QUADRATIC THEORY

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have had experience of factorising. (common factor, difference of two squares and trinomial quadratics).

1. Factorise fully

(a) $98 - 8x^2$ (b) $5s^2 - 5t^2$ (c) $98 - 2x^2$

(d) $75x^2 - 243$ (e) $72 - 18x^2$ (f) $12x - 3x^3$

(g) $81 - x^4$ (h) $27w - 12w^3$ (i) $64a^4 - 4$

(j) $50x^3 - 2x$ (k) $5r^3 - 20r$ (l) $32p^5 - 2p$

2. Factorise fully

(a) $2x^2 - 7x + 3$ (b) $2x^2 + 11x + 12$ (c) $3x^2 + 10x + 8$

(d) $x^2 + x - 6$ (e) $6x^2 + 7x + 2$ (f) $x^2 - 3x + 2$

(g) $5x^2 + 4x - 1$ (h) $7x^2 + 16x + 4$ (i) $2x^2 + 7x - 15$

(j) $x^2 - 2x - 15$ (k) $4x^2 + 13x + 3$ (l) $12x^2 - 4x - 1$

(m) $8x^2 + 2x - 3$ (n) $8x^2 + 6x - 9$ (o) $9x^2 + 15x + 4$

3. Factorise fully

(a) $6 - x - x^2$ (b) $20 + 11x - 3x^2$ (c) $3 + x - 2x^2$

(d) $15 - 7x - 2x^2$ (e) $4 - 7x - 2x^2$ (f) $15 - 2x - x^2$

4. Factorise fully

(a) $3x^2 + 6x - 24$ (b) $15x^2y + 5x$ (c) $2x^2 - 32$

(d) $5x^3 - 45x$ (e) $18x^2 - 6x - 12$ (f) $12x^2y + 8xy^3$

(g) $10x^2 + 25x - 15$ (h) $6x^3 + 30x^2 + 36x$ (i) $7x^2 - 28$

(j) $2x^2 - 10x + 12$ (k) $3x^3 + 21x^2 + 30x$ (l) $6x^3 - 54x$

R2 I can find the roots of a Quadratic Equation by factorising.

1. Solve each of these quadratic equations

(a) $x^2 + 7x + 12 = 0$ **(b)** $x^2 - 4 = 0$ **(c)** $n^2 + 3n + 2 = 0$

(d) $5x^2 + 15x = 0$ **(e)** $p^2 + 11p + 24 = 0$ **(f)** $12a - 3a^2 = 0$

(g) $s^2 + 6s + 8 = 0$ **(h)** $r^2 - 25 = 0$ **(i)** $n^2 + 5n + 6 = 0$

2. Solve each of these quadratic equations

(a) $x^2 - 11x + 24 = 0$ **(b)** $4x^2 - 9 = 0$ **(c)** $n^2 + 3n - 10 = 0$

(d) $5x^2 + 3x = 0$ **(e)** $p^2 - 10p + 24 = 0$ **(f)** $5a^2 - 20 = 0$

(g) $2n^2 + 7n + 3 = 0$ **(h)** $5r^2 + 7r + 2 = 0$ **(i)** $3n^2 - 4n + 1 = 0$

(j) $2r^2 - r - 6 = 0$ **(k)** $6s^2 - 18s - 18 = 6$ **(l)** $7r^2 - 14r = -7$

(m) $n^2 + 8n = -15$ **(n)** $5r^2 - 44r + 120 = -30 + 11r$

R3 I can find the roots of a Quadratic Equation using the Quadratic Formula.

1. Solve these equations giving your answer to 2 significant figures.

(a) $x^2 - 3x - 1 = 0$ **(b)** $2x^2 + 5x + 1 = 0$ **(c)** $5x^2 - 7x - 2 = 0$

2. Solve these equations giving your answer to 3 significant figures.

(a) $3x^2 - 10x = -2$ **(b)** $2x^2 = 6x - 3$ **(c)** $4x^2 + x = 1$

R4 I have revised how to determine where quadratic graphs cut the axes, Turning Points and the Axis of Symmetry.

