POLYNOMIALS

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can factorise a polynomial expression using the factor theorem.

1. Show that $x = -4$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.

2. (a) Show that $(x + 2)$ is a factor of $f(x) = 2x^3 + 3x^2 - 5x - 6$.

(b) Hence factorise $f(x)$ fully.

3. (a) Show that $(x + 1)$ is a factor of $f(x) = 2x^3 - 3x^2 - 3x + 2$.

(b) Hence factorise $f(x)$ fully.

4. Show that $(x - 2)$ is a factor of $f(x) = x^3 - 5x^2 + 2x + 8$.

(a) Factorise $x^3 - 5x^2 + 2x + 8$ fully

(b) Solve $x^3 + 2x = 5x^2 - 8$

5. Factorise fully

a) $x^3 - 7x + 6$
b) $2x^3 + 3x^2 - 2x - 3$
c) $2x^3 - x^2 - 13x - 6$
d) $3x^3 + 8x^2 - 5x - 6$
e) $2x^4 + 6x^3 + 6x^2 + 2x$
f) $x^5 + x^4 - x - 1$

NR2 I can evaluate an unknown coefficient of a polynomial by applying the remainder and/or the factor theorem.

1. $f(x) = 2x^3 + ax^2 + bx + 4$.
   Given that $(x - 2)$ is a factor of $f(x)$, and the remainder when $f(x)$ is divided by $(x - 5)$ is 54, find the values of *a* and *b*.

2. Find p if $(x - 4)$ is a factor of $x^3 - 9x^2 + px - 28$.

3. Given that (x + 1) is a factor of $2x^3 + 3x^2 + px - 6$
   (a) Find the value of p
   (b) Hence or otherwise, solve $2x^3 + 3x^2 + px - 6 = 0$

4. Find the value of k if (x+5) is a factor of $3x^4 + 15x^3 - kx^2 - 9x + 5$

5. Given that (x-1) is a factor of $x^3 + x^2 - (t + 1)x - 4$, find the value of 't' and hence factorise fully.

6. Given that x = 3 is a root of the equation $x^4 - 3x^3 + px - 5$, find p.

7. When $x^4 - 3x^3 + px - 5$ is divided by (x+3) the remainder is 16.

   Find the value of p.

NR3 I can solve a polynomial equation to determine where a curve cuts the x-axis.

1. A function is defined on the set of real numbers by $f(x) = (x + 3)(x^2 + 4)$.

Find where the graph of $y = f(x)$ cuts:

(a) the x-axis;

(b) the y-axis. **(Non-calculator)**

2. A function is defined by the formula $g(x) = 2x^3 - 7x^2 - 17x + 10$ where x is a real number.

(a) Show that $(x - 5)$ is a factor of $g(x)$, and hence factorise $g(x)$ fully.

(b) Find the coordinates of the points where the curve with equation $y = g(x)$ crosses the x and y-axes. **(Non-calculator)**

3. A function is defined by the formula $f(x) = 5x - x^3$.

Find the exact values where the graph of $y = f(x)$ meets the x and y-axes.

(Non-calculator)

4. $h(x) = x^3 - 5x^2 + 3x + 9$

(a) (i) Show that $(x + 1)$ is a factor of $h(x)$.

(ii) Hence or otherwise factorise $h(x)$ fully.

