Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have revised percentage appreciation and depreciation questions using real life examples.

1. £3000 is invested at an interest rate of 4% per annum.

(a) What is the value of the investment after 5 years?

(b) After how many years will the investment be over £4000?

2. The population of Mexico in 2000 was 104 million. At the time, the rate of population was set to increase by 1.2% per annum.

If this rate remained at 1.2%, what was the predicted population in 2010. Round your answer to 3 significant figures.

3. A patient was injected with 120ml of a drug to reduce inflammation. Every hour 8% of the drug passes out of her bloodstream. Calculate the amount of the drug in the bloodstream after 6 hours.

Round your answer to 2 significant figures.

4. An investigation was carried out into the effect of temperature on fermentation in yeast.

The volume of gas produced (cm^3) by fermenting yeast increases by 23% as the temperature increases by 1^oC.

When the temperature was 15^oC the volume of gas was $5cm^3$. Calculate the volume when the temperature was 20^oC.

Round your answer to 1 decimal place.

5. A student carried out an investigation into the effect of temperature on the growth of her Asiatic Lilly Plant. When the temperature was 10^oC the height of the Lilly plant was 20cm.

It is expected to grow at a rate of 1% each week for the next four weeks. Calculate the height of the Lilly plant after four weeks.

6. A record of a human male's height from the age of 1 to age 15 was recorded. Each year his height increased by 6%. If his starting height was 80cm, what was his height aged 15?

Round your answer to the nearest cm

7. The pH level in milk changes as it sours. At the start of an experiment the pH level was 6.7. It is expected to decrease by 0.5% each hour for the next 50 hours.

Calculate the pH level after 50 hours to 2 significant figures.

8. The number of bacteria grown in a fermenter over a 24 hour period were recorded. The results calculated are 15% growth in the first 4 hour cycle, then 55% in the second four hour cycle then 80% in the third four hour cycle. The number of bacteria at the start of the experiment was 20 billion/mm^3. Calculate the bacteria present at the end of the third cycle.

Round your answer to 2 significant figures.

R2 I can set up a recurrence relation from given information.

1. A family take out a loan of £3000. The interest charged on this works out as 1.2% per calendar month. They set up a payment plan of £500 per month.

(a) Write down a recurrence relation for the amount they owe.

(b) How much will the family owe after 3 months?

(c) How many payments will it take for the loan to be repaid?

2. An investor saves £50000 in an account, gaining 4.5% interest per year. They withdraw £1800 every year.

(a) Write down a recurrence relation for the amount of money in the account.

(b) Find how much they would have in this savings account after 5 years.

3. The air pressure in a used car tyre was 35 p.s.i. This is above its recommended minimum pressure of 30 p.s.i. The tyre loses 12% of its air pressure every month. The owner has been refilling the tyre with air at a rate of 3 p.s.i. every month.

(a) Find a recurrence relation showing the air pressure of the tyre.

(b) What is the air pressure of the tyre after 2 months?

(c) After how many months would the tyre end up below the recommended minimum pressure?

R3 I can calculate a required term in a recurrence relation.

1. For each recurrence relation find, rounding you answer to 2 decimal places where applicable;

(a) $u_2 : \ u_{n+1} = 0.2u_n + 4, \ \ u_0 = 3$

(b) $u_3 : \ u_{n+1} = 0.1u_n + 5, \ \ u_0 = 7$

(c) $u_4 : \ u_{n+1} = -0.5u_n + 20, \ \ u_0 = 16$

(d) $u_3 : \ u_{n+1} = -u_n - 7, \ \ u_0 = 1$

(e) $u_2 : \ u_n = 0.9u_{n-1} + 450, \ \ u_0 = 2$

2. A sequence in defined by the recurrence relation $u_{n+1} = 0.3u_n + 6, \ \ u_0 = 100$

(a) Calculate the value of u_4

(b) Calculate the value of u_{10} , round your answer to 2 decimal places.

3. A sequence in defined by the recurrence relation $v_{n+1} = 1.2v_n - 8, \ \ v_0 = 150$

(a) Calculate the value of v_3

(b) Calculate the value of v_{11} , round your answer to 2 decimal places.

