Straight Line

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1	I have revised National 5 straight line.

1. Find the gradient of the line joining each pair of points

   (a) $T(3, 2)$ and $R(4, 4)$

   (b) $A(-1, 3)$ and $Q(4, 8)$

   (c) $C(-3, -2)$ and $S(7, 3)$

   (d) $V(0, 3)$ and $L(-3, 9)$

   (e) $B(1, 4)$ and $H(-1, -2)$

   (f) $G(-3, 4)$ and $W(-1, 8)$

   (g) $K(9, -2)$ and $N(5, -12)$

   (h) $X(-7, -4)$ and $E(-3, -2)$

2. Write down the gradient and y-intercept of each the line.

   (a) $y = 3x + 2$

   (b) $y = \frac{5}{8}x - 7$

   (c) $y = 2 - 3x$

   (d) $y = 4 - \frac{3}{4}x$

   (e) $y = x - 3$

   (f) $y = \frac{1}{2}x + 9$

   (g) $y = 7 - 2x$

   (h) $y = 11 - \frac{1}{3}x$

   (i) $y = 5x + 1$

   (j) $y = \frac{7}{9}x - 4$

   (k) $y = 1 - x$

   (l) $y = 43 - \frac{5}{7}x$

3. Rearrange the equation of each line so that it is in the form $y = mx + c$ and write down its gradient and y-intercept.

   (a) $3y - 5x = 3$

   (b) $4x + 3y = 9$

   (c) $2x - y = -12$

   (d) $5y + 2x = 0$

(e) $2y - 6x + 15 = 0$ **(f)** $4x - 3y - 7 = 0$

(g) $5x + 2y + 6 = 0$ **(h)** $8y + 4x - 11 = 0$

4. Write down the equation, in the form $y = mx + c$ where possible, of each straight line described.

(a) The straight line with gradient of -2 and passing through the point $(3, -2)$.

(b) A straight line passes through the point $(0, 7)$, with a gradient of 6.

(c) A straight line parallel to the x-axis and passes through $(-2, 4)$.

(d) A straight line passes through the point $(0, 11)$, with a gradient of -2.

(e) A straight line parallel to the y-axis and passes through $(5, 1)$.

(f) A straight line has a gradient of $\frac{1}{2}$ and passes through the point $(-1, 4)$.

(g) The straight line with gradient of 4 and passing through the point $(-1, 9)$.

(h) A straight line passes through the point $(0, -3)$, with a gradient of 2.

(i) A straight line parallel to the x-axis and passes through $(4, -3)$.

(j) A straight line passes through the point $(0, -4)$, with a gradient of $\frac{2}{3}$.

(k) A straight line parallel to the y-axis and passes through $(-3, -1)$.

(l) A straight line has a gradient of $-\frac{1}{2}$ and passes through the point $(-1, 4)$.

3

I can find the Distance between 2 points- using the Distance Formula.

1.   Use the distance formula to calculate the length of the straight line joining each pair of points. Leave your answer as a surd.

(a) $A(1,5)$ and $B(3,3)$ (b) $P(-7,1)$ and $Q(3,8)$

(c) $C(-3,-5)$ and $D(7,1)$ (d) $V(0,3)$ and $W(-7,9)$

(e) $G(7,3)$ and $H(-1,-2)$ (f) $R(-2,3)$ and $S(-1,8)$

(g) $K(9,-5)$ and $L(2,-12)$ (h) $X(-7,-3)$ and $Y(-1,-2)$

R3   **I can use the Midpoint Formula.**

1.   Find the midpoint of each pair of points

(a) $A(-3,1)$ and $B(1,3)$ (b) $P(1,4)$ and $Q(9,8)$

(c) $C(3,-3)$ and $D(-6,1)$ (d) $V(-7,1)$ and $W(3,9)$

(e) $G(2,4)$ and $H(-2,-2)$ (f) $R(-6,2)$ and $S(-2,8)$

(g) $K(-3,-3)$ and $L(3,-11)$ (h) $X(0,-4)$ and $Y(-4,-2)$

2.   The Line CD has the midpoint $(5,3)$ and the point C has coordinates $(-3,2)$.

Find the coordinates of D.