For each of the quadratic functions given below

(i) Write down the points where the graph of $y = f(x)$ cuts the axes.

(ii) State the equation of the axis of symmetry of $y = f(x)$.

(iii) Write down the coordinates of the turning point of $y = f(x)$.

(iv) Sketch the graph of $y = f(x)$.

1. $f(x) = 5x^2 + 20x$ **2.** $f(x) = x^2 + 6x + 8$

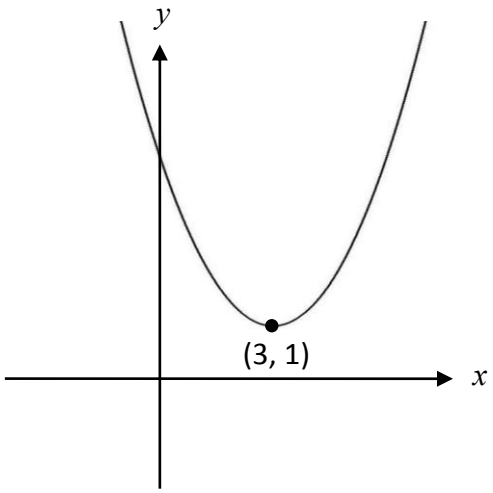
3. $f(x) = 20 - 5x^2$ **4.** $f(x) = 15 - 2x - x^2$

R5 I can determine the equation of a Quadratic from its graph.

1. The equations of the quadratic functions whose graphs are shown below are of the form $y = (x + a)^2 + b$ or $y = b - (x + a)^2$, where a and b are integers.

Write down the equation of each graph.

(a)



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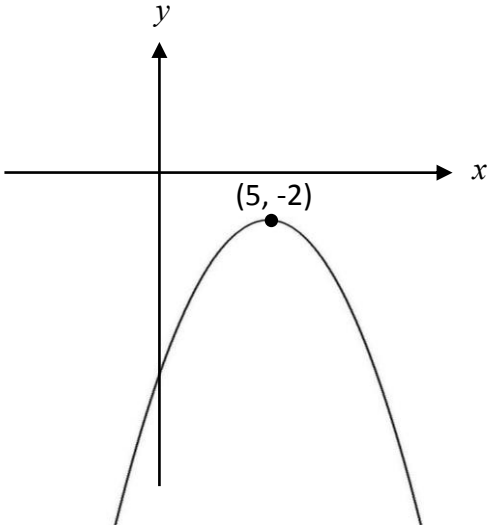

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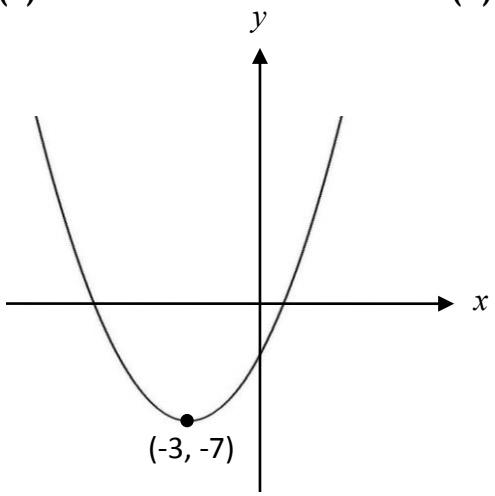