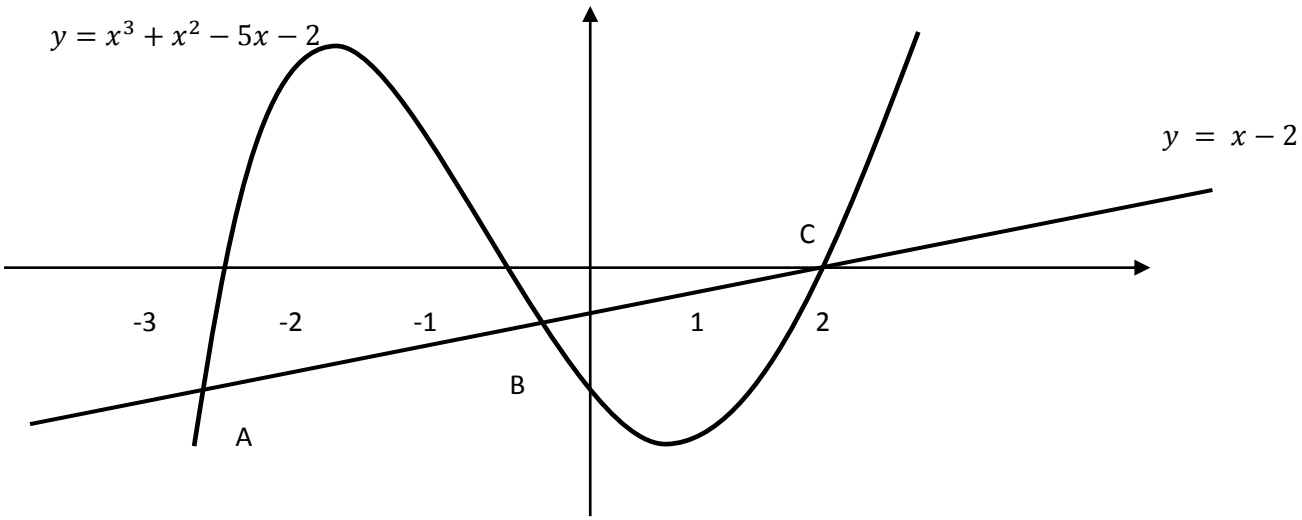
(b) One of the turning points of the graph of $y = h(x)$ lies on the x-axis.

Write down the coordinates of this turning point. **(Non-calculator)**

5. Find where the graph of $y = x^4 + 6x^3 - 12x^2 - 88x - 96$ meets the x and y-axes. **(Non-calculator)**

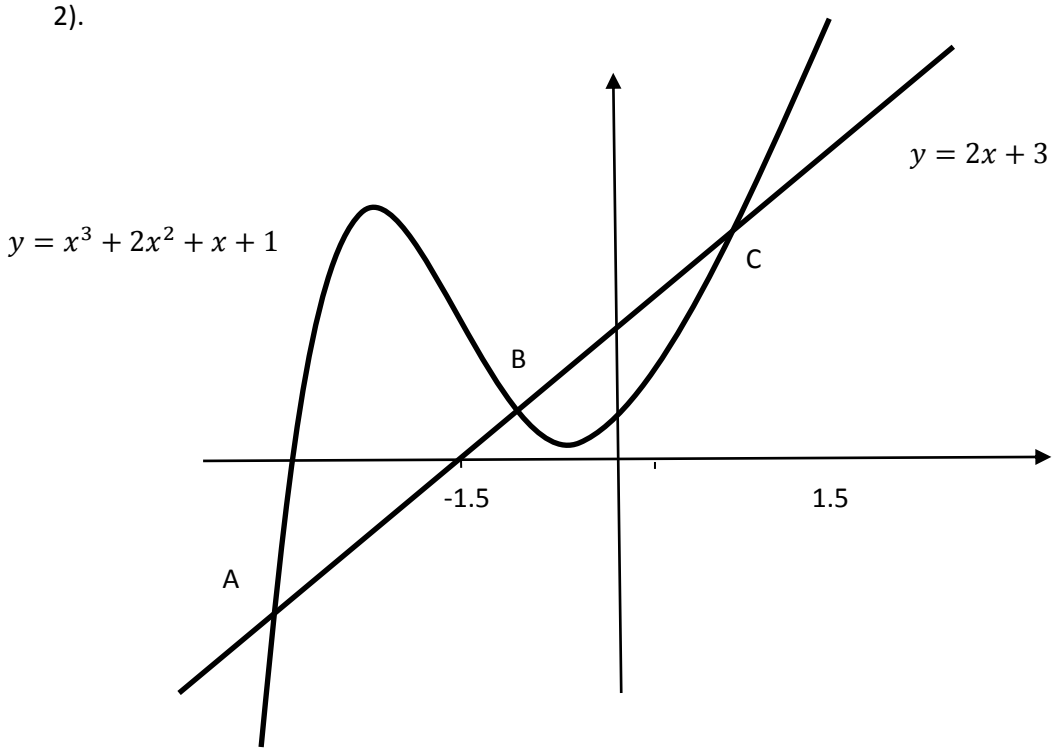
NR4 I can find points of intersection by solving polynomial equations.