3.   The Line AB has the midpoint $(-2,7)$ and the point A has coordinates $(3,-7)$.

Find the coordinates of B.

4.   The Line EF has the midpoint $(-5,3)$ and the point F has coordinates $(3,11)$.

Find the coordinates of E.

R4   **I can calculate the gradient of perpendicular lines.**

1.   Write down the gradient of the line perpendicular to the gradient given

(a) $m = 3$

(b) $m = -2$

(c) $m = 6$

(d) $m = \frac{1}{3}$

(e) $m = -\frac{1}{4}$

(f) $m = \frac{1}{5}$

(g) $m = -\frac{2}{3}$

(h) $m = \frac{5}{4}$

(i) $m = -\frac{3}{5}$

2.   Write down the gradient of the line perpendicular to the given line

(a) $y = 5x + 2$

(b) $y = \frac{2}{3}x - 7$

(c) $y = 2 - 3x$

(d) $y = 4 - \frac{1}{2}x$

(e) $y = 3x - 3$

(f) $y = x + 9$

(g) $y - 4x + 12 = 0$

(h) $3x - y - 8 = 0$

(i) $3x - 2y + 7 = 0$

(j) $8y + 4x - 2 = 0$

R5   **I can find the point of intersection of straight lines.**

1.   Find the point of intersection between each pair of lines

(a) $3x + 4y = -7$;   and $2x + y = -3$

(b) $y = -x + 12$;   and $y = x - 4$

(c) $y = -x$;   and $4x + 3y = 3$

(d) $2x - 5y = 1$;   and $4x - 3y = 9$

(e) $y = -x + 2$;   and $y = 2x - 2$

(f) $x + y = 5$;   and $x - y = 2$

(g) $2x + 4y = 7$;   and $4x - 3y = 3$

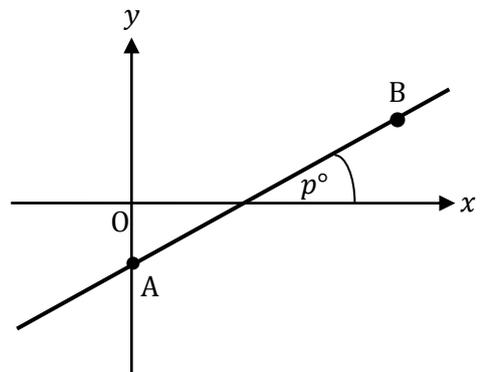(h) $2x + 5y = 16$;   and $x - y = 1$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1   **I can apply $m = \tan\theta$ in the context of a problem.**

1.   Find the equation of the line AB, where A is the point $(-3, 0)$ and the angle BAO is $30°$.
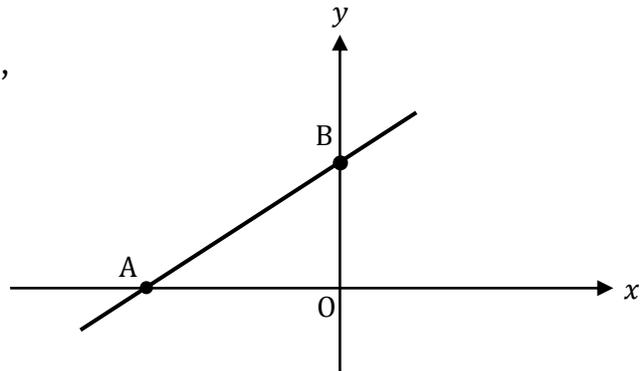
2.   Find the size of the angle $p°$ that the line joining the points $A(0, -2)$ and $B(4\sqrt{3},\ 2)$ makes with the positive direction of the x-axis.

3.   A straight line has equation $3x + 2y - 1 = 0$.

This line is inclined to the x-axis by an angle of $a°$.

Find the size of angle $a°$.

1. Find the equation of the straight line through the point $(-1, 5)$ which is parallel to the line with equation $3x - y + 1 = 0$.

2. Find the equation of the straight line which passes through the point $(-1, 4)$ and is perpendicular to the line with equation $4x + y - 3 = 0$.

3. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 6$ and which passes through the point $(2, -1)$.

4. The point P has coordinates $(1, 12)$. The straight lines with equations $x + 3y - 7 = 0$ and $2x + 5y = 11$ intersect at Q.