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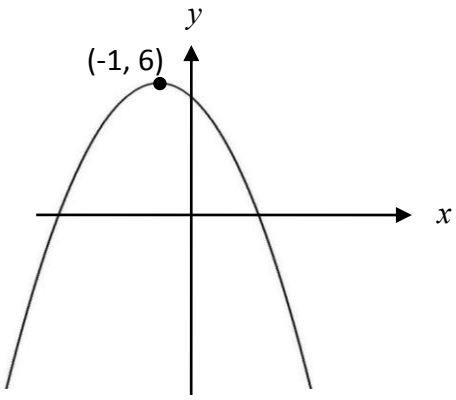
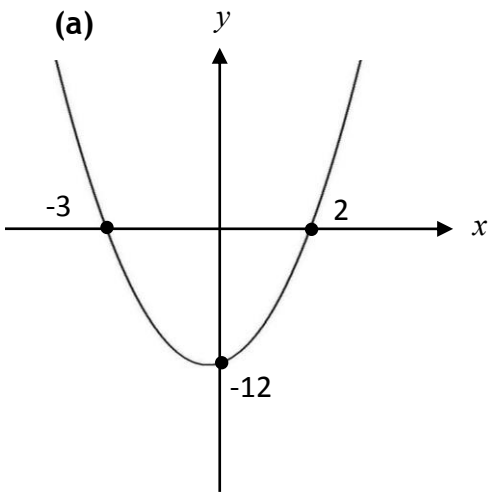
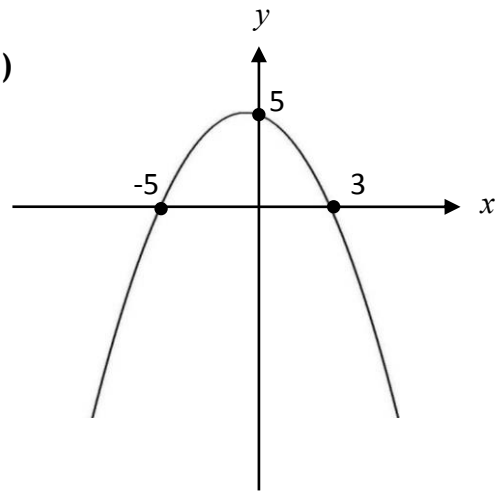


2. Find the equation of each of the parabolas below in the form $y = ax^2 + bx + c$

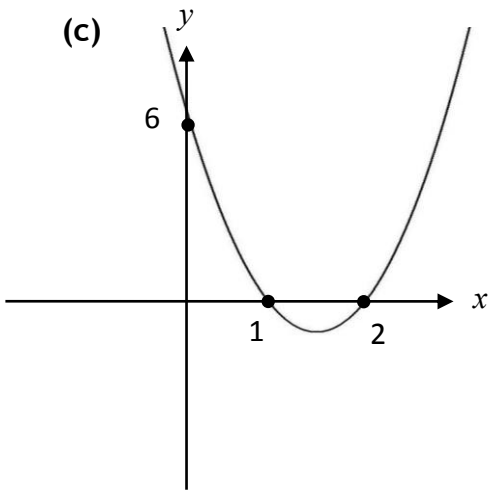
(a)



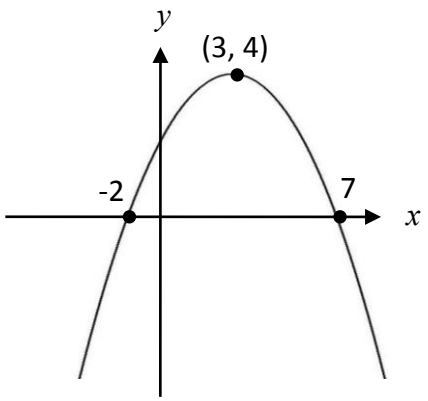
(b)



(c)



(d)



R6 I can solve Quadratic inequalities using a sketch of the graph.

