of logarithms and exponentials	Use the laws from N5 and from Exercise 1B e.g. $\log_5 x + \log_5 3 = \log_5 21$ $\log_5 3x = \log_5 21$ $3x = 21$ $x = 7$			
1G	Solving with different bases and with constants	Laws of logarithms can only be used if they have the same base number. e.g. $\log_3 x - \log_3 5 = \log_9 81$ $\log_3\left(\frac{x}{5}\right) = \log_9 9^2$ $\log_3\left(\frac{x}{5}\right) = 2\log_9 9$ $\log_3\left(\frac{x}{5}\right) = 2$ $\frac{x}{5} = 3^2$ $x = 5 \times 9 = 45$			
1H/1I	Applications of exponential and logarithmic functions	Knowledge of logarithmic and exponential functions can be used to solve a wide range of real-life problems. e.g. In a biology experiment it was found that cells are dying according to the formula $C_t = C_0 e^{-kt}$ where C_t is the number of cells after *t* days and C_0 is the initial number of cells. a. The experiment started with 250000 cells and half the cells died after 8 days. $-8k = \log_e 0.5$ $k = \frac{\log_e 0.5}{-8} = 0.0866$			

Chapter	Topic	Skills			
		b. Calculate the time taken to reduce to 20% of the initial population $50\,000 = 250\,000e^{-0.0866t}$ $0.2 = e^{-0.0866t}$ $-0.0866t = \log_e 0.2$ $t = \frac{\log_e 0.2}{-0.0866} = 18.585$ So approximately 18.6 days			
1J	Interpreting experimental data	In science experiments, growth and decay functions can be interpreted through the application of logarithms. These functions can be of the form: • $y = kx^n$ • $y = ab^x$ e.g. This graph represents a function of the form $y = kx^n$  Determine the values of k and n. $y = kx^n$ $\log_{10} y = \log_{10} kx^n$ $\log_{10} y = \log_{10} k + \log_{10} x^n$ $\log_{10} y = n\log_{10} x + \log_{10} k$ Gradient of straight line = n $n = \frac{8.3-0.3}{2-0} = 4$ y-intercept of straight line = $\log_{10} k$ $\log_{10} k = 0.3 \;\rightarrow\; k = 10^{0.3} = 2$ e.g. .g. This graph represents a function of the form $y = ab^x$  Determine the values of a and b. $y = ab^x$ $\log_{10} y = \log_{10} ab^x$ $\log_{10} y = \log_{10} a + \log_{10} b^x$			

Chapter	Topic	Skills			
		Gradient of straight line = $\log_{10} b$ $m=\frac{1.3-0.1}{0-3}=\frac{1.2}{-3}=-0.4$ $\log_{10} b=-0.4 \rightarrow k=10^{-0.4}=0.398...=0.4$ y-intercept of straight line = $\log_{10} a$ $\log_{10} a=1.3 \rightarrow a=10^{1.3}=19.952...=20$			

Chapter	Topic	Skills			
2 Manipulating Trigonometric Expressions					
2A	Working with the sine, cosine and tangent ratios of an angle x	In this topic, use the CAST diagram and the trigonometric graphs from National 5 Mathematics			



Trigonometric identities from National 5:

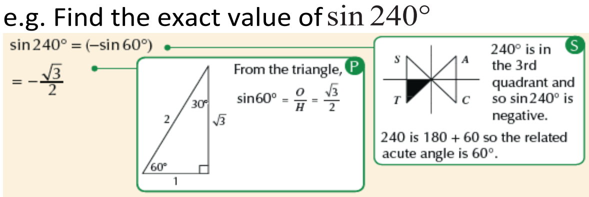
- $\sin^2 a+\cos^2 a=1$
- $\frac{\sin a}{\cos a}=\tan a$

The triangles shown in the table give the exact values for 30°, 45° and 60°.

	angle	sin	cos	tan
	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
	45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

For 0°, 90° and multiples of 90°, use the graphs to read values for sin, cos and tan.

e.g. Find the exact value of $\sin 240°$

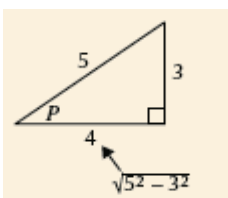


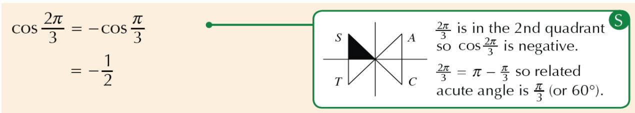
240° in 3rd quadrant: 240 = 180 + 60

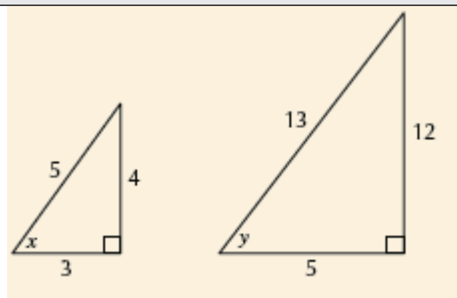
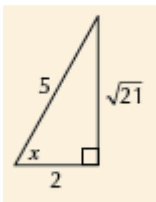
Sine ratio is negative in 3rd quadrant

$$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$$

e.g. Given that P is an acute angle with $\sin P=\frac{3}{5}$:

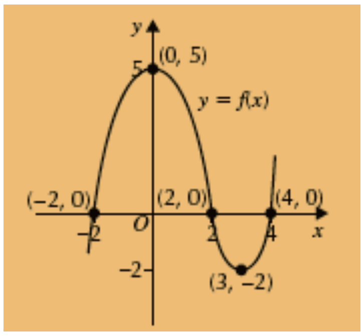
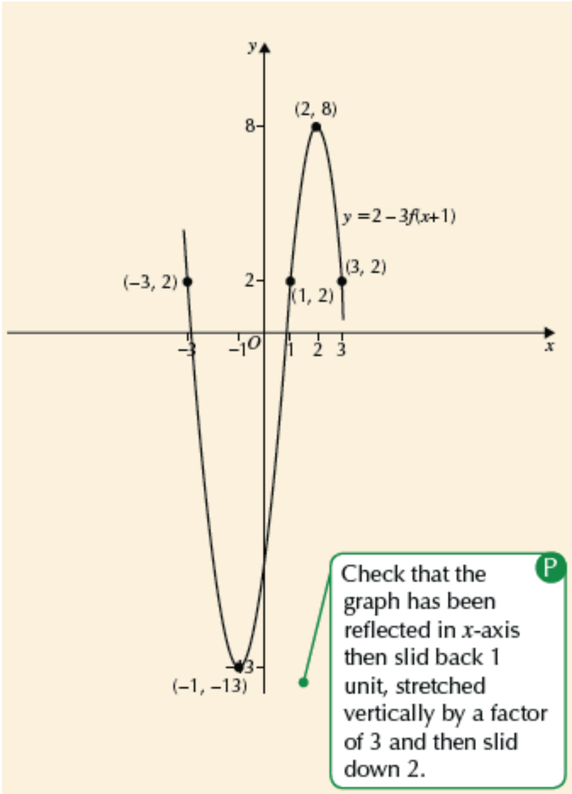