    (a) Find the gradient of PQ.

    (b) Hence show that PQ is perpendicular to only one of the lines.

5. ABCD is a parallelogram.

    A is the point $(3, 0)$, B is the point $(5, 6)$ and D lies on the x-axis. The diagonal BD is perpendicular to side AB.

    (a) Show that the equation of BD is $x + 3y - 23 = 0$.
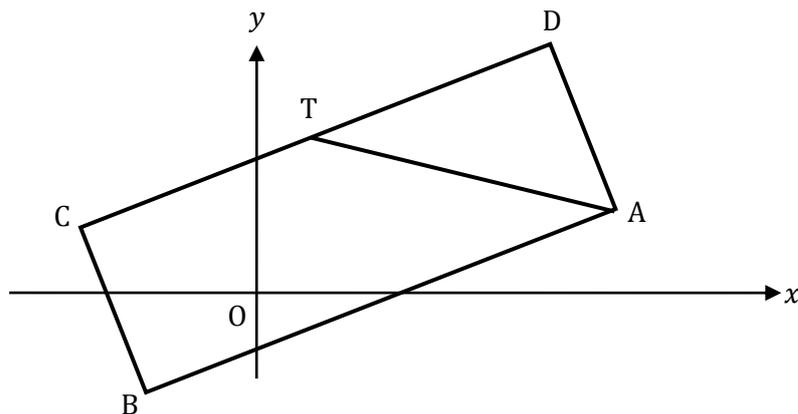
    (b) Hence find the coordinates of C and D.
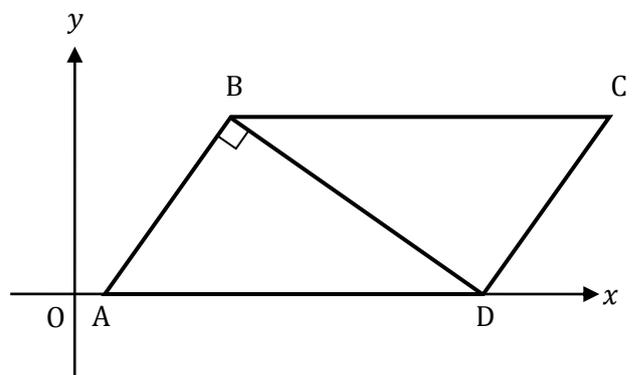
1.   A quadrilateral has vertices
$A(-2, 8)$, $B(6, 12)$, $C(7, 5)$
and $D(1, -3)$ as shown in the
diagram.
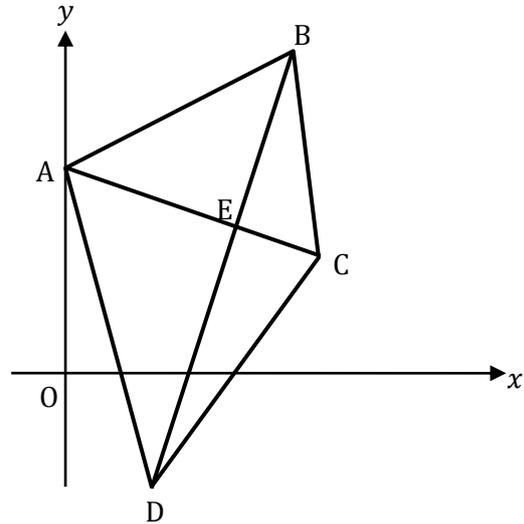
    (a)   Find the equation of
diagonal BD.

    (b)   The equation of diagonal
AC is $x + 3y = 22$. Find
the coordinates of E, the
point of intersection of
the diagonals.

    (c)   **(i)**   Find the equation of the perpendicular bisector of AB.

         (ii)   Show that this line passes through E.

2.   The diagram shows rectangle ABCD with $A(7, 1)$ and $D(5, 5)$.

    (a)   Find the equation of AD.

    (b)   The line from A with equation $x + 3y = 10$ intersects with CD at T.

        Find the coordinates of T.

    (c)   Given that T is the midpoint of CD, find the coordinates of C and B.