1. Solve the following quadratic inequalities

(a) $(x + 2)(x - 3) > 0$ **(b)** $x(x - 7) < 0$ **(c)** $-x(x - 3) \geq 0$

(d) $-(x + 4)(x + 2) > 0$ **(e)** $(x + 5)(x - 5) \leq 0$ **(f)** $(x - 2)(x - 8) > 0$

2. Solve the following quadratic inequalities

(a) $\boldsymbol{x^2 + x - 2 > 0}$ **(b)** $2x^2 - 5x - 3 < 0$ **(c)** $3x^2 + 7x + 2 \leq 0$

(d) $\boldsymbol{x^2 - 2 > 0}$ **(e)** $2x^2 + 10x \geq 0$ **(f)** $\boldsymbol{x^2 + x > 0}$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can complete the square.

1. **(a)** Show that the function $f(x) = 3x^2 + 30x + 73$ can be written in the form $f(x) = a(x + b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise find the coordinates of the turning point of function $f(x)$.

(Non-calculator)

2. **(a)** Show the function $f(x) = 9 - 8x - x^2$ can be written in the form $f(x) = p(x + q)^2 + r$ where p, q and r are constants.

(b) Hence or otherwise find the minimum value of $g(x) = \frac{1}{f(x)}$.

(Non-calculator)

3. The cost, c pence of running a car for 20 miles at an average speed of x mph is given by $c = \frac{1}{4}x^2 - 25x + 875$

(a) Express c in the form $p(x - q)^2 + r$

(b) Find the most economical average speed and hence the cost for 20 miles at this speed

4. The height h metres, of a toy rocket is given by $h = 60 + 10t - t^2$ where t seconds is the time of flight

(a) Express h in the form $p(t + q)^2 + r$

(b) Find the maximum height of the rocket and the time taken to reach it

5. **(a)** Show that the function $f(x) = 4x^2 + 16x - 5$ can be written in the form $f(x) = a(x + b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise, find the coordinates of the turning point of the function f.

(Non-calculator)

6. **(a)** Express $f(x) = 10 - 6x - 3x^2$ in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants.

(b) Find the nature and the coordinates of the turning point of the function.

(Non-calculator)

7. **(a)** Express $f(x) = 2x^2 + 5x - 3$ in the form $f(x) = a(x + p)^2 + q$.

(b) Hence or otherwise sketch the graph of $y = f(x)$.

(Non-calculator)

NR2 I can find the nature of the roots of a Quadratic using the discriminant.

1. **(a)** Determine the nature of the roots of equation $2x^2 + 4x - k = 0$ when $k = 6$.

(b) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.

(Non-calculator)

2. Prove that for all values of k, that the equation $x^2 - 2x + k^2 + 2 = 0$ has no real roots

3. Find the nature of the roots of the equation $(p-1)^2 + 3p^2 = 6p - 11$.

4. (a) Prove that the roots of the equation,

$(9p^2 - 4qr)x^2 + 2(q + r)x - 1 = 0, where\ p, q, r \in Q$

are real for all values of p, q and r.

(b) Show also that if $q = r$ the roots are rational.

NR3 I can use the discriminant to find an unknown value.

1. Find the value of k for which equation $2x^2 - 3k = 4x^2 + k^2 - 2k$ has equal roots. $k \neq 0$

(Non-calculator)

2. Find the smallest integer value of k for which

$f(x) = (x-2)(x^2 - 2x + k)$ has equal roots.

(Non-calculator)

3. Find the values of k which ensures the following equation has equal roots

$$\frac{(x-3)^2}{x^2+3} = k.$$

4. Find two values of p for which the equation

$$p^2x^2 + 2(p+1)x + 4 = 0$$

has equal roots and solve the equation for x in each case.

5. If the roots of the equation $(x-1)(x+k) = -9$ are equal, find the values of k.

6. Find k, if the roots of the quadratic equation

$$2x^2 + (4k+2)x + 2k^2 = 0$$

are not real.

