DIFFERENTIATION

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can differentiate algebraic functions which can be simplified to an expression in powers of x, including terms expressed as surds.

1. Find the derivative of the following

(a) $y = x^3$ **(b)** $y = x^5$ **(c)** $f(x) = x^4$

(d) $y = x^{12}$ **(e)** $f(x) = x^9$ **(f)** $y = x^{14}$

(g) $y = x^8$ **(h)** $y = x^7$ **(i)** $y = x^{24}$

(j) $f(x) = x^{100}$ **(k)** $y = x^{50}$ **(l)** $y = x^{44}$

2. Find the derivative of the following

(a) $y = x^{-2}$ **(b)** $y = x^{-8}$ **(c)** $f(x) = x^{-4}$

(d) $y = x^{-6}$ **(e)** $f(x) = x^{-19}$ **(f)** $y = x^{-5}$

(g) $y = x^{-3}$ **(h)** $y = x^{-10}$ **(i)** $y = x^{-20}$

(j) $f(x) = x^{-100}$ **(k)** $f(x) = x^{-50}$ **(l)** $f(x) = x^{-54}$

3. Find the derivative of the following

(a) $y = 2x^3$ **(b)** $y = 5x^4$ **(c)** $f(x) = 10x^7$

(d) $y = 6x^6$ **(e)** $f(x) = \frac{1}{2}x^6$ **(f)** $y = \frac{2}{3}x^9$

(g) $y = \frac{2}{5}x^{18}$ **(h)** $f(x) = \frac{2}{x^2}$ **(i)** $y = \frac{3}{x^3}$

(j) $y = \frac{25}{x^4}$ **(k)** $y = 7x^3$ **(l)** $y = \frac{3}{x^{-2}}$

(m) $f(x) = \frac{15}{x^{-3}}$ **(n)** $f(x) = 3x^{-5}$ **(o)** $y = \frac{6}{x^{-4}}$

(p) $y = \frac{14}{x^{-4}}$ **(q)** $y = 9x^4$ **(r)** $f(x) = \frac{2}{7x^{-1}}$

(s) $y = \frac{3}{3x^{-5}}$ **(t)** $f(x) = 12x^{-20}$ **(u)** $y = \frac{5}{2x^{-8}}$

4. Find the derivative of the following

(a) $y = x^{\frac{1}{2}}$ **(b)** $f(x) = x^{\frac{2}{3}}$ **(c)** $y = x^{\frac{3}{4}}$

(d) $y = x^{\frac{5}{6}}$ **(e)** $y = x^{\frac{1}{4}}$ **(f)** $f(x) = x^{\frac{2}{5}}$

(g) $f(x) = x^{\frac{6}{7}}$ **(h)** $y = x^{\frac{3}{7}}$ **(i)** $f(x) = x^{\frac{2}{9}}$

(j) $f(x) = x^{\frac{1}{5}}$ **(k)** $f(x) = x^{\frac{3}{11}}$ **(l)** $y = x^{\frac{9}{13}}$

5. Find the derivative of the following

(a) $y = \sqrt{x}$ **(b)** $y = \sqrt{x^3}$ **(c)** $f(x) = \sqrt{x^5}$

(d) $f(x) = \sqrt[3]{x}$ **(e)** $y = \sqrt[3]{x^2}$ **(f)** $y = \sqrt[5]{x^4}$

(g) $f(x) = \frac{1}{\sqrt{x}}$ **(h)** $f(x) = \frac{1}{\sqrt[3]{x^2}}$ **(i)** $y = \frac{1}{\sqrt[3]{x^4}}$

(j) $y = \frac{1}{\sqrt[5]{x^3}}$ **(k)** $y = \frac{2}{\sqrt[3]{x^8}}$ **(l)** $f(x) = \frac{3}{\sqrt[4]{x^3}}$

(m) $y = \frac{1}{2\sqrt[3]{x^2}}$ **(n)** $y = \frac{2}{3\sqrt[4]{x^3}}$ **(o)** $f(x) = \frac{3}{5\sqrt[3]{x^7}}$

6. Find the derivative of the following

(a) $y = x^3 + 3x^2 + 5x$ **(b)** $y = 3x^5 + 2x^4 - x$ **(c)** $y = x^2 + 6x - 1$