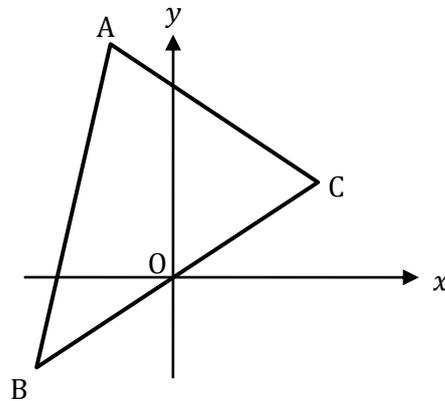**3.** Triangle ABC has vertices A(4, 1), B(−4, 17) and C(18, 21).

Medians AP and CR intersect at the point T(6, 13).

(a) Find the equation of median BQ.

(b) Verify that T lies on BQ.
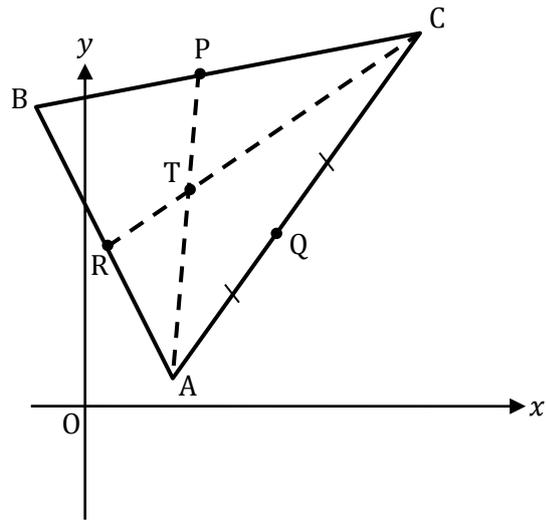
4. Triangle ABC has vertices A(−2, 6), B(−4, −2) and C(4, 2) as shown. Find

(a) the equation of the line p, the median from C of triangle ABC.

(b) the equation of the line q, the perpendicular bisector of BC.

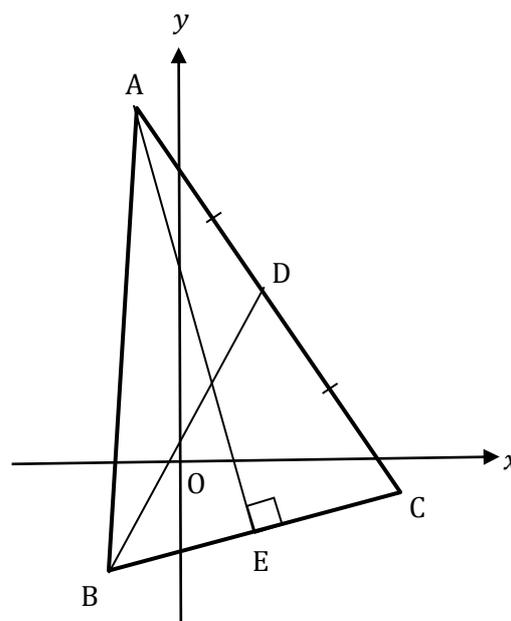**(c)** the coordinates of the point of intersection of the lines p and q.

5. Triangle ABC has vertices A(1, 2), B(11, 2) and C(7, 6) as shown.

(a) Write down the equation of l_1, the perpendicular bisector of AB.

(b) Find the equation of l_2, the perpendicular bisector of AC.

**(c)** Find the point of intersection of the lines l_1 and l_2.
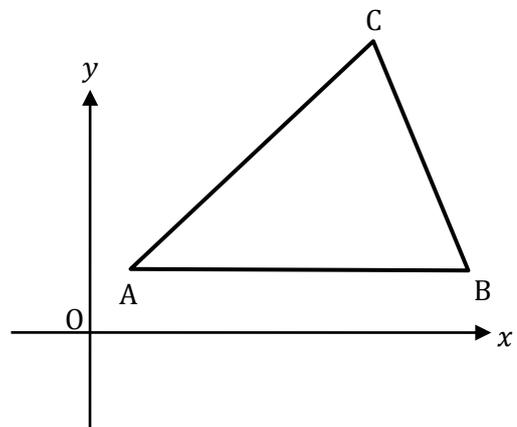
6. Triangle ABC has vertices A($-2, 12$), B($-3, -5$) and C($6, -2$) as shown.

