

## INTEGRATION

### Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

**R1** I can evaluate the definite integral of a polynomial functions with integer limits.

1. Find

(a)  $\int_0^1 (x^2 - 3x + 4) dx$       (b)  $\int_0^1 (4x^2 + 3x) dx$       (c)  $\int_0^1 (x^3 + 2x^2 - 1) dx$

(d)  $\int_0^2 (2x - 1)(x + 2) dx$       (e)  $\int_{-1}^1 2x^2 (2x + 1) dx$       (f)  $\int_{-2}^1 (2x^3 - x^2 + 3x) dx$

2. Find

(a)  $\int_{-1}^1 (5x^3 - 2x) dx$       (b)  $\int_{-1}^1 (3x^2 - 4x + 2) dx$       (c)  $\int_{-1}^1 (3x + 2)(x - 2) dx$

(d)  $\int_0^2 (3x^2 + 8x - 5) dx$       (e)  $\int_{-2}^0 (x - 3)^2 dx$       (f)  $\int_{-1}^0 (x^2 - 2x + 7) dx$

(g)  $\int_0^3 x(x - 2)(x - 3) dx$       (h)  $\int_{-2}^2 (x + 2)(x - 2) dx$       (i)  $\int_1^4 (x - 1)(x - 2) dx$

**R2** I can evaluate the definite integral of a function with limits in radians, surds or fractions.

1. Evaluate

(a)  $\int_0^\pi \cos 2x dx$       (b)  $\int_0^{\pi/2} \cos 2x dx$       (c)  $\int_0^\pi \sin 2x dx$

(d)  $\int_0^{\pi/4} \sin 2x \, dx$       (e)  $\int_0^{\pi/3} \cos 3x \, dx$       (f)  $\int_0^{2\pi} \cos \frac{1}{2}x \, dx$

2. Evaluate

(a)  $\int_0^{\pi} (\sin t + \cos t) \, dt$       (b)  $\int_0^{\pi/4} \sin 4t + \cos 4t \, dt$

(c)  $\int_0^{\pi/4} \cos \left( 2t + \frac{\pi}{2} \right) \, dt$       (d)  $\int_{\pi/6}^{\pi/4} \sin \left( 2t - \frac{\pi}{3} \right) \, dt$

3. Evaluate

(a)  $\int_0^{1/2} (x^3 + 12x^2 + 7) \, dx$       (b)  $\int_{-1}^{1/2} (3x^2 - 4x) \, dx$       (c)  $\int_0^{2/3} (9x^2 + 8) \, dx$

(d)  $\int_{-1/2}^1 (9x^2 + 2x - 1) \, dx$       (e)  $\int_0^{\sqrt{3}} (2x + 4) \, dx$       (f)  $\int_1^{\sqrt{3}/2} (10 - 2x) \, dx$

**R3** I can apply a standard integral of the form  $f(x) = (px + q)^n$  with  $n \neq -1$ .

1. Find

(a)  $\int (x + 2)^8 \, dx$       (b)  $\int (2x + 4)^3 \, dx$       (c)  $\int (5x + 7)^4 \, dx$

(d)  $\int (2x - 1)^5 \, dx$       (e)  $\int 6(5 - 4x)^6 \, dx$       (f)  $\int (10 - x)^{-10} \, dx$

(g)  $\int 3(4x + 1)^{-3} \, dx$       (h)  $\int 2(5x - 9)^{-5} \, dx$       (i)  $\int (3 - 7x)^{-4} \, dx$

(j)  $\int (x - 1)^{\frac{1}{2}} \, dx$       (k)  $\int (2x - 1)^{\frac{1}{3}} \, dx$       (l)  $\int (2x - 1)^{\frac{1}{4}} \, dx$

(m)  $\int (2x - 2)^{\frac{1}{2}} \, dx$       (n)  $\int (3x + 4)^{\frac{2}{3}} \, dx$       (o)  $\int (7 + 3x)^{\frac{3}{4}} \, dx$

2. Find

(a)  $\int \frac{1}{(5x+3)^5} dx$

(b)  $\int \frac{dx}{(3x-2)^4}$

(c)  $\int \frac{3}{(4-2x)^6} dx$

(d)  $\int \frac{2 dx}{(x-2)^3}$

(e)  $\int \frac{3 dx}{(4x+2)^4}$

(f)  $\int \frac{1}{(5x-2)^{\frac{1}{2}}} dx$

3. Find

(a)  $\int \sqrt{4x+2} dx$

(b)  $\int 6\sqrt{3x+1} dx$

(c)  $\int \sqrt{9-5x} dx$

(d)  $\int \sqrt[3]{2x-3} dx$

(e)  $\int \sqrt[3]{6x-2} dx$

(f)  $\int \sqrt[4]{2x+4} dx$

(g)  $\int \frac{1}{\sqrt{3x-4}} dx$

(h)  $\int \frac{dx}{\sqrt{x+8}}$

(i)  $\int \frac{2dx}{\sqrt{2x-5}}$

**R4 I can integrate  $\sin^2 x$  and  $\cos^2 x$  by first making a substitution.**

1. Find

(a)  $\int \sin^2 x dx$

(b)  $\int \cos^2 x dx$

(c)  $\int 2\sin^2 x dx$

(d)  $\int 2\cos^2 x dx$

## Section B

This section is designed to provide examples which develop Course Assessment level skills