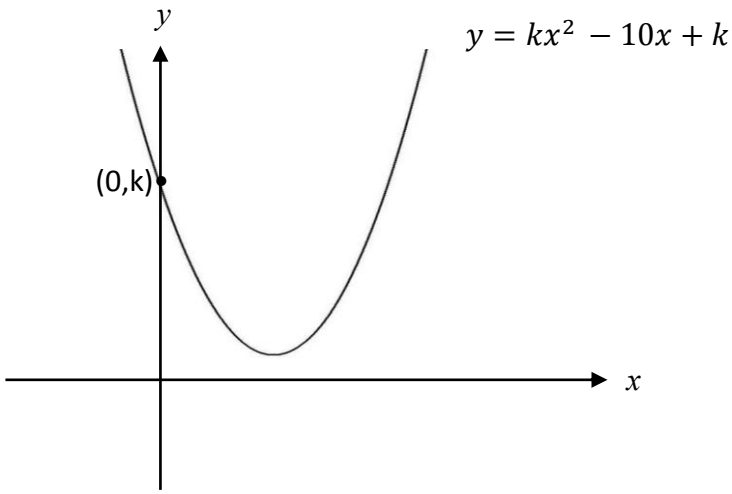
7. Find the range of values of k for which $(k+1)x^2 + 4kx + 9 = 0$ has no real roots.

8. Find the range of values of m for which, $2x^2 + 5mx + m = 0$, has two real and distinct roots.

9. For what range of values of k does the equation $x^2 - 2kx + 2 = k$ have real roots.

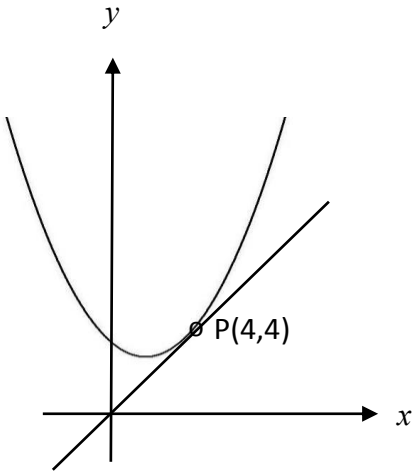
10. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 10x + k$ does not cut the $x-$ axis.

(Non-calculator)



NR4 I can apply the condition for tangency.

1. The point P(4,4) lies on the parabola $y = x^2 + mx + n$



(a) Find a relationship between m and n.

(b) The tangent to the parabola at point P is the line $y = x$.

Find the value of m.

(c) Using your values for m and n, find the value of the discriminant of $x^2 + mx + n = 0$. What feature of the above sketch is confirmed by this value?

2. Show that $y = 17 - 7x$ is a tangent to the parabola $y = -x^2 - x + 8$ and find the point of contact.

(Non-calculator)

3. The line $y = -8x + k$ is a tangent to the parabola $y = 6x - x^2$.

Find the equation of the tangent.

(Non-calculator)

4. **(a)** Show that the line $y = x + 5$ is a tangent to the curve with equation $y = \frac{1}{4}x^2 + 3x + 9$.

(b) Find the point of contact of the tangent to the curve.

NR5 I have experience of cross topic exam standard questions.

1. Figure 1 shows the sketch of the graph $f(x) = (x-3)^2 + 2$.

The graph cuts the y-axis at A and has a minimum turning point at B.

(a) Write down the coordinates of A and B.

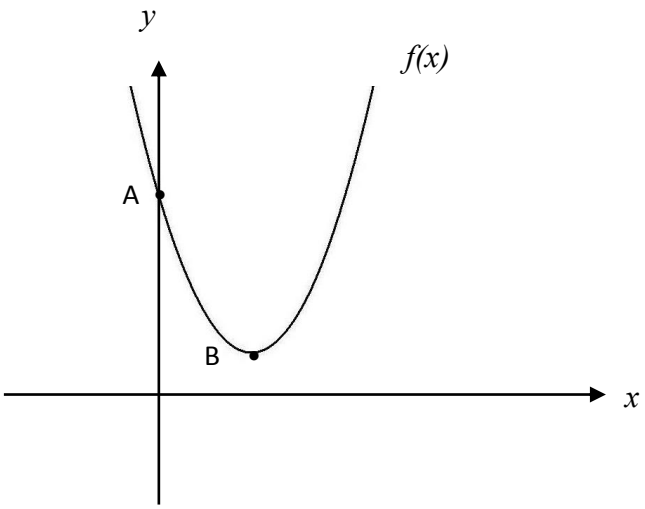


Figure 1

(b) Figure 2 shows the sketch of f(x) and $g(x) = 11 + 6x - x^2$.