1).



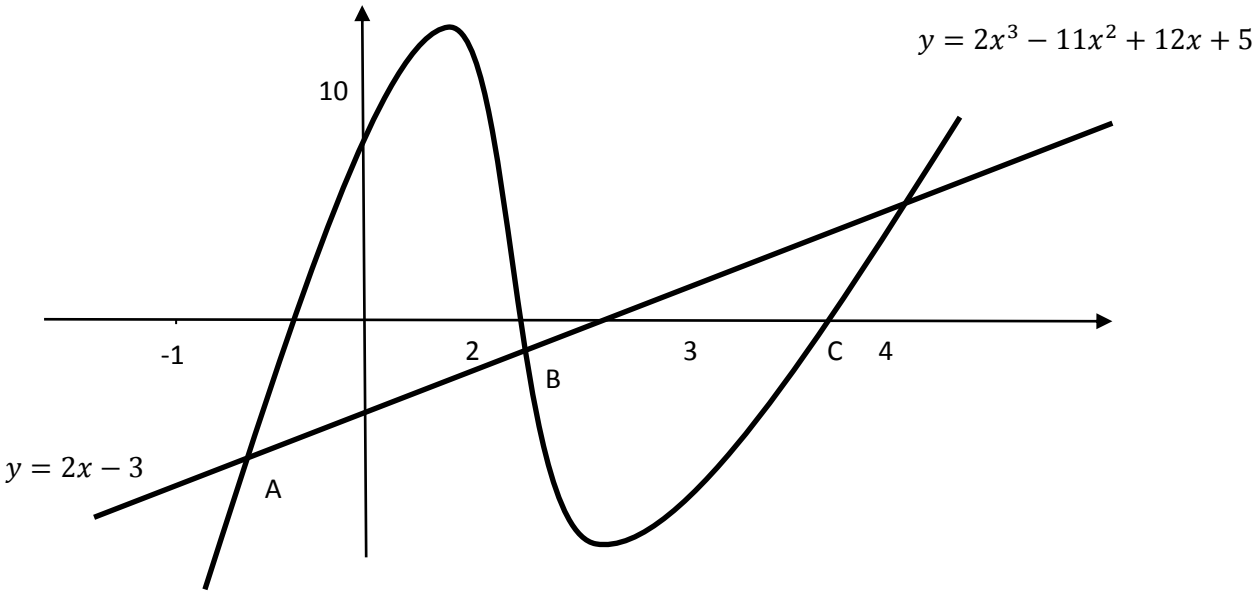
Find the coordinates of the points of intersection namely A, B and C where the line $y = x - 2$ meets the graph $y = x^3 + x^2 - 5x - 2$. (**NB the diagram is not drawn to scale**)

2).



Find the coordinates A, B and C of the points of intersection between the line $y = 2x + 3$ and the curve $y = x^3 + 2x^2 + x + 1$.