**NR1 I can evaluate one of the limits of a definite integral given the value of the definite integral.**

1. Find a, when  $a > 0$

(a)  $\int_0^a (2x+2) dx = 8$

(b)  $\int_0^a x^2 dx = \frac{64}{3}$

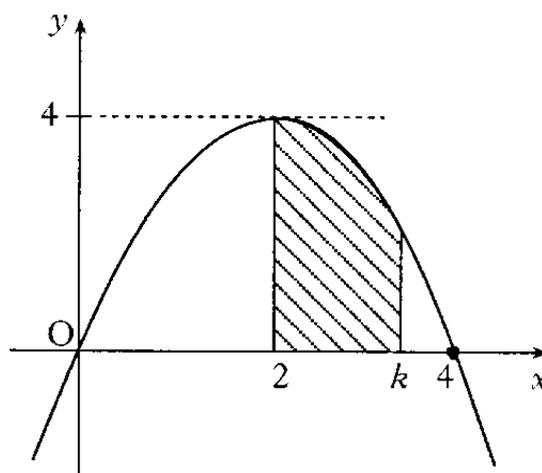


2. The parabola shown crosses the  $x$ -axis at  $(0, 0)$  and  $(4, 0)$ , and has a maximum at  $(2, 4)$ .

The shaded area is bound by the parabola, the  $x$ -axis and the lines  $x = 2$  and  $x = k$ .

- (a) Find the equation of the parabola.
- (b) Hence show that the shaded area,  $A$ , is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$



### Integration and the wave function

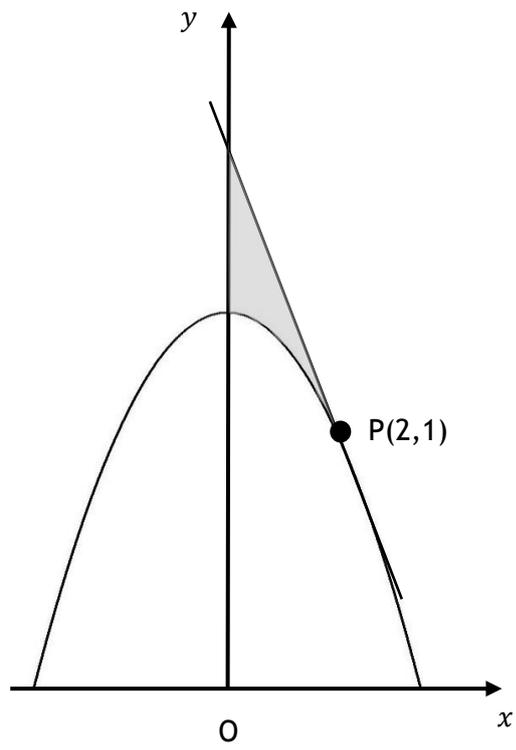
3. (a) The expression  $3 \sin x - 5 \cos x$  can be written in the form  $R \sin(x + a)$  where  $R > 0$  and  $0 \leq a \leq 2\pi$ .
- Calculate the values of  $R$  and  $a$ .

- (b) Hence find the value of  $t$ , where  $0 \leq t \leq 2$ , for which

$$\int_0^t (3 \sin x - 5 \cos x) dx = 3$$

### Integration and Differentiation

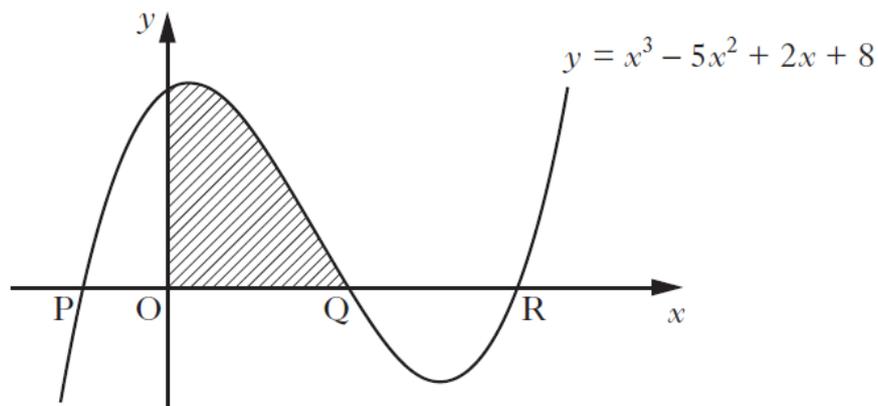
4. (a) Find the equation of the tangent to the parabola  $y = 5 - x^2$  at  $P(2, 1)$
- (b) Calculate the area of the shaded region bounded by the tangent, the parabola and the  $y$  axis.



## Integration and polynomials

5. (a) (i) Show that  $(x - 4)$  is a factor of  $x^3 - 5x^2 + 2x + 8$ .  
(ii) Factorise  $x^3 - 5x^2 + 2x + 8$  fully.  
(iii) Solve  $x^3 - 5x^2 + 2x + 8 = 0$ .

- (b) The diagram shows the curve with equation  $y = x^3 - 5x^2 + 2x + 8$



The curve crosses the  $x$ -axis at  $P$ ,  $Q$  and  $R$ .

Determine the shade area.