(d) $f(x) = x^{\frac{2}{3}} + 4x^2$ **(e)** $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}$ **(f)** $y = 5x^{-2} - 3x^{\frac{1}{2}}$

(g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2$ **(h)** $y = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$ **(i)** $y = \frac{3}{5\sqrt[2]{x^5}} + 5$

(j) $y = \frac{2}{3\sqrt[4]{x^3}} + 2x^2 + x$ **(k)** $y = 5x^2 - \frac{1}{\sqrt[3]{x^2}}$ **(l)** $y = 4x^{-1} - 4x^{\frac{2}{3}}$

(m) $f(x) = 5x^3 - 6x^{-\frac{1}{2}}$ **(n)** $f(x) = 4x^2 + \frac{6}{\sqrt[3]{x}}$ **(o)** $y = x^2 - 5 - \frac{1}{x^2}$

7. Find the derivative of the following

(a) $y = (x+1)(x+2)$ **(b)** $y = (x+2)(x-3)$ **(c)** $y = (x+2)^2$

(d) $y = (x-3)(x+4)$ **(e)** $y = (x-5)(2x-2)$ **(f)** $y = x(x-4)$

(g) $y = (2x-3)(x+4)$ **(h)** $y = x^2(x-2)$ **(i)** $y = \frac{1}{x^2}(x^3+2x)$

(j) $y = \frac{1}{x}(x^2+x)$ **(k)** $y = (\frac{1}{x}+1)^2$ **(l)** $y = \frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}}-1)$

8. Find the derivative of the following

(a) $y = \frac{x^2+3x+5}{x}$ **(b)** $y = \frac{2x^3+x^2+x}{x}$ **(c)** $y = \frac{x^4-6x+x^3}{x^2}$

(d) $y = \frac{x+5}{x}$ **(e)** $y = \frac{3+x^3}{x^2}$ **(f)** $y = \frac{x+2}{\sqrt{x}}$

(g) $y = \frac{(x+1)(x+2)}{x}$ **(h)** $y = \frac{(x-1)(x+3)}{x^2}$ **(i)** $y = \frac{3x^2+5x+1}{2x^2}$

R2 I am able to differentiate expressions which contain terms involving $sinx$ and $cosx$, expressed in radians.

1. Find the derivative of the following

(a) $y = \sin x$ **(b)** $y = \cos x$ **(c)** $f(x) = 2\sin x$

(d) $y = 5\sin x$ **(e)** $y = -2\cos x$ **(f)** $f(x) = -2\sin x$

(g) $y = -6\sin x$ **(h)** $y = \cos x + 2\sin x$ **(i)** $y = 2\sin x - \cos x$

(j) $y = 5\cos x - 3\sin x$ **(k)** $y = 2\cos x + 4\sin x$ **(l)** $y = 9\sin x + \cos x$

(m) $y = 3\cos x - 5\sin x$ **(n)** $y = 9\sin x + 7\cos x$ **(o)** $y = \sin x + \cos x$

(p) $y = 3\sin x - 5\cos x$ **(q)** $y = 9\cos x + 7\sin x$ **(r)** $y = \cos x - \sin x$

R3 I can differentiate a composite function using the chain rule.

1. Find the derivative of the following

(a) $y = (x+5)^2$ **(b)** $y = (x-8)^5$ **(c)** $y = (x+2)^3$

(d) $y = (2x-3)^2$ **(e)** $y = (3x-1)^5$ **(f)** $y = (4x+7)^4$

2. Find the derivative of the following

(a) $y = (x+5)^{-2}$ **(b)** $y = (3x-5)^{-3}$ **(c)** $y = (x+2)^{-8}$

(d) $y = \frac{1}{(2x-2)^2}$ **(e)** $y = \frac{1}{(3x+3)^5}$ **(f)** $y = \frac{1}{(6x-1)^4}$

(g) $y = \sqrt{(x+3)}$ **(h)** $y = \sqrt{(x-2)}$ **(i)** $y = \sqrt{(x+2)^3}$

(j) $y = \frac{1}{\sqrt{(x+2)}}$ **(k)** $y = \frac{1}{\sqrt{(x-10)}}$ **(l)** $y = \frac{2}{\sqrt{(x+2)^5}}$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can determine the equation of a tangent to a curve, at a given point by differentiation.