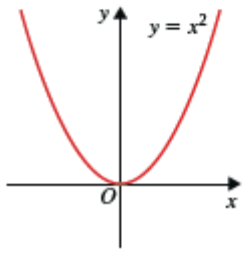
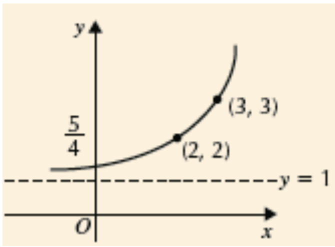
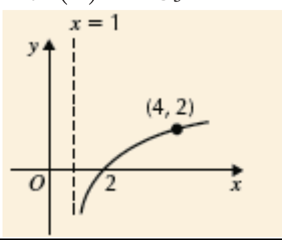
Chapter	Topic	Skills			
		$\sqrt{5^2 - 3^2} = 4$ Using SOH CAH TOA ratios: $\cos P = \frac{4}{5}, \; \tan P = \frac{3}{4}$			
2B	Working with angles measured in radians	Angles can also be measured in radians. Use the link 180° = π radians e.g. 120° = $\frac{120\pi}{180} = \frac{2\pi}{3}$ radians $\frac{5\pi}{4}$ radians = $\frac{5 \times 180}{4}$ =225° Use the above link to find exact values of ratios for angles measured in radians e.g. $\cos\left(\frac{2\pi}{3}\right) = \cos 120°$  120° in 2nd quadrant: 120 = 180 - 60 Cosine ratio is negative in 2nd quadrant $\cos 120° = -\cos 60° = -\frac{1}{2}$			
2C	Working with the sine, cosine and tangent of compound angles	The addition formulae for any angles A and B are: $\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$ e.g. expand and simplify $\sin(P+30°)$ $\sin(P+30°) = \sin P \cos 30° + \cos P \sin 30°$ $= \sin P \times \frac{\sqrt{3}}{2} + \cos P \times \frac{1}{2}$ $= \frac{\sqrt{3}\sin P + \cos P}{2}$			
2D	Finding exact values ratios using the addition formulae	Use the addition formulae in Exercise 2B e.g. $\cos 15° = \cos(45-30)°$ $= \cos 45° \cos 30° + \sin 45° \sin 30°$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ e.g. Given that $\sin x = \frac{4}{5}$ and $\cos x = \frac{5}{13}$, where x and y are both acute angles, find the exact value of $\cos(x-y)$			

Chapter	Topic	Skills			
		 $\cos(x-y) = \cos x \cos y + \sin x \sin y$ $= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$ $= \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$			
2E	The double angle formulae	Derived from the addition formulae. For any angle A: $\sin 2A = 2\sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $= 1 - 2\sin^2 A$ e.g. Find the exact value of $\cos 2x$ and $\sin 2x$ where x is an acute angle with $\cos x = \frac{2}{5}$  $\cos 2x = \cos^2 x - \sin^2 x$ $= \left(\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2$ $= \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}$ $\sin 2x = 2\sin x \cos x$ $= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5}$ $= \frac{4\sqrt{21}}{25}$			
2F/2G	Proving trigonometric identities	Prove trigonometric identities are true by working with the expression on one side of the equals sign, and demonstrate it is the same as the other side e.g. Show that $\frac{\sin(a+b)}{\cos a \cos b} = \tan a + \tan b$ LHS $\frac{\sin(a+b)}{\cos a \cos b}$ $= \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b}$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
		$\frac{\sin(a+b)}{\cos a \cos b}$ $= \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b}$ $= \frac{\sin a \cos b}{\cos a \cos b} + \frac{\cos a \sin b}{\cos a \cos b}$ $= \frac{\sin a}{\cos a} + \frac{\sin b}{\cos b} = \tan a + \tan b$ So LHS = RHS			
2I	Working with the wave function $a\cos x + b\sin x$	$a\cos x + b\sin x$ can also be expressed as $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$ $k = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$			
2J	Express $a\cos x + b\sin x$ in the form $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$	e.g. express $3\cos x + 4\sin x$ in the form $k\cos(x - \alpha)$ $k = \sqrt{3^2 + 4^2} = 5$ $k\cos(x - \alpha) = k\cos x\cos\alpha + k\sin x\sin\alpha$ $3\cos x + 4\sin x = k\cos x\cos\alpha + k\sin x\sin\alpha$ $k\cos\alpha = 3$ and $k\sin\alpha = 4$ so $\tan\alpha = \frac{4}{3}$ $\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$ (1st quadrant as $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ are all positive) $\therefore 3\cos x + 4\sin x = 5\cos(x - 53.1)°$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
3 Identifying and sketching related functions					
3A	Sketching the graphs of related functions	Starting with a graph, $y = f(x)$ you are expected to sketch the graphs of related functions: • $y = f(x) + a$ is a vertical translation. When $a > 0$ the graph slides up by a, and when $a < 0$ the graphs slides down by a • $y = f(x + b)$ is a horizontal translation. When $b > 0$ the graph slides to the left by b, and when $b < 0$ the graphs slides to the right by b • $y = -f(x)$ is a reflection in the x-axis • $y = f(-x)$ is a reflection in the y-axis • $y = cf(x)$ is a vertical scaling. When $c > 1$ It is stretched vertically and when $0 < c < 1$ it is compressed vertically • $y = f(dx)$ is a horizontal scaling. When $d > 1$ It is compressed horizontally and when $0 < d < 1$ it is stretched horizontally			

Chapter	Topic	Skills			
		e.g. Part of the graph of $y=f(x)$ is shown  Use this to sketch $y=2-3f(x+1)$. Use rules similar to BODMAS i.e. stretch, compress or reflect before translations 			
3B	Expressing a quadratic function of the form ax^2+bx+c in completed square form	A quadratic function expressed as $a(x+p)^2+r$ can be used to help sketch the graph of a quadratic function e.g $4x^2+8x+3=4(x^2+2x)+3$ $=4(x^2+2x+1-1)+3=4((x+1)^2-1)+3$ $=4(x+1)^2-4+3=4(x+1)^2-1$			
3C	Working with the graphs of quadratic functions expressed in completed square form	The graph of $y=x^2$ is a parabola with a key point at (0,0), the minimum turning point.			

Chapter	Topic	Skills			
		 Use graph transformations in Exercise 3A to determine turning point e.g. the graph of $y = 2(x-5)^2 - 3$ has a minimum turning point at $(5, -3)$			
3D	Working with the graphs of exponential functions	Using the graphs of exponential functions from Exercise 1A, graphs of related functions can be drawn by considering the transformation on the key points $(0,1)$ and $(1,a)$ e.g. sketch the graph of $y = 2^{(x-2)} + 1$ If $f(x) = 2^x$, this is a sketch of $y = f(x-2)+1$:  Y intercept: $y = 2^{(0-2)} + 1 = 2^{-2} = 1 = \frac{1}{4} + 1 = \frac{5}{4}$			
3E	Working with the graphs of logarithmic functions	Using the graphs of logarithmic functions from Exercise 1B, graphs of related functions can be drawn by considering the transformation on the key points $(1,0)$ and $(a,1)$ e.g. sketch the graph of $y = 2\log_3(x-1)$ If $f(x) = \log_3 x$, this is a sketch of $y = 2f(x-1)$ 			
3F	Working with trigonometric graphs	Using the graphs of trigonometric functions from National 5, graphs of related functions $y = a\sin(bx+d)+c$ and $y = a\cos(bx+d)+c$ can be drawn by considering the transformation on the key points on the graphs of $y = \sin x$ and $y = \cos x$. Note that: • a is the amplitude of the function • there are b cycles in 360° e.g. the diagram shows a graph of the form $y = p\sin(x+q)+r$. Find the values of p, q and r			

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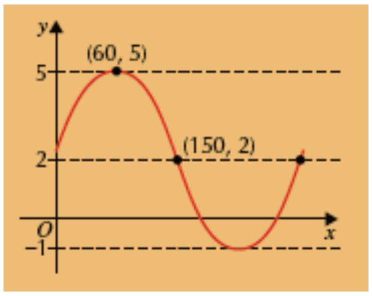
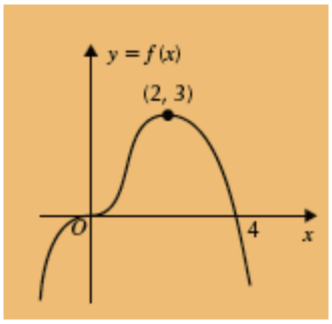
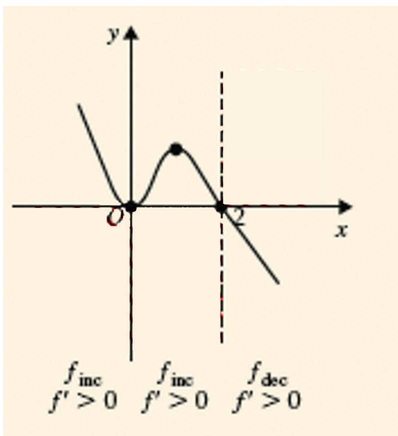
<tr><th>Chapter</th><th>Topic</th><th>Skills</th><th></th><th></th><th></th></tr>
<tr><td></td><td></td><td>