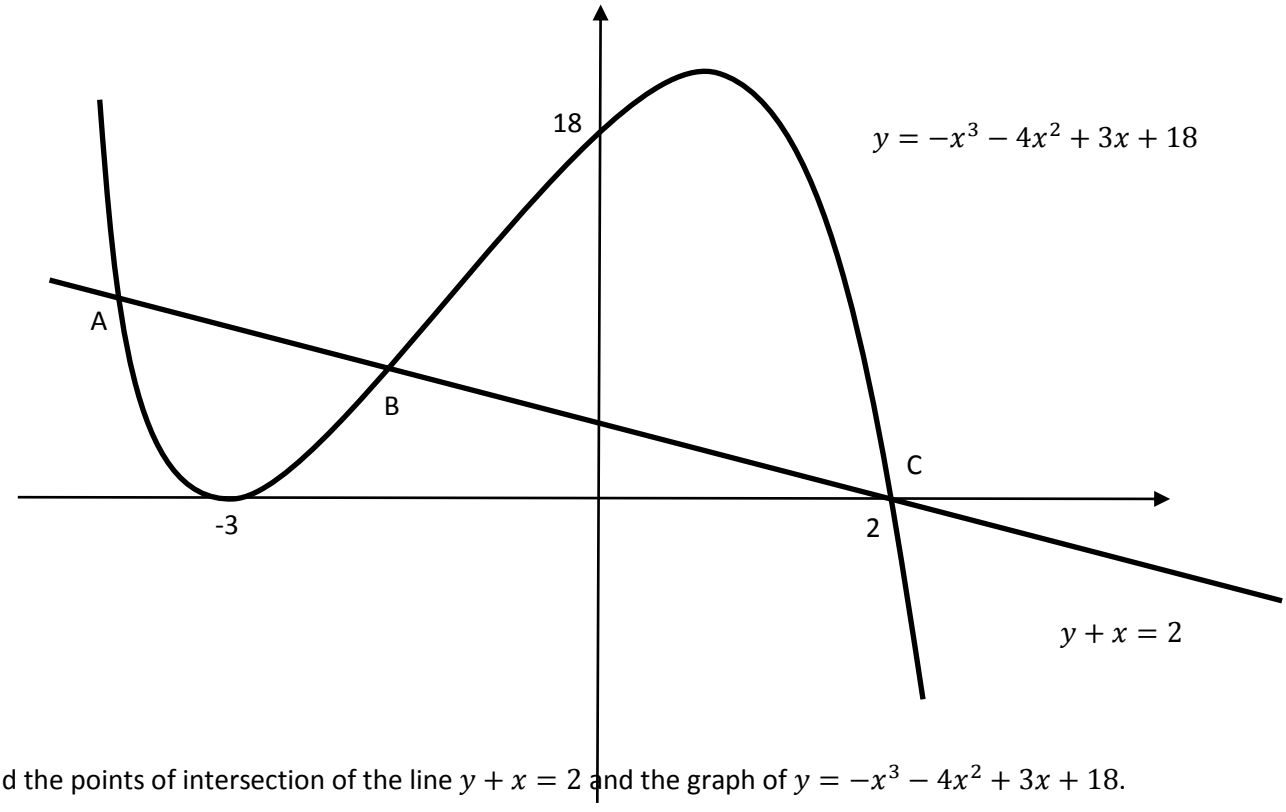
3) (NB diagram not to scale)



a) Fully factorise the polynomial $2x^3 - 11x^2 + 10x + 8$

b) Hence or otherwise find the coordinates of the points of intersection of the line $y = 2x - 3$ and the graph $y = 2x^3 - 11x^2 + 12x + 5$

4)



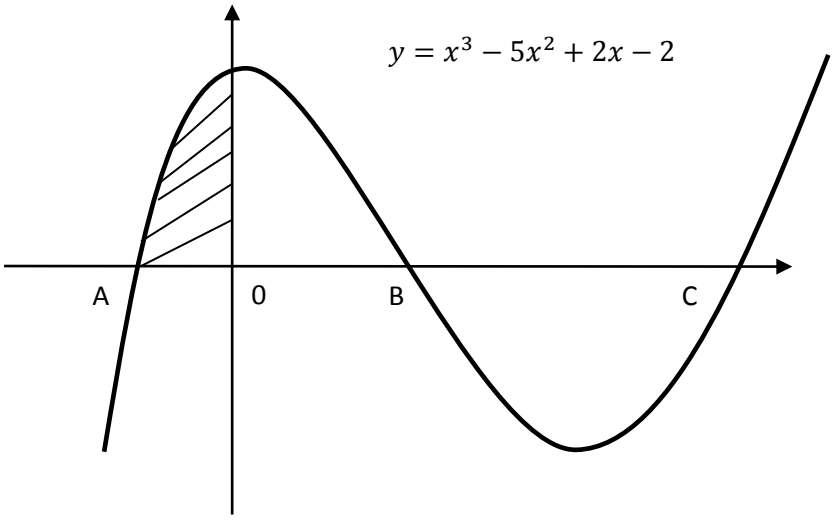
Find the points of intersection of the line $y + x = 2$ and the graph of $y = -x^3 - 4x^2 + 3x + 18$.

NR5 I have experience of cross topic exam standard questions.

1. Given that $f'(x) = 6x^2 + 2x - 128$ and $(x - 8)$ is a factor of $f(x)$, find
   a) Formula for $f(x)$
   b) Hence factorise $f(x)$ fully
   c) Solve $f(x) = 0$.