1. Find the equation of the tangent to the curve at the given point

(a) $y = x^2$ at (1,4)

(b) $y = x^2 + 2x$ at (0,2)

(c) $y = x^3 + 3$ at (2,8)

(d) $y = x^2 - 6x + 5$ at x = 2

(e) $y = \sqrt{x} + 1$ at x = 1

(f) $y = x^2 + 2x - 3$ at (0,2)

(g) $f(x) = 5x^2 - 6x^{\frac{1}{2}}$ at x = 1

(h) $y = (x + 4)^2$ at (2,-2)

(i) $y = \frac{x^2 + 5x + 10}{x^2}$ at x = 1

(j) $y = \frac{1}{\sqrt{(x+1)}}$ at (3,3)

2. A curve has equation $y = 3x^2 - 4x$. At the point T the tangent to this curve has a gradient of 2, find the coordinates of T and hence the equation of the tangent.

3. A curve has equation $f(x) = 2x^2 + 8x - 3$. A tangent to this curve has a gradient of -4, find the equation of this tangent.

4. A tangent to the equation $y = \frac{2}{\sqrt{x}}$ has a gradient of -1, find the equation of this tangent.

5. A curve has equation $y = x^3 - 6x$. There are two tangents with a gradient 6. Find the equation of both tangents

NR2 **I can determine use the stationary points of a curve and state their nature.**

1. Find the coordinates of the Stationary points and determine their nature

(a) $y = x^3 - 6x^2 + 9x$ **(b)** $y = x^3 - 3x^2 - 9x + 12$

(c) $y = x^4 - 4x^3$ **(d)** $y = x^3 - 3x + 2$

(e) $y = x^3 - 12x + 2$ **(f)** $y = 2x^3 - 7x^2 + 4x + 1$

(g) $y = 3x^4 + 16x^3$ **(h)** $y = 8x^3 - x^2 + 11$

(i) $y = \frac{x^3 + 4x + 16}{x}$ **(j)** $y = (x - 2)(x^2 + 1)$

NR3 **I can sketch the graph of an algebraic function by determining stationary points and where it cuts the axes.**

1. Sketch the graph of the following functions, stating clearly where it cuts the x & y axis

(a) $y = x^2 - 6x$ **(b)** $y = x^2 + 5x + 6$ **(c)** $y = x^2 - 5x$

(d) $y = 2x^3 - 3x^2 - 36x$ **(e)** $y = (x - 1)^2(x + 2)$ **(f)** $y = 12 - 4x - x^2$

(g) $y = x^3 - 3x^2$ **(h)** $y = x(x^2 - 4)$ **(i)** $y = x^2(2x - 1)$

(j) $y = 3x^4 + 9x^3$ **(k)** $y = x^3 + 3x^2$ **(l)** $y = 4x^3 - 2x^4$

(m) $y = 2x^3 - 3x^2$ **(n)** $y = 3x^2 + 5x - 2$ **(o)** $y = 2x^2 + 9x + 4$

NR4 I can determine where a function is strictly increasing/decreasing.

1. State whether the function is increasing or decreasing

(a) $y = x^2 - 4x$ at x = 3 **(b)** $y = x^3 - 3x + 2$ at x = -1

(a) $y = x^2 - 10x + 4$ at x = -2 **(b)** $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2$ at x = 4