Find the area enclosed by the two curves.

(Non-calculator)

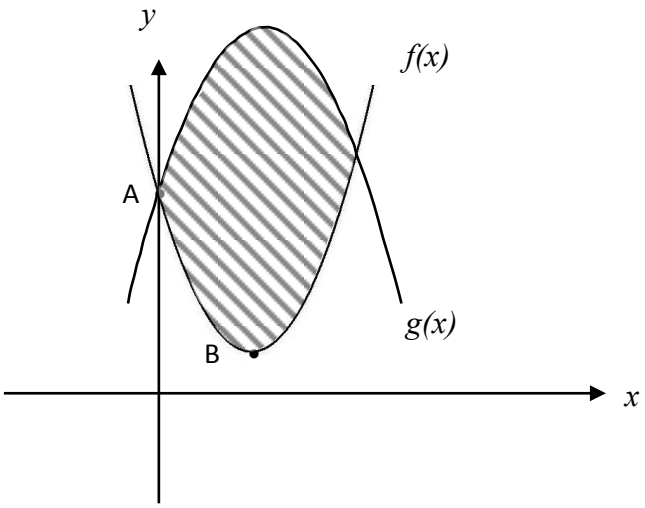
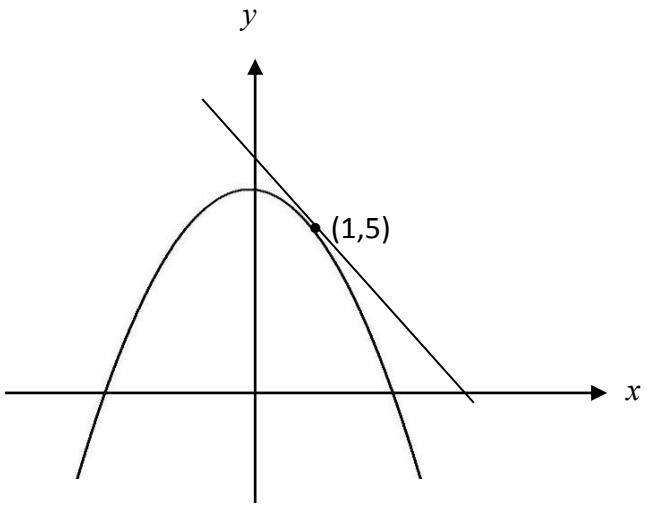


Figure 2

2. **(a)** Find the equation of the tangent to the parabola with equation $y = 7 + x - 2x^2$ at the point (1,5).



(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 2x - 10y + 26 = 0$.

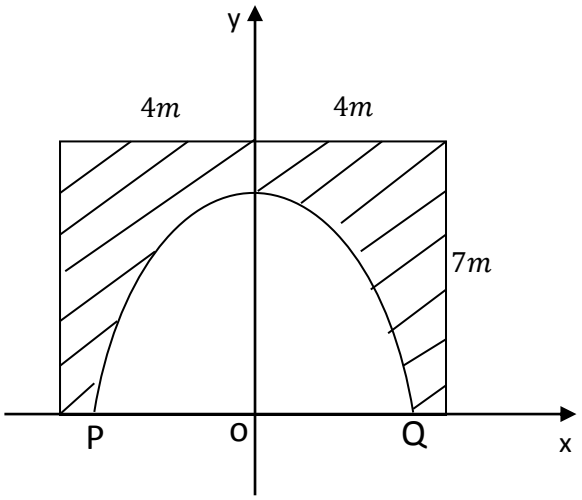
(Non-calculator)

3. **(a)** The points $A(2,6),\ B(-4,21)$ and $C(7,k)$ are on the same straight line.