Amplitude = 3 $\therefore\ p = 3$
Vertical translation upwards by 2 of $y = 3\sin x$
$\therefore r = 2$
Horizontal translation 30° to the left $\therefore q = 30$</td><td></td><td></td><td></td></tr>
<tr><td>3G</td><td>Sketching the graph of $y = f'(x)$</td><td>Sketching graphs of $y = f'(x)$ links to Chapter 8.
• $f'(x) = 0$ when $f(x)$ is stationary
• $f'(x) > 0$ when $f(x)$ is increasing
• $f'(x) < 0$ when $f(x)$ is decreasing
e.g. the diagram shows the graph of $y = f(x)$


Sketch the graph of $y = f'(x)$


• $f(x)$ stationary when $x = 0$ and $x = 2$ $\rightarrow$ roots of $y = f'(x)$
• $f(x)$ increasing when $x < 0$ and $0 < x < 2$ $\rightarrow$ $y = f'(x)$ above x-axis
• $f(x)$ increasing when $x > 2$ $\rightarrow y = f'(x)$ below x-axis</td><td></td><td></td><td></td></tr>
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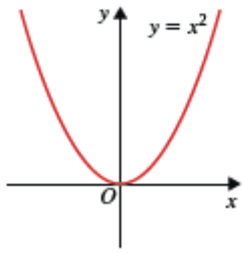
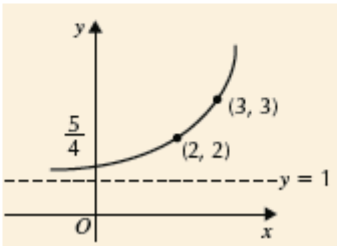
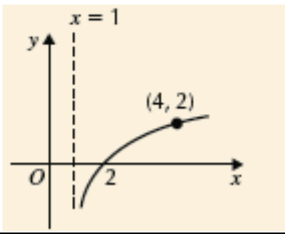
Chapter	Topic	Skills			
		 Use graph transformations in Exercise 3A to determine turning point e.g. the graph of $y=2(x-5)^2-3$ has a minimum turning point at $(5,-3)$			
3D	Working with the graphs of exponential functions	Using the graphs of exponential functions from Exercise 1A, graphs of related functions can be drawn by considering the transformation on the key points $(0,1)$ and $(1,a)$ e.g. sketch the graph of $y=2^{(x-2)}+1$ If $f(x)=2^x$, this is a sketch of $y=f(x-2)+1$:  Y intercept: $y=2^{(0-2)}+1=2^{-2}=1=\frac{1}{4}+1=\frac{5}{4}$			
3E	Working with the graphs of logarithmic functions	Using the graphs of logarithmic functions from Exercise 1B, graphs of related functions can be drawn by considering the transformation on the key points $(1,0)$ and $(a,1)$ e.g. sketch the graph of $y=2\log_3(x-1)$ If $f(x)=\log_3 x$, this is a sketch of $y=2f(x-1)$ 			
3F	Working with trigonometric graphs	Using the graphs of trigonometric functions from National 5, graphs of related functions $y=a\sin(bx+d)+c$ and $y=a\cos(bx+d)+c$ can be drawn by considering the transformation on the key points on the graphs of $y=\sin x$ and $y=\cos x$. Note that: • a is the amplitude of the function • there are b cycles in 360° e.g. the diagram shows a graph of the form $y=p\sin(x+q)+r$. Find the values of p, q and r			

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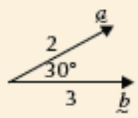
4 Determining composite and inverse functions					
4A	Determining the domain and range of a function	A function $f(x)$ from a set of numbers (the domain) to a new set of numbers (the range) is a rule by which each member of the domain is linked to exactly one member of the range. Some functions place a restriction on the domain so that each input value has a corresponding output value e.g. $f(x)=\frac{2}{x-1}$, $x \neq 1$ (An input of $x=1$ gives $\frac{2}{0}$ which is undefined e.g. $f(x)=\sqrt{x-2}$, $x \geq 2$ (An input of $x<2$ gives the root of a negative value which is undefined)			
4B	Determining the formula for a composite function	$g(f(x))$ is the composition of two functions, with the f function applied to a set of numbers followed by the g function. Substitute one function into the other. e.g. Let $f(x)=3x+4$ and $g(x)=x^2$ $g(f(x))=g(3x+4)=(3x+4)^2$ $f(g(x))=f(x^2)=3x^2+4$			
4C	Inverse of a functions	If $f(g(x))=g(f(x))=x$, then g is the inverse function of f, and is denoted as $f^{-1}(x)$. e.g. find the inverse of $f(x)=3x-2$ $y=3x-2$ $3x-2=y \quad \Rightarrow 3x=y+2 \quad \Rightarrow x=\frac{y+2}{3}$ $\Rightarrow f^{-1}(y)=\frac{y+2}{3} \Rightarrow f^{-1}(x)=\frac{x+2}{3}$			
4D	Extension to include logarithmic and exponential functions	Apply process in Exercise 4B to establish composite logarithmic functions			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
5 Determining vector connections					
5A	Working with vectors	For any vector, **v**, there exists a vector one unit long that has the same direction as **v**. e.g. If $v=\begin{pmatrix}5\\12\end{pmatrix}$ then $\lvert v\rvert=\sqrt{5^2+12^2}=13$ The unit vector in the direction of v is $\frac{1}{13}\begin{pmatrix}5\\12\end{pmatrix}=\begin{pmatrix}\frac{5}{13}\\\frac{12}{13}\end{pmatrix}$			

Chapter	Topic	Skills			
		There are three special unit vectors **i**, **j** and **k** representing a vector 1 unit long in the direction of the positive x, y and z axes. A vector in component form can also be expressed in terms of **i**, **j** and **k** . This is called basis vector form. e.g. $v=\begin{pmatrix}4\\-2\\3\end{pmatrix}=4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$			
5B	Position vectors and coordinates	The position of a point in 2D or 3D can be given in coordinate form (x,y,z) or as a position vector $\overrightarrow{OP}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ which is the vector from origin O to point P. Position vectors can be used to derive a formula for the vector $\overrightarrow{AB}$ $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=-\mathbf{a}+\mathbf{b}=\mathbf{b}-\mathbf{a}$			
5C	Calculating the coordinates of an internal division point of a line	Convert the ratio into a fraction to determine the new coordinates of a point between the given coordinate points. e.g. Given points A(4, -6, 12) and B(4, 4, -3), P divides AB in the ratio 3:2. $\overrightarrow{AB}=\begin{pmatrix}4\\4\\-3\end{pmatrix}-\begin{pmatrix}4\\-6\\12\end{pmatrix}=\begin{pmatrix}0\\10\\-15\end{pmatrix}$ $\overrightarrow{AP}=\frac{3}{5}\overrightarrow{AB}=\frac{3}{5}\begin{pmatrix}0\\10\\-15\end{pmatrix}=\begin{pmatrix}0\\6\\-9\end{pmatrix}$ $\overrightarrow{AP}=\mathbf{p}-\mathbf{a}\rightarrow\mathbf{p}=\overrightarrow{AP}+\mathbf{a}=\begin{pmatrix}0\\6\\-9\end{pmatrix}+\begin{pmatrix}4\\-6\\12\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$ P has coordinated (4, 0, 3)			
5D	Determining the resultant of vector pathways in 3 dimensions	Determine an unbroken route from the starting point to the finishing point of a resultant vector using the named vectors within a diagram. Simplify your answer where necessary.			
5E	Working with parallel vectors and collinearity	For any non-zero scalar k, $k\mathbf{u}$ is parallel to $\mathbf{u}$. Three points are collinear if they lie on the same straight line e.g. Show that A(5, 6, -1), B(6,3,2) and C(8, —3, 8) are collinear $\overrightarrow{AB}=\begin{pmatrix}1\\-3\\3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}2\\-6\\6\end{pmatrix}=2\begin{pmatrix}1\\-3\\3\end{pmatrix}$ $2\overrightarrow{AB}=\overrightarrow{BC}\therefore\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel. They share common point B $\therefore$ collinear			