(c) Find the smallest value of n for which $v_n > 1500$

4. A sequence in defined by the recurrence relation $u_n = 1.05u_{n-1} - 20, \ \ u_1 = 200$

(a) Calculate the value of u_5

(b) Find the smallest value of n for which $u_n < 50$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can solve problems involving the limit of a recurrence relation.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = ku_n - 5,\ u_0 = 0$$

(a) Given that $u_2 = -7$, find the value of k.

(b) (i) Why does this sequence tend to a limit as $n \to \infty$.

(ii) Find the value of this limit.

(Non-calculator)

2. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 20,\ u_0 = 0.$$

(a) Determine the value of u_1, u_2 and u_3.

(b) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of the limit.

(Non-calculator)

3. The terms of a sequence satisfy

$$u_{n+1} = ku_n + 4.$$

(a) Find the value of k which produces a sequence with a limit of 5.

(b) A sequence satisfies the recurrence relation

$$u_{n+1} = mu_n + 4,\ u_0 = 3.$$

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 8$, find the value of m which produces a sequence with no limit.

(Non-calculator)

4. Two unique sequences are defined by the following recurrence relations

$$u_{n+1} = ku_n + 6 \quad \text{and} \quad u_{n+1} = k^2u_n + 9,$$

where k is a constant.

(a) If both sequences have the same limit, find the value of k.

(b) For both sequences $u_0 = 80$. Find the difference between their first terms.

(Non-calculator)

5. A recurrence relation is given as

$$u_{n+1} = 0 \cdot 3u_n + 21$$

(a) Given that $u_1 = 36$, find the initial value, u_0 of this sequence.

(b) Hence find the difference between u_0 and the limit of this sequence.

(Non-calculator)

6. Two sequences are generated by the recurrence relations

$$u_{n+1} = 0 \cdot 2u_n + 4 \cdot 8$$

$$v_{n+1} = kv_n + 4$$

The 2 sequences approach the same limit as $\rightarrow \infty$.

(a) Evaluate this limit.

(b) Hence determine the value of k.

(Non-calculator)

NR2	**Given $u_{n-1} = au_n + b$ (or equivalent), I can find the values of a and b from applying my knowledge of recurrence relations.**

1. A recurrence relation is defined by

$$u_{n+1} = au_n + b,$$

where $-1 < a < 1$ and $u_0 = 25$.

(a) If $u_1 = 30$ and $u_2 = 31$, find the values of a and b.

(b) Find the limit of this recurrence relation as $\to \infty$.

(Non-calculator)

2. A sequence is defined by

$$u_{n+1} = -\frac{1}{3}u_n$$

with $u_0 = -18$

(a) Write down the values of u_1 and u_2.

A second sequence is given by $3, 5, 11, 29, \ldots.$ It is generated by the recurrence relation

$$v_{n+1} = pv_n + q$$

with $v_0 = 3$.

(b) Find the values of p and q.

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate the limit.

(ii) Why does the other sequence not have a limit?

(Non-calculator)

3. For the recurrence relation

$$u_{n+1} = mu_n + c,$$

it is known that $u_0 = 2$, $u_1 = 4$ and $u_2 = 7$

Find the values of m, c and u_3.

(Non-calculator)

4. The first three terms of a sequence are 5, 11 and 29.

The sequence is generated by the recurrence relation

$$u_{n+1} = au_n + b,$$

where $u_1 = 5$.

Find the values of a and b.

(Non-calculator)

5. A sequence of numbers is defined by the recurrence relation

$$u_{n+1} = ku_n + c,$$

where k and c are constants.

(a) Given that $u_1 = 100$, $u_2 = 90$ and $u_3 = 84$, find algebraically, the values of k and c.

(b) Hence find the limit of this sequence.

(Non-calculator)

6. For the recurrence relation

$$u_{n+1} = au_n + b,$$

it is known that $u_0 = 6$, $u_1 = 12$ and $u_2 = 21$.