Find the value of k.

(b) Find the equation of the tangent to the curve, $y = 2 + 3x - x^2$, at the point where $x = 2$.

4. The concrete on the 7 metre by 8 metre rectangular facing of the entrance to a tunnel is to be re-plastered.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 metre.

The roof is in the form of a parabola with equation $y = 3 - \frac{1}{3}x^2$.

(a) Find the coordinates of the points P and Q.

(b) Calculate the total cost of re-plastering the facing at £8 per square metre.

5. Diagram 1 below shows a rectangular sheet of plastic moulded into a parabolic shape and pegged to the ground to form a cover for a storm shelter. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.

Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres).

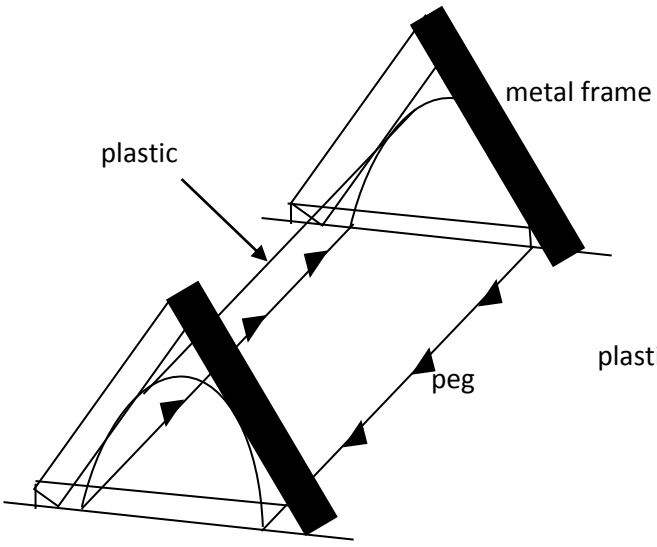


Diagram 1

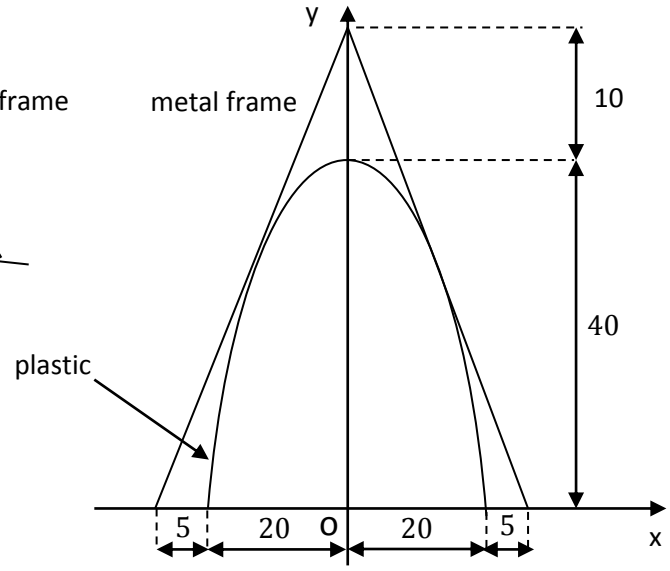


Diagram 2

(a) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{10}$, $-20 \leq x \leq 20$.

(b) Show that the triangular frame touches the cover without disturbing the parabolic shape.

6. The gradients of two straight lines are the solutions of the quadratic equation, $px^2 - 2x + (2p + 1) = 0$.

Calculate p if,

(i) the lines are perpendicular.

(ii) the lines are parallel.