Chapter	Topic	Skills			
6 Working with vectors					
6A	Working with the zero vector	If a pathway starts and ends at the same point, the resultant vector is 0, the zero vector, which is $\begin{pmatrix}0\\0\end{pmatrix}$ in 2 dimensions and $\begin{pmatrix}0\\0\\0\end{pmatrix}$ in 3 dimensions.			
6B	Working with the scalar product of 2 vectors	There are two scalar product formulae: Given that **a** and **b** are two non-zero vectors with ϑ the angle between their positive directions, the scalar product is defined as $\mathbf{a}\bullet\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\vartheta$ e.g.  $\underset{\sim}{a}\bullet\underset{\sim}{b} = \lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos\theta°$ $= 2\times3\times\cos 30°$ $= 2\times3\times\frac{\sqrt{3}}{2}$ $= 3\sqrt{3}$ Given two position vectors $a = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $b = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, the scalar product can be calculating using: $\mathbf{a}\bullet\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$			
6C	Using the scalar product to calculate the angle between two vectors	Re-arrange the first definition of scalar product: $\cos\vartheta = \frac{\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ Then substitute the component form of the definition for $\mathbf{a}\bullet\mathbf{b}$: $\cos\vartheta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ e.g. calculate the angle between the vectors $u = \begin{pmatrix}4\\2\\5\end{pmatrix}$ and $v = \begin{pmatrix}3\\1\\2\end{pmatrix}$ $\lvert u\rvert = \sqrt{4^2 + 2^2 + 5^2} = \sqrt{45}$ $\lvert v\rvert = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$ $\mathbf{u}\bullet\mathbf{v} = 12 + 2 + 10 = 24$ $\cos\vartheta = \frac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert} = \frac{24}{\sqrt{45}\times\sqrt{14}}$ $\vartheta = \cos^{-1}\left(\frac{24}{\sqrt{45}\times\sqrt{14}}\right) = 17.0°$			

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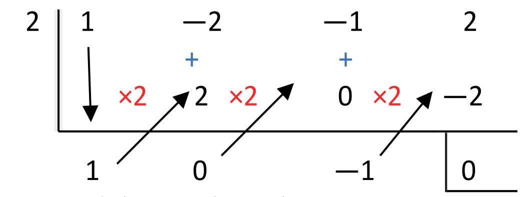
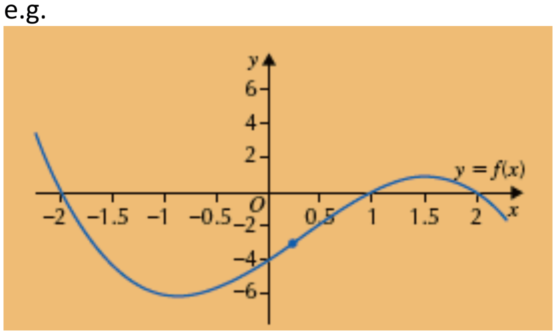
Chapter	Topic	Skills			
6D	Working with perpendicular vectors	If vector a is perpendicular to b, then $\vartheta = 90°$ $\mathbf{a} \cdot \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 90° = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \times 0 = 0$ e.g. Given than $u = 3\mathbf{i} + 2\mathbf{j}$ and $v = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ show that u and v are perpendicular. $\mathbf{u} \cdot \mathbf{v} = 3 \times 2 + 2 \times (-3) + 0 \times 4 = 6 - 6 + 0 = 0$ $\mathbf{u} \cdot \mathbf{v} = 0 \therefore$ perpendicular			
6E	Working with the properties of the scalar product	Use these properties to calculate the scalar product of more complex calculations: • $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$ (commutative) • $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ (distributive) • $\mathbf{a} \cdot \mathbf{a} = \lvert\mathbf{a}\rvert^2$			

Chapter	Topic	Skills			
7 Solving Algebraic Equations					
7A	Introduction to polynomials	A polynomial is a sum or difference of algebraic terms when all the indices are whole numbers. The degree of a polynomial ins the value of the highest power. If $f(a) = 0$ then $(x-a)$ is a factor of $f(x)$			
7B/C/D	Factorising a cubic polynomial	Synthetic division uses "nesting" to simplify the process of dividing. Express the polynomial in order from highest to lowest power, and find a factor $(x-a)$ which evaluates to $f(a) = 0$ e.g. a. Show that $(x-2)$ is a factor of $f(x) = x^3 - 2x^2 - x + 2$  Since $f(2) = 0$, $(x-2)$ is a factor (x=2 is a root) b. Fully factorise $f(x)$ The values at the bottom of the table represent the coefficients of the remaining quotient, starting one degree lower than $f(x)$ $f(x) = (x-2)(x^2-1)$ (no x term) $f(x) = (x-2)(x-1)(x+1)$			
7E	Calculating unknown coefficients	Use the factor theorem to determine unknown coefficients in a polynomial. If there are more than one unknown coefficients within a question, find two equations using the information in the question then solve using simultaneous equations.			

Chapter	Topic	Skills			
7F	The remainder theorem	Use synthetic division or substitution to determine the remainder when dividing a polynomial.			
7G	Determining the points of intersection of the graph with the x axis	Roots of the graph are points at which the graph of the function intersects the x-axis (when y=0). Factorise using synthetic division then solve (see Exercise 7I)			
7H	Determining the function of a polynomial from its graph	Reverse the process used to find roots in Exercise 7G. For example, a cubic function can be expressed as $f(x)=k(x-a)(x-b)(x-c)$ where a, b and c are the roots and k is a numerical common factor affecting the amplitude. Use other information given in the sketch to determine k e.g.  Roots are x=—2, x=1 and x=2 Factors are $(x+2)$, $(x-1)$ and $(x-2)$ $y=k(x+2)(x-1)(x-2)$ Substitute (0,—4): $-4=k(0+2)(0-1)(0-2)$ $-4=4k \rightarrow k=-1$ So $f(x)=-(x+2)(x-1)(x-2)$			
7I	Solving a polynomial equation	Ensure one side of the equation is equal to zero. Factorise and solve by setting each factor equal to zero (similar to process of solving quadratic equations). e.g. Solve $x^3+x^2-10x+8=0$ $\begin{array}{c\|cccc} & x^3 & x^2 & x & c \\ 1 & 1 & 1 & -10 & 8 \\ & \downarrow & 1 & 2 & -8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array}$ $(x-1)(x^2+2x-8)=0$ $(x-1)(x-2)(x+4)=0$ $x-1=0,\ x-2=0,\ x+4=0$ $x=1,\ 2,\ -4$			
7J	Determining the point of intersection of a straight line and a curve (or two curves)	Substitute one equation into the other (as the y-coordinates are the same at the point(s) of intersection. Rearrange and solve the equation, then substitute each x-coordinate into one of the original equations to find the y-coordinate			