2. (a) (i) Show that $(x - 2)$ is a factor of $x^3 - 5x^2 + 2x + 8$.
   (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.
   (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

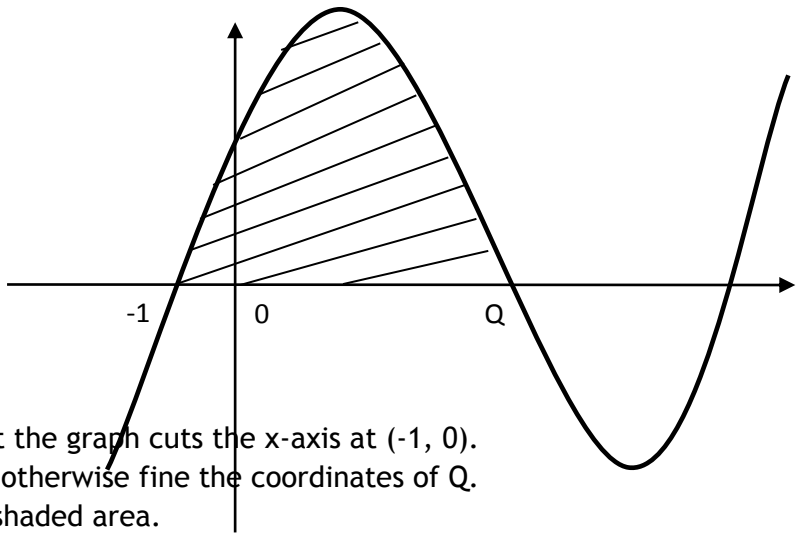
   b)The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



   The curve crosses the x-axis at A, B and C.
   Determine the shaded area.

3. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$

$y = x^3 - 4x^2 + x + 6$



   a) Show that the graph cuts the x-axis at (-1, 0).
   b) Hence or otherwise fine the coordinates of Q.
   c) Find the shaded area.

4. Functions $f, g \; and \; h$ are defined on the set of real numbers by

   - $f(x) = x^3 + 1$
   - $g(x) = 2x + 1$
   - $h(x) = 17 - 11x$

a) Find $g\big(f(x)\big)$

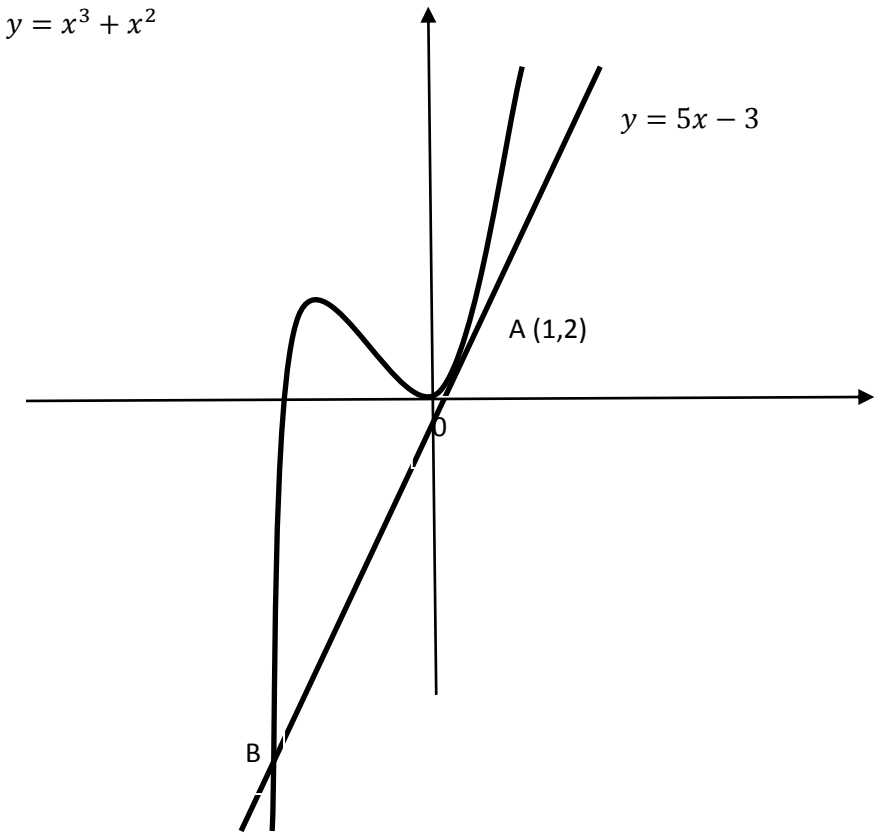
b) Show that $g\big(f(x)\big) + xh(x) - 9 = 2x^3 - 11x + 17x - 6$