(a) Find the equation of the median BD.

(b) Find the equation of the altitude AE.

(c) Find the coordinates of the point of intersection of BD and AE.

7. The vertices of triangle ABC are A($7, 8$), B($-3, -2$) and C($5, -6$) as shown.

The broken line represents the perpendicular bisector of BC.

(a) Show that the perpendicular bisector of BC is $y = 2x - 6$.

(b) Find the equation of the median from C.

(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.
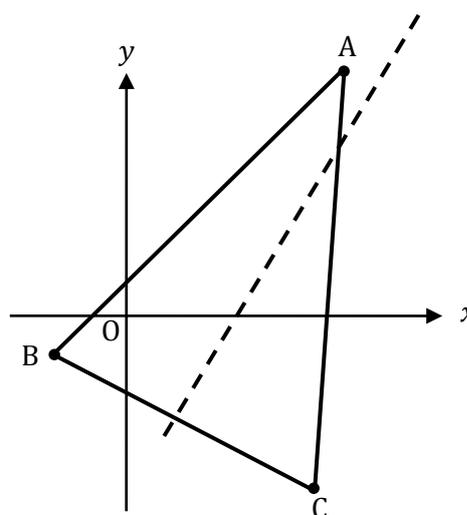
8. Triangle PQR has vertex P on the x-axis as shown.

Q and R are the points $(3, 6)$ and $(7, -2)$ respectively. The equation of PQ is $3x - 7y + 12 = 0$

(a) State the coordinates of P.

(b) Find the equation of the altitude from P.

(c) The altitude from P meets the line QR at T. Find the coordinates of T.
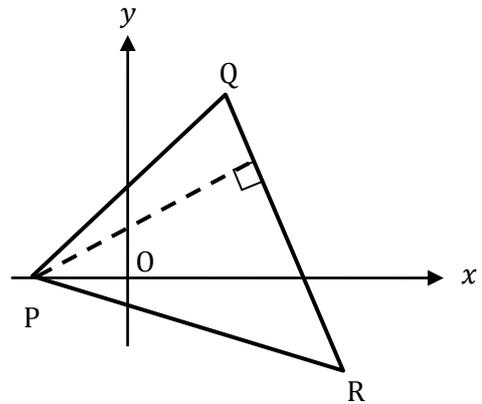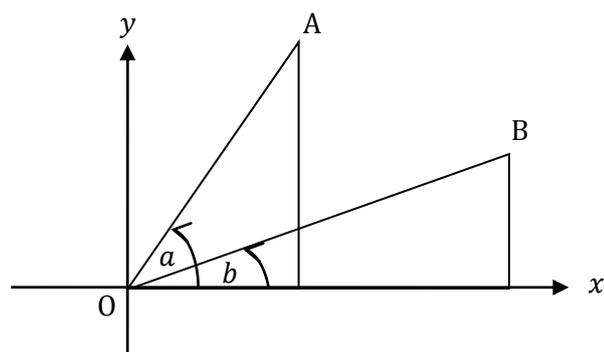
Straight line and trigonometry

1.   **(a)**   The diagram below show a right angled triangle, where the line OA
           has equation $5x - 3y = 0$.
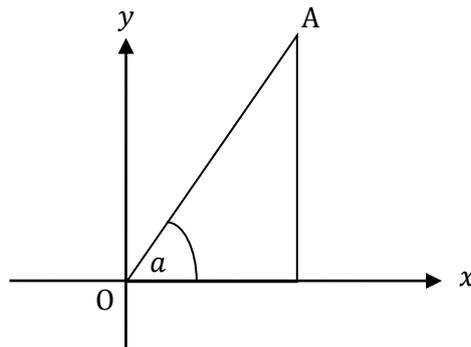
   (i)   Show that $\tan a = \frac{5}{3}$.

   (ii)   Find the value of $\sin a$ and $\cos a$.

   (b)   A second right angled triangle is added as shown.

   The line OB has equation $x - 2y = 0$.

   Find values of $\sin b$ and $\cos b$.

   (c)   **(i)**   Find the value of $\sin(a - b)$.

   (ii)   Find the value of $\cos(a + b)$.