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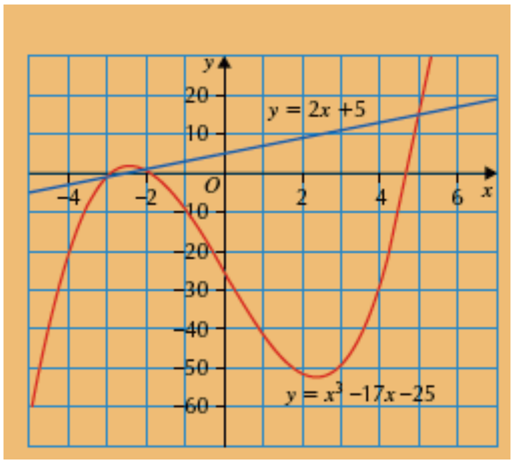
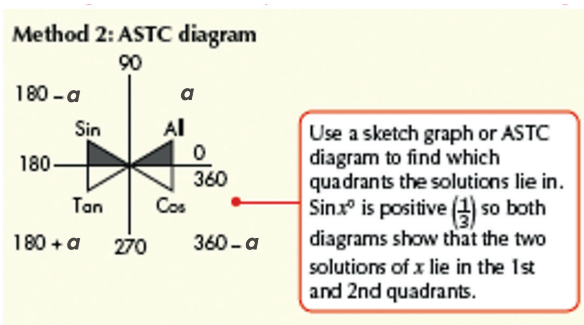
Chapter	Topic	Skills			
		e.g. determine the coordinates of the points of intersection of this line and curve  $x^3-17x-25=2x+5$ $x^3-19x-30=0$ $x^3 \quad x^2 \quad x \quad c$ $-2 \mid 1 \quad 0 \quad -19 \quad -30$ $\downarrow \quad -2 \quad 4 \quad 30$ $1 \quad -2 \quad -15 \quad \boxed{0}$ $(x+2)(x^2-2x-15)=0$ $(x+2)(x+3)(x-5)=0$ $x=-2,\ -3,\ 5$ $y=2(-2)+5,\ 2(-3)+5,\ 2(5)+5=1,\ -1,\ 15$ Points of intersection at (−2,1), (−3, −1) and (5,15).			
7K	Using the discriminant to determine an unknown coefficient in a quadratic equation	For $ax^2+bx+c=0$: • If $b^2-4ac=0$ there are two real, equal roots • If $b^2-4ac>0$ there are two real, distinct roots • If $b^2-4ac<0$ there are no real roots e.g. the equation $3x^2+kx+3=0$ has equal roots. $b^2-4ac=0 \rightarrow k^2-4\times3\times3=0$ $k^2-36=0 \rightarrow (k-6)(k+6)=0$ $k=6,\ -6$			

Chapter	Topic	Skills			
8A	Solving trigonometric equations in degrees	In this topic, use the CAST diagram and the trigonometric graphs from National 5 Mathematics  			

Chapter	Topic	Skills			
		You will now be required to work in intervals beyond $0° \le x \le 360°$ and to work with: • exact value ratios for the sine, cosine and tangent of 30°, 60°, 45° and their related angles • exact values for the sine, cosine and tangent of angles that are multiples of 90° from their graphs			
8B	Solving linear equations with compound angles	• Solve to find the compound angle • Use the result for the compound angle to find x e.g. solve $5\tan 2x° = 3,\ \ 0 \le x \le 360$ $5\tan 2x° = 3 \rightarrow \tan 2x° = \frac{3}{5}$ Using CAST diagram $2x° = \tan^{-1}\left(\frac{3}{5}\right) = 31.0,\ 180 + 31 = 31.0, 211.0$ Add 360° to each as it represents $0 \le 2x \le 720$ $2x° = 31.0,\ 211.0,\ 391.0,\ 571.0$ $\therefore x = 15.5,\ 105.5,\ 195.5,\ 285.5$			
8C	Solving quadratic trigonometric equations	Factorise the quadratic equation, then solve each trig equation separately e.g. solve $3\sin^2 x° - 2\sin x° - 1 = 0,\ 0 \le x \le 360$ $(3\sin x + 1)(\sin x - 1) = 0$ $3\sin x + 1 = 0 \rightarrow \sin x = -\frac{1}{3}$ $a = \sin^{-1}\left(\frac{1}{3}\right) = 19.5°$ Using CAST: $x = 180 + 19.5,\ 180 - 19.5 = 199.5,\ 340.5$ $\sin x - 1 = 0 \rightarrow \sin x = 1$ Using sine graph: $x = 90$ $\therefore$ Solution $x = 90,\ 199.5,\ 340.5$			
8D	Solving trigonometric equations in radians	Use the same method as working in degrees e.g. $2\sin x + 1 = 0,\ 0 \le x \le 2\pi$ $2\sin x = -1 \rightarrow \sin x = -\frac{1}{2}$ $a = \sin^{-1}\left(\frac{1}{2}\right) = 30° = \frac{\pi}{6}$ $x = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6} = \frac{7\pi}{6},\ \frac{11\pi}{6}$			
8E	Solving quadratic trigonometric equations using radians	Combine the methods required in Exercise 8C and Exercise 8D to solve quadratic trigonometric equations working in radians.			
8F/G	Solving equations using standard trigonometric identities	• Substitute the relevant identity (see Exercise 2E • Factorise and solve each linear equation separately $2\sin 2x° + 3\sin x° = 0,\ 0 \le x \le 360$ $2(2\sin x° \cos x°) + 5\sin x° = 0$ $4\sin x° \cos x° + 5\sin x° = 0$ e.g. Solve $\sin x°(4\cos x° + 5) = 0$			

Chapter	Topic	Skills			
		$\sin x° = 0 \rightarrow x° = 0, 180, 360$ $\cos x = -\frac{5}{4} \rightarrow$ no solutions ($y = \cos x$ has a minimum value of —1)			
8H	Solving equations of the form $a\cos x + b\sin x + c = 0$	• Express $a\cos x + b\sin x$ in the form $k\sin(x \pm a)$ or $k\cos(x \pm a)$ from Exercise 2J • Solve using method in Exercise 8B for compound angles.			

Chapter	Topic	Skills			
9 Differentiating Functions					
9A	Differentiating a functions already expressed in powers of x	Use these rules to differentiate: • $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$ • $y = ax^n \Rightarrow \frac{dy}{dx} = anx^{n-1}$ • $y = f(x) + g(x) \Rightarrow \frac{dy}{dx} = f'(x) + g'(x)$ e.g. differentiate $y = x^4 + 3x^{-2} + x^{\frac{1}{2}}$ $\frac{dy}{dx} = 4x^3 - 6x^{-3} + \frac{1}{2}x^{-\frac{1}{2}}$			
9B	Differentiating a function which is not already an expression in powers of x	• Use the laws of indices to convert your expression into differentiable form $\frac{1}{x} = x^{-n}$ and $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ e.g. $f(x) = \frac{1}{4x^8} + 6\sqrt{x}$ $f(x) = \frac{1}{4}x^{-8} + 6x^{\frac{1}{2}}$ $f'(x) = \frac{1}{4} \times -8x^{-9} + 6 \times \frac{1}{2}x^{-\frac{1}{2}} = -2x^{-9} + 3x^{-\frac{1}{2}}$ • Multiply out brackets e.g. $y = (3x-2)(x+5)$ $y = 3x^2 + 13x - 10 \rightarrow \frac{dy}{dx} = 6x + 13$ • Simplify quotient e.g. $f(x) = \frac{x^3 - 4x + 5}{x^2}$ $f(x) = \frac{x^3}{x^2} - \frac{4x}{x^2} + \frac{5}{x^2} = x - 4x^{-1} + 5x^{-2}$ $f'(x) = 1 + 4x^{-2} - 10x^{-3}$			