c) (i) Show that ($x - 2$) is a factor of $2x^3 - 11x + 17x - 6$

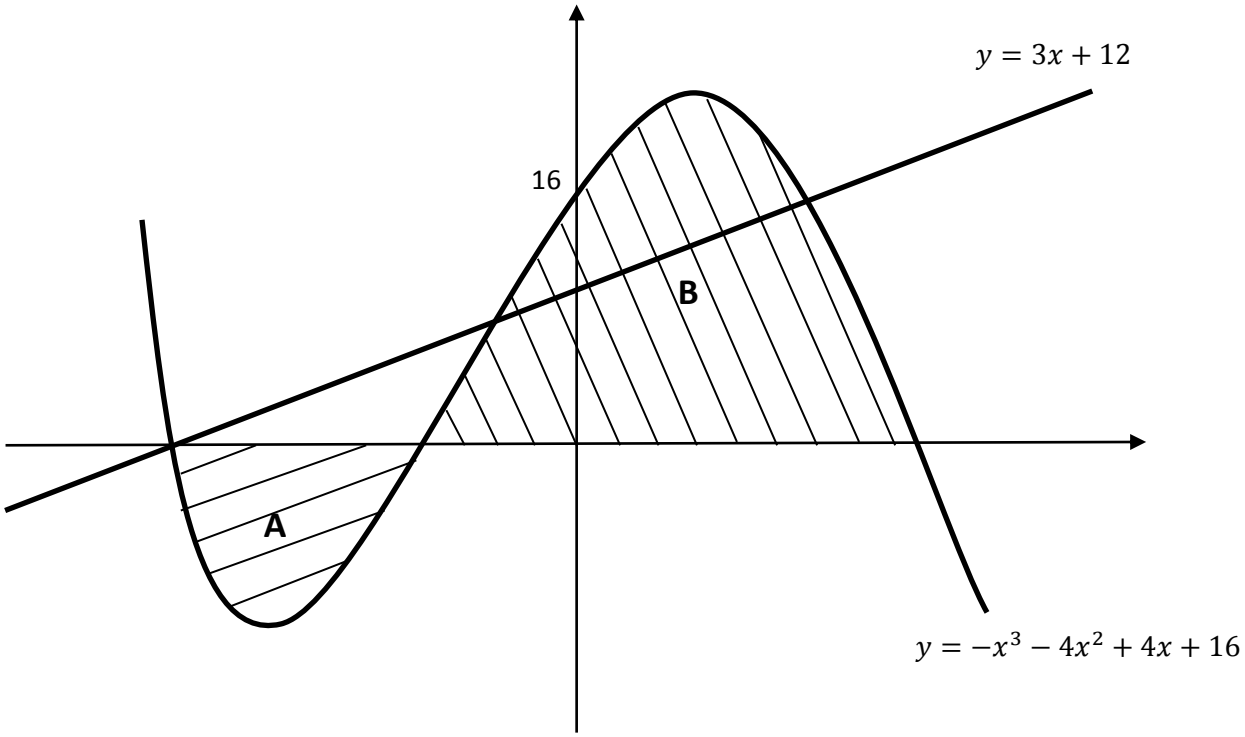
   (ii) Factorise $2x^3 - 11x + 17x - 6$

d) Hence or otherwise solve $g\big(f(x)\big) + xh(x) = 9$

5. The line with equation y = 5x – 3 is a tangent to the curve with equation $y = x^3 + x^2$ at A (1,2), as shown on the diagram. Show that the point B is (-3,-18)



6.



a) Find where the graph $y = -x^3 - 4x^2 + 4x + 16$ crosses the x axis.
b) Calculate the points of intersection of the line $y = 3x + 12$ and the graph.
c) Find the shaded area A
d) Find the shaded area B