(a) $y = 4x^2 + 5x + 7$ at x = 1 **(b)** $y = 2x^4 - 4x^2 + 12$ at x = -1

2. For each function state the intervals in which it is increasing AND decreasing

(a) $y = x^2 - 5x + 12$ **(b)** $y = 2x^2 + x + 3$

(a) $y = 8 + 2x - x^2$ **(b)** $y = x^3$

(a) $y = 3x - x^3$ **(b)** $y = x^2(3 - 2x)$

3. Show that the function $y = x^3 - x^2 + x$ is never decreasing

4. Show that the function $y = 2x^5 + 5$ is never decreasing

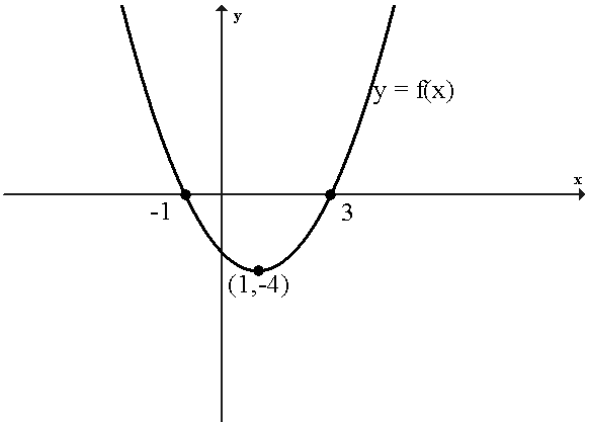
5. Show that the function $y = 12x^2 - 6x - 8x^3$ is never increasing

6. Show that the function $y = -x^3 - 3x^2 - 3x$ is never increasing

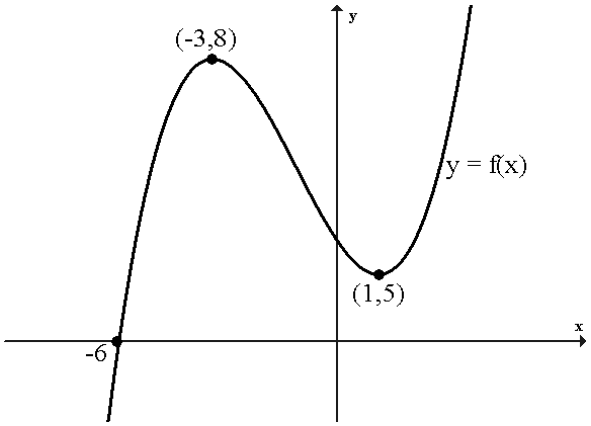
NR5 I can sketch graphs of derivatives.

1. For each graph of f(x) sketch f'(x)

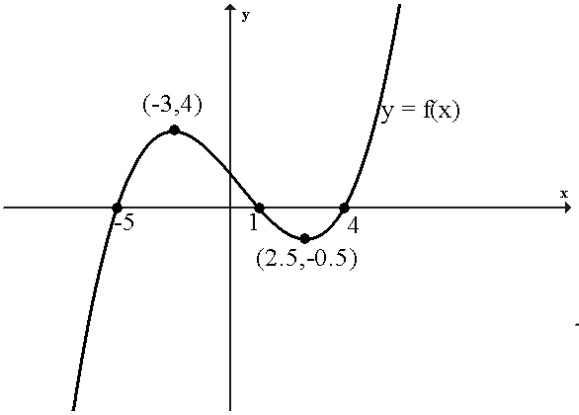
(a)



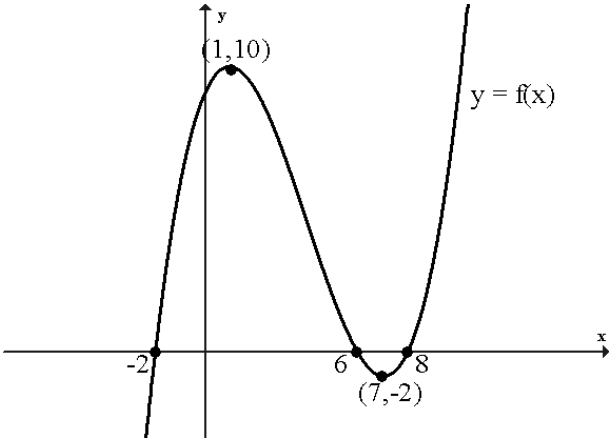
(b)



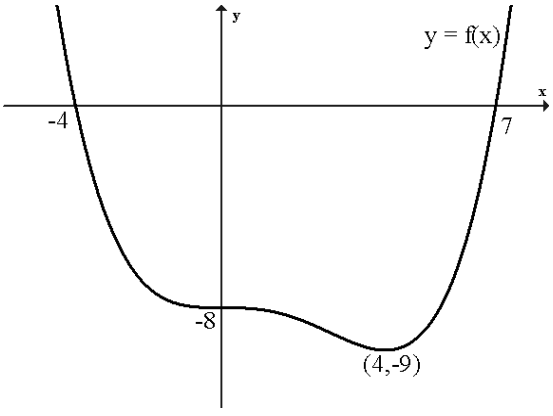
(c)