Chapter	Topic	Skills	Green	Yellow	Red
9D	Evaluating derivatives – rates of change and gradient of tangent to curves	• Differentiate • Substitute given value for x e.g. a parabola has equation $y=x^2-4x+6$ with a tangent drawn at P$(4,6)$. Find the gradient of the tangent $\frac{dy}{dx}=2x-4$ When x=4 $\frac{dy}{dx}=2(4)-4=4$			
9E	Differentiate $k\sin x$, $k\cos x$	$f(x)=k\sin x \to f'(x)=k\cos x$ $f(x)=k\cos x \to f'(x)=-k\sin x$			
9G	Differentiate a composite function using the chain rule	$y=f(g(x)) \Rightarrow \frac{dy}{dx}=f'(g(x))\times g'(x)$ e.g. differentiate $y=(5x-2)^7$ $\frac{dy}{dx}=7(5x-2)^6\times 5=35(5x-2)^6$			
9H	Differentiate $\sin ax$, $\cos ax$	$f(x)=\sin ax \to f'(x)=a\cos ax$ $f(x)=\cos ax \to f'(x)=-a\sin ax$ Note: only true when x is measured in radians			

Chapter	Topic	Skills	Green	Yellow	Red
10 Using differentiation to investigate the nature and properties of functions					
10A	Determine the equation of a tangent to a curve	• Find the gradient of the tangent: differentiate then substitute x coordinate into derivative (see Exercise 9D) • Substitute this gradient and the given coordinate into $y-b=m(x-a)$ e.g. find the equation of the tangent to the curve $y=x^2-4x+7$ at the point (3,4) $\frac{dy}{dx}=2x-4$. When x=3, $\frac{dy}{dx}=2(3)-4=2$ $y-4=2(x-3) \to y=2x-2$			
10B	Determine where a function is strictly increasing or decreasing	• When $f'(a)>0$ the function $f(x)$ is strictly increasing at the point where $x=a$ • When $f'(a)<0$ the function $f(x)$ is strictly decreasing at the point where $x=a$ • When $f'(a)=0$ the function $f(x)$ is stationary at the point where $x=a$			
10C	Determine the stationary points of a function	• Differentiate the function • Solve $\frac{dy}{dx}$ to find the x coordinates of the stationary point • Substitute into the original function to find the y-coordinates • Draw a nature table for each point • Communicate your answer			

<table>
<tr><th>Chapter</th><th>Topic</th><th>Skills</th><th></th><th></th><th></th></tr>
<tr><td></td><td></td><td>
e.g. Find the stationary points of
$y=\frac{1}{3}x^3-2x^2-12x+15$
$\frac{dy}{dx}=x^2-4x-12$
$x^2-4x-12=0 \rightarrow (x+2)(x-6)=0$
$x=-2,6$
$y=\frac{1}{3}(-2)^3-2(-2)^2-12(-2)+15=18\frac{1}{3}$
$y=\frac{1}{3}(6)^3-2(6)^2-12(6)+15=-67$
Stationary points are $\left(-2,18\frac{1}{3}\right)$ and $(6,-67)$
<table>
<tr><td>x</td><td>…</td><td>−2</td><td>…</td><td>…</td><td>6</td><td>…</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>+</td><td>0</td><td>−</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>Shape</td><td>/</td><td>—
Maximum SP</td><td>\</td><td>\</td><td>—
Minimum SP</td><td>/</td></tr>
</table>
$\therefore$ Maximum SP at $\left(-2,18\frac{1}{3}\right)$ and minimum SP at $(6,-67)$
</td><td></td><td></td><td></td></tr>
<tr><td>10D</td><td>Sketching the graph of an algebraic fraction</td><td>
• Find the points of intersection with the x- and y-axes (see Exercise 7G)
• Find stationary points and determine their nature (see Exercise 10A)
• Consider the behaviour of the function for large positive and negative values of x
• Sketch the graph using the above information
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Chapter	Topic	Skills			
11 Integrating Functions					
11A	Integrating a function already expressed in powers of x	Use these rules to integrate: • $\int x^n dx=\frac{x^{n+1}}{n+1}+c$ • $\int ax^n dx=\frac{ax^{n+1}}{n+1}+c$ • $\int(f(x)+g(x))dx=\int f(x)dx+\int g(x)dx$ • $\int kdx=kx+c$ e.g. $\int(x^2-4x^{-3}+2)dx$ $=\frac{x^3}{3}-\frac{4x^{-2}}{-2}+2x+c=\frac{1}{3}x^3+2x^{-2}+2x+c$			
11B	Integrating a function which is not already an expression in powers of x	• Use the laws of indices to convert your expression into integrable form $\frac{1}{x}=x^{-n}$ and $\sqrt[n]{x^m}=x^{\frac{m}{n}}$ • Multiply out brackets • Simplify quotients			

Chapter	Topic	Skills	Green	Yellow	Red
11C	Integrate functions of the form $(x+q)^n$ and $(px+q)^n$	• $\int (x+q)^n = \frac{(x+q)^{n+1}}{n+1} + c$ e.g. $\int (x+1)^5\, dx = \frac{(x+1)^6}{6} + c$ $\int (px+q)^n = \frac{(px+q)^{n+1}}{p(n+1)} + c$ e.g. $\int (3x+7)^4\, dx$ $= \frac{(3x+7)^5}{3\times 5} + c = \frac{1}{15}(3x+7)^5 + c$			
11D/E	Integrate functions of the form $p\sin x$, $p\cos x$, $p\sin(qx+r)$, $p\cos(qx+r)$	$\int \sin x dx = -\cos x + c$ $\int \cos x dx = \sin x + c$ $\int \sin(qx+r)\,dx = -\frac{1}{q}\cos(qx+r) + c$ $\int \cos(qx+r)\,dx = \frac{1}{q}\sin(qx+r) + c$ Note: only true when x is measured in radians			
11G	Solve differential equations of the form $\frac{dy}{dx} = f(x)$	Integrate the differential equation to find the new function, then substitute the given information to find a value for c e.g. Given that $\frac{dy}{dx} = x^2 - 6$ and when x=—3, y=7. Express y in terms of x $y = \int (x^2 - 6)dx \rightarrow y = \frac{1}{3}x^3 - 6x + c$ $7 = \frac{1}{3}(-3)^3 - 6(-3) + c \rightarrow 7 = -9 + 18 + c$ $c = -2 \rightarrow y = \frac{1}{3}x^3 - 6x - 2$			

Chapter	Topic	Skills	Green	Yellow	Red
12 Using integration to calculate definite integrals					
12A	Evaluating definite integrals	Definite integration is defined by: $\int_a^b f(x)dx = F(b) - F(a)$ where a and b are the limits of the integral e.g. $\int_1^3 (3x^2 - 2x)dx = \left[\frac{3x^3}{3} - \frac{2x^2}{2}\right]_1^3 = \left[x^3 - x^2\right]_1^3$ $= (3^3 - 3^2) - (1^3 - 1^2) = 18 - 0 = 18$			
12B	Evaluating definite integrals for trigonometric functions	Use integration from Exercise 11D to calculate definite integrals for trigonometric functions. These results are only true when x is measured in radians			

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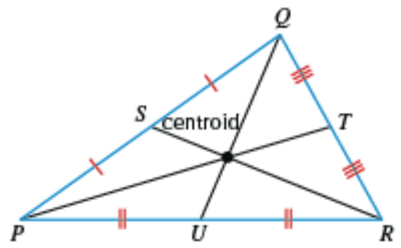
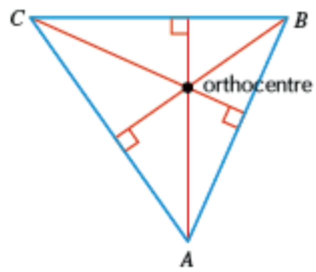
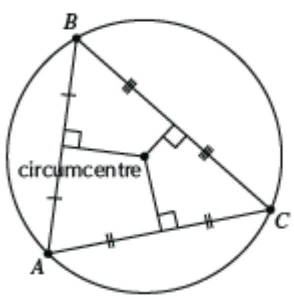
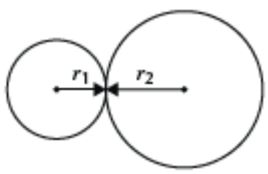
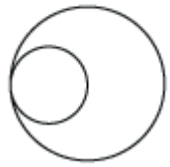
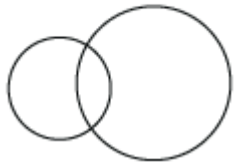
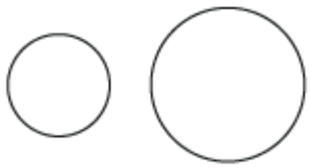
Chapter	Topic	Skills	☐	☐	☐
		e.g. $\int_0^{\frac{\pi}{2}} 4\cos x dx = [4\sin x]_0^{\frac{\pi}{2}}$ $= \left(4\sin\frac{\pi}{2}\right) - 4\sin 0 = 4 - 0 = 4$			