(a) Find the values of a and b.

(b) Hence find the value of u_3.

(Non-calculator)

NR3 I can use recurrence relations to solve real life problems.

1. A scientist studying a large colony of bats in a cave has noticed that the fall in the population over a number of years has followed the recurrence relation

$$u_{n+1} = 0{\cdot}75u_n + 150,$$

where n is the time in years and 150 is the average number of bats born each year during a concentrated breeding week. He estimates the colony size at present to be 1800 bats with the breeding week just over.

(a) Calculate the estimated bat population in 4 years time immediately **after** that year's breeding.

(b) The scientist knows that if in the long term the colony drops, at any time, below 700 bats it is in serious trouble and will probably be unable to sustain itself.

Is this colony in danger of extinction?

Explain your answer with words and appropriate working.

(Calculator)

2. A new 24 volt lead acid battery is being tested as a possible power source for a battery-powered wheelchair.

The battery, which has an initial capacity of 22Ah (ampere hours), is being **artificially** drained over a 12 hour period to represent 1 month of use and an operating distance of 300 miles.

It has been found that by the end of each draining period the battery has lost 28% of its initial capacity at the start of that session.

After each 12 hour draining period the battery is hooked up to a super-charger for 12 hours which allows it to regain 4 Ah of capacity.

(a) What is the capacity of the battery immediately after its fifth re-charging period?

The battery is unusable if its capacity falls below 14.5 Ah.

(b) By considering the limit of a suitable sequence, make a comment on the durability and lifespan of the battery.

(Calculator)

3. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0.4 metres. In response to this warning he decides to trim 15% off the height of the trees at the start of any year.

(a) If he adopts the "15% pruning policy", to what height (to 2 decimal places) will he expect the trees to grow in the long run?

(b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition?

(Calculator)

4. A new '24 hour antibiotic' is being tested on a patient in hospital.

It is known that over a 24 hour period the amount of antibiotic remaining in the bloodstream is reduced by 70%. On the first day of the trial, an initial 220 mg dose is given to a patient at 7 a.m.

(a) After 24 hours and just prior to the second dose being given, how much antibiotic remains in the patient's bloodstream?

The patient is then given a further 220 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(b) A recurrence relation of the form un+1 = aun + b can be used to model this course of treatment.

Write down the values of a and b.

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this antibiotic over an extended period of time?

(Calculator)

5. Biologists calculate that when the concentration of a particular chemical in a sea loch reaches 7 milligrams per litre (mg/l), the level of pollution endangers the life of the fish.

A factory wishes to release waste containing the chemical into the loch. It is claimed that the discharge will not endanger the fish.

The Local Authority is supplied with the following information:

- The loch contains none of this chemical at present.
- The factory manager has applied to discharge waste once per week which will result in an increase in concentration of 2.5mg/l of the chemical in the loch.
- The natural tidal action will remove 40% of the chemical from the loch every week.

(a) If the Local Authority allows the factory to go ahead with the discharge, what will the level of concentration of the chemical in the loch be in the long run?

(b) A local MSP is concerned that the level of discharge is too high and the factory is not granted permission. The factory manager is told that in the long run there has to be no more than 6mg/l of the chemical in the loch.

How much of the chemical can the factory now discharge every week?

(Calculator)

6. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radiotherapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Muller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses 55% of its mass every 12 hours*
- *the maximum recommended mass in the bloodstream is 120mgs*
- *100mgs is the smallest mass detectable by the Geiger-Muller counter*

An initial dose of 50mgs of the isotope is injected into a patient and top-up injections of 60mgs are given every 12 hours.

(a) After how many top-up injections will the Geiger-Muller counter be able to detect the isotope?

Your answer must be accompanied by appropriate working.

(b) **(i)** Set up a linear recurrence relation to model this situation.

(ii) Comment on the long-term suitability of this plan.

Your answer must be accompanied by appropriate working.

(Calculator)

NR4 I have experience of cross topic exam standard questions.

1. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relations

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(a) Why do these sequences have a limit?

(b) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x.