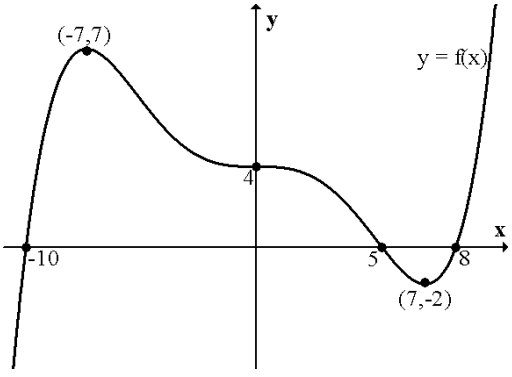
(d)



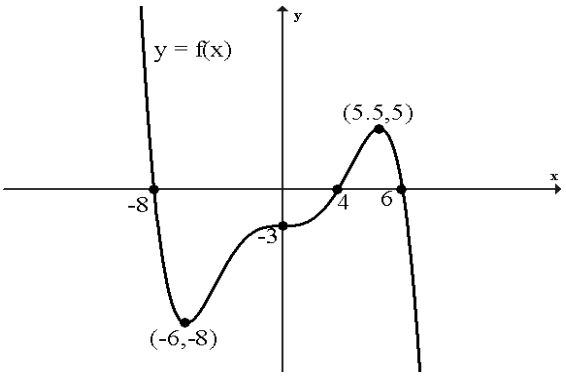
(e)



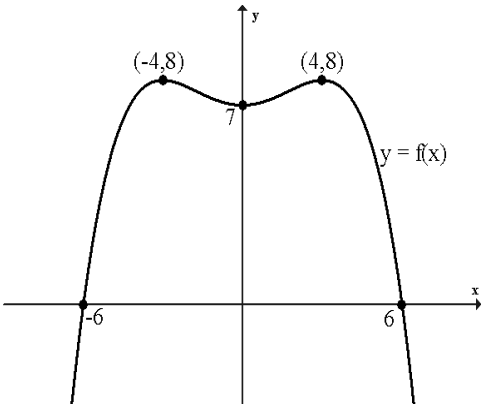
(f)



(g)



(h)



NR8 I can differentiate $sinkx$ and $coskx$ (including applications of the chain rule).

1. Find the derivative of the following

(a) $y = \sin 2x$ **(b)** $y = \cos 2x$ **(c)** $f(x) = 2\sin 3x$

(d) $y = 2\sin 4x$ **(e)** $y = -2\cos 3x$ **(f)** $f(x) = -2\sin 2x$

(g) $y = \sin(2x + 1)$ **(h)** $y = \cos(3x - 2)$ **(i)** $f(x) = 2\sin 2x$

(g) $y = sin^2 x$ **(h)** $y = cos^3 x$ **(i)** $f(x) = \frac{1}{sin^2 x}$

(j) $f(x) = \frac{1}{cos^2 x}$ **(k)** $f(x) = \frac{1}{sin^3 x}$ **(l)** $f(x) = \frac{1}{2sin^3 x}$

(m) $y = \sin x^2$ **(n)** $y = \cos x^2$ **(o)** $y = 2\sin x^3$

NR9 I have experience of cross topic exam standard questions.

1. **(a)** Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$

(b) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}$

2. **(a)** Given that (x - 1) is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic

(b) Show that the curve with equation $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ has only 1 Stationary point.

Find the x coordinate and determine the nature of this point.

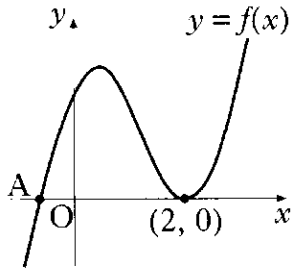
3.

The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

(*a*) Find the x-coordinate of the maximum turning point.

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$.

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$.



4.

(*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

5.

A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.

(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.

(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.

6.

Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(a) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$.

(b) Solve $p'(x) = q'(x)$.