Chapter	Topic	Skills	☐	☐	☐
13 Applying algebraic skills to rectilinear shapes					
13A	Parallel lines	Parallel lines have equal gradients e.g. find the equation of a line that passes through the point $(4,-2)$ and parallel to $3x+2y=7$ $3x+2y=7 \rightarrow 2y=-3x-7$ $y=-\frac{3}{2}x-\frac{7}{2} \rightarrow m=-\frac{3}{2}$ $y-(-2)=-\frac{3}{2}(x-4)$ $y+2=-\frac{3}{2}x+6 \rightarrow 2y+3x=8$			
13B	Collinearity	A set of points that lie on the same straight line are collinear. e.g. show that A(—5, —4), B(1,4) and C(4,8) are collinear $m_{AB}=\frac{4-(-4)}{1-(-5)}=\frac{8}{6}=\frac{4}{3}$ $m_{BC}=\frac{8-4}{4-1}=\frac{4}{3}$ $m_{AB}=m_{BC}$ ∴AB and BC are parallel Share common point B, so collinear			
13C	Perpendicular lines	Any two straight line that meet are right angles are perpendicular to one another. Their gradients will multiply to give -1: $m \times m_{\perp} = -1$ e.g. The gradient of a line perpendicular to a line with a gradient of $\frac{3}{5}$ has gradient: $-\frac{5}{3}$ $\left(\frac{3}{5}\times-\frac{5}{3}=-1\right)$			
13D	Gradients and angles	The angle between a line and the positive direction of the x axis can be found using the formula: $m=\tan\vartheta$			
13E	Special lines in a triangle	A median joins a vertex to the midpoint of the opposite side. Find the midpoint of the opposite side using the midpoint formula $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$. Use the vertex and the midpoint to find the equation of the straight line.			

Chapter	Topic	Skills	☐	☐	☐
		 Every triangle has three medians. They all pass through the same point (called the centroid) An altitude is a line through a vertex perpendicular to the opposite side. Find the gradient of the opposite side, and use $m \times m_{\perp} = -1$ to establish the gradient of the altitude. Use the vertex and the gradient to find the equation of the straight line.  Orthocentre is inside the triangle. Every triangle has three altitudes. They all pass through the same point (called the orthocentre). A line that cuts another in half at right angles is called the perpendicular bisector. Find the midpoint using the formula, and find the gradient of the original line to find the gradient of the perpendicular bisector using $m \times m_{\perp} = -1$. Use your midpoint and the perpendicular gradient to find the equation of the line.  Every triangle has three perpendicular bisectors. They all pass through the same point (called the circumcentre). The intersection of two straight lines can be calculated using simultaneous equations.			
		The intersection of two straight lines can be calculated using simultaneous equations.			

Chapter	Topic	Skills	🟩	🟨	🟥
14 Applying algebraic skills to circles					
14A	The equation of a circle $(x-a)^2+(y-b)^2=r^2$	The equation $x^2+y^2=r^2$ represents a circle with centre $(0,0)$, radius r and where (x,y) represents any point on the circumference of the circle. The equation $(x-a)^2+(y-b)^2=r^2$ represents a circle with centre (a,b), radius r and where (x,y) represents any point on the circumference of the circle.			
14B	The general equation of the circle	$x^2+y^2+2gx+2fy+c=0$ represents a circle with centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$. For a circle to exist, $g^2+f^2-c>0$. e.g. does $x^2+y^2-4x+2y+8=0$ represent a circle? $2g=-4 \rightarrow g=-2,\ 2f=2 \rightarrow f=1$ $g^2+f^2-c=(-2)^2+1^2-8=-3<0$ So this does **not** represent a circle			
14C	Use properties of tangency when solving problems	When a straight line touches a circle at exactly one point, the line is a tangent to the circle. To show that a line is a tangent to a circle, substitute the equation of the straight line into the circle equation, rearrange so that one side of the equation equals zero, and use your discriminant (b^2-4ac) to demonstrate there is only one point of contact. To find the point of contact, factorise your re-arranged equation and solve for one coordinate, then substitute into the straight line equation for the other coordinate. To find the equation of a tangent: find the gradient of the radius between the centre and the given point of contact, then find the gradient of the tangent as it is perpendicular to the radius at this point. Use the perpendicular gradient and the point of contact to find the equation of the straight line. e.g. find the equation of the tangent to the circle $x^2+y^2-4x-2y-3=0$ Centre $(2,1)$ $m=\frac{3-1}{4-2}=1$ $m_{radius} \times m_{tangent}=-1$ $m_{\perp}=1$ $y-3=-(x-4) \rightarrow y+x=7$			
14D	Intersecting circles	Compare the distance between the two centres (d) and the sum of the radii (r_1+r_2):  			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
		Circles meet externally as $d = r_1 + r_2$  Circles meet internally as $d = r_1 - r_2$  Circles intersect at two points as $d < r_1 + r_2$  Circles do not touch as $d > r_1 + r_2$  Circles do not touch: one is inside the other as $d < r_1 - r_2$			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
15 Modelling situations using sequences					
15A	Nth term formulae	A sequence is an ordered list of numbers generated by a set rule. Each number is called a term. Terms are denoted by $u_1, u_2, \ldots u_n, u_{n+1}, \ldots$ A formula for the nth term requires a substitution of the position number (n) to the formula to generate the sequence e.g. $u_n = 2n - 1$ generates the sequence 1, 3, 5, 7,.... by substituting values for n (1,2, 3, ...)			
15B	Determining a recurrence relation from given information	A recurrence relation is defined by a starting value together with a rule that shows the connection between a term and the one following it in the sequence. A linear recurrence relation is written as $u_{n+1} = mu_n + c$ where m and c are constant e.g. $u_{n+1} = 2u_n - 1$			
15C	Using a recurrence relation to calculate a required term	Starting with the initial term (u_0) use the recurrence relation to calculate successive terms. e.g. $u_{n+1} = 2u_n - 1$ and $u_0 = 2$, calculate u_3 $u_1 = 2 \times 2 - 1 = 3$ $u_2 = 2 \times 3 - 1 = 5$ $u_3 = 2 \times 5 - 1 = 9$			

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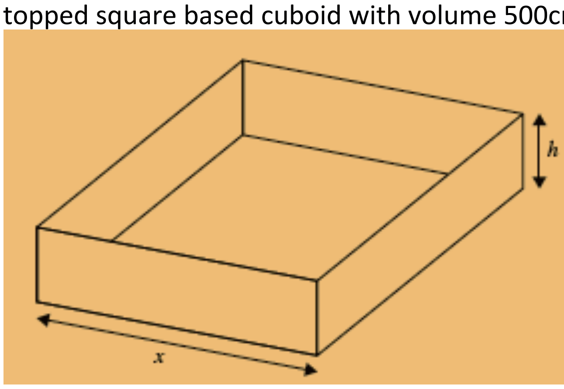
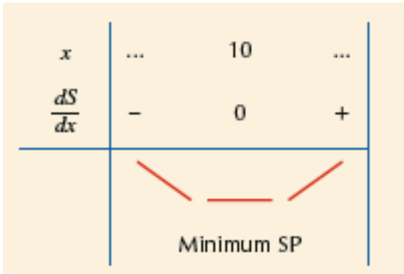
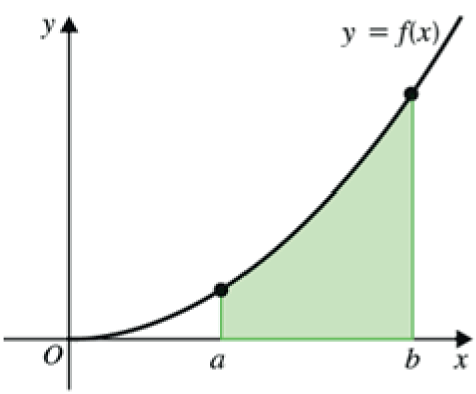
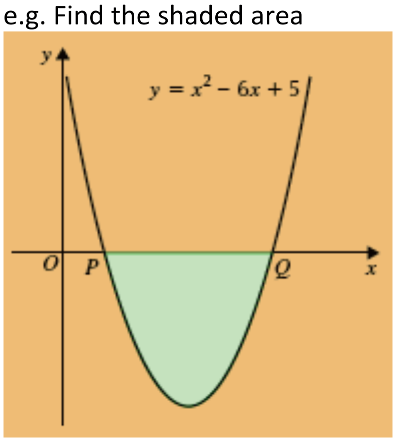
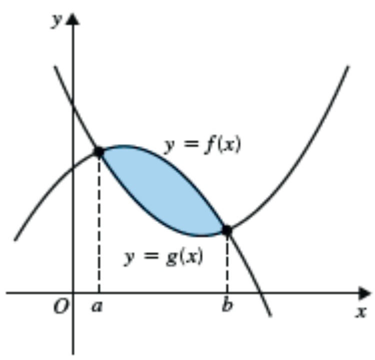
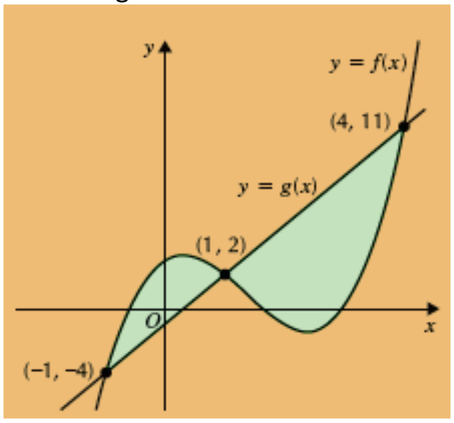
Chapter	Topic	Skills			
		e.g. A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$. Find the values of m and c if $u_1 = 32$, $u_2 = 20$ and $u_3 = 14$. $20 = 32m + c$ $14 = 20m + c$ $6 = 12m \rightarrow m = 0.5$ $c = 14 - 20(0.5) = 4$			
15D	The limit of a sequence	A recurrence relation of the form $u_{n+1} = mu_n + c$ converges if $-1 < m < 1$. Eventually each successive term will have the same value. This is called the limit. To calculate the limit we can substitute L for limit into our recurrence relation: $L = mL + c$ This can be rearranged to: $L = \frac{c}{1-m}$ e.g. Explain why $u_{n+1} = 0.7u_n + 15$ converges to a limit as $n \rightarrow \infty$ and find the value of the limit Converges as $-1 < 0.7 < 1$ $L = \frac{c}{1-m} = \frac{15}{1-0.7} = \frac{15}{0.3} = 50$			

Chapter	Topic	Skills			
16 Applying Differential Calculus					
16A	Determine the greatest and least values of an algebraic function on a given interval	To determine the greatest and least values of a function $f(x)$ on a closed interval: • find the stationary points and determine their nature (see Exercise 10C) • find the value of the function at the endpoints of the interval (determine $f(a)$ and $f(b)$) • make a sketch to represent the stationary points and endpoints • communicate the greatest and least values			
16B	Determining the optimal solution to a given problem	Differentiation can solve real-life problems that involve maximising or minimising a given quantity. e.g A small baking tin is in the shape of an open-topped square based cuboid with volume 500cm^3.  a. Show that the surface area, S, of metal needed to make the tin is given by $S = x^2 + \frac{2000}{x}$.			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
		$V = lbh$ $500 = x \times x \times h = x^2 h \rightarrow h = \frac{500}{x^2}$ $S = x^2 + xh \times 4 = x^2 + 4xh$ $S = x^2 + 4x\left(\frac{500}{x^2}\right) = x^2 + \frac{2000}{x}$ b. Find the dimensions of the tin that minimise the surface area $S = 2000x^{-1} + x^2$ $\frac{dS}{dx} = -2000x^{-2} + 2x = 2x - \frac{2000}{x}$ $2x - \frac{2000}{x} = 0 \rightarrow 2x = \frac{2000}{x}$ $2x^3 = 2000 \rightarrow x^3 = 1000 \rightarrow x = 10$  Minimum surface area when x=10 $x = 10,\ h = \frac{500}{10^2} = 5$ Dimensions are 10cm by 10cm by 5cm			
16C	Solving problems involving the rate of change	Given a function that describes displacement with respect to time, we can obtain a function for velocity by differentiation. Given a function that describes velocity with respect to time, we can obtain a function for acceleration by differentiation.			

Chapter	Topic	Skills	☐ (green)	☐ (yellow)	☐ (red)
17 Applying Integral Calculus					
17A	Find the area between the curve and the x axis	 The area under a curve $y = f(x)$ between $x = a$ and $x = b$ is given by: $\int_a^b f(x)dx = F(b) - F(a)$			

Chapter	Topic	Skills			
		• If the curve lies entirely above the x-axis between $x=a$ and $x=b$, expect a positive answer when evaluating the integral. • If the curve lies entirely below the x-axis between $x=a$ and $x=b$, expect a negative answer when evaluating the integral. Convert this to a positive value (be careful with communication) • If the curve lies entirely below the x-axis between $x=a$ and $x=b$, expect a negative answer when evaluating the integral. Convert this to a positive value (be careful with communication) • If part of the curve lies above the x axis and part lies below, evaluate the two separate integrals and then calculate the area. e.g. Find the shaded area  Find the x-coordinates of P and Q: $x^2-6x+5=0 \rightarrow (x-1)(x-5)=0 \rightarrow x=1,5$ $\int_1^5 (x^2-6x+5)\,dx = \left[\frac{1}{3}x^3-3x^2+5x\right]_1^5$ $=\left(\frac{1}{3}(5)^3-3(5)^2+5(3)\right)-\left(\frac{1}{3}(1)^3-3(1)^2+5(1)\right)$ $=\left(-\frac{25}{3}\right)-\left(\frac{7}{3}\right)=-\frac{32}{3}$ $\therefore \text{Area} = \frac{32}{3}\text{ square units}$			
17B	Find the area between a straight line and a curve or between two curves				

Chapter	Topic	Skills	☐	☐	☐
		The area between two curves $y=f\left(x\right)$ and $y=g\left(x\right)$ between $x=a$ and $x=b$ is given by $\int_a^b \left(f\left(x\right)-g\left(x\right)\right)dx$ On the interval $a<x<b$, $f\left(x\right)>g\left(x\right)$ so $y=f\left(x\right)$ is known as the upper curve and $y=g\left(x\right)$ is known as the lower curve In the diagram below:  In the interval $-1<x<1$ $f\left(x\right)$ is the upper curve In the interval $1<x<4$ $g\left(x\right)$ is the upper curve. Two separate calculations are required: For $-1<x<1$: $\int_{-1}^{1}\left(f\left(x\right)-g\left(x\right)\right)dx$ For $1<x<4$ $\int_1^4\left(g\left(x\right)-f\left(x\right)\right)dx$			