Vectors
Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have revised National 5 vectors and 3D coordinate.

1. If vector $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find the resultant vector of:

(a) $\boldsymbol{a}+\boldsymbol{b}$ **(b)** $\boldsymbol{a}-\boldsymbol{b}$ **(c)** $3\boldsymbol{a}+\boldsymbol{b}$

(d) $\boldsymbol{a}-2\boldsymbol{b}$ **(e)** $5\boldsymbol{a}-3\boldsymbol{b}$ **(f)** $2\boldsymbol{a}+4\boldsymbol{b}$

2. If vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$, find the resultant vector of

(a) $\boldsymbol{a}+\boldsymbol{b}$ **(b)** $\boldsymbol{a}-\boldsymbol{b}$ **(c)** $2\boldsymbol{a}+3\boldsymbol{b}$

(d) $5\boldsymbol{a}-\boldsymbol{b}$ **(e)** $3\boldsymbol{a}-2\boldsymbol{b}$ **(f)** $\boldsymbol{a}+4\boldsymbol{b}$

3. If vector $\boldsymbol{p} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ and vector $\boldsymbol{q} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$, find the resultant vector of

(a) $\boldsymbol{p}+\boldsymbol{q}$ **(b)** $\boldsymbol{p}-\boldsymbol{q}$ **(c)** $\boldsymbol{p}+2\boldsymbol{q}$

(d) $2\boldsymbol{p}-\boldsymbol{q}$ **(e)** $3\boldsymbol{p}-5\boldsymbol{q}$ **(f)** $4\boldsymbol{p}+3\boldsymbol{q}$

4. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$, find:

(a) $|\boldsymbol{p}|$ **(b)** $|\boldsymbol{q}|$ **(c)** $|\boldsymbol{p}+\boldsymbol{q}|$

(d) $|\boldsymbol{p}-\boldsymbol{q}|$ **(e)** $|3\boldsymbol{p}-\boldsymbol{q}|$ **(f)** $|2\boldsymbol{p}+3\boldsymbol{q}|$

5. Three vectors are defined as $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{EF} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$, find:

(a) $|\overrightarrow{AB}|$ **(b)** $|\overrightarrow{CD}|$ **(c)** $|\overrightarrow{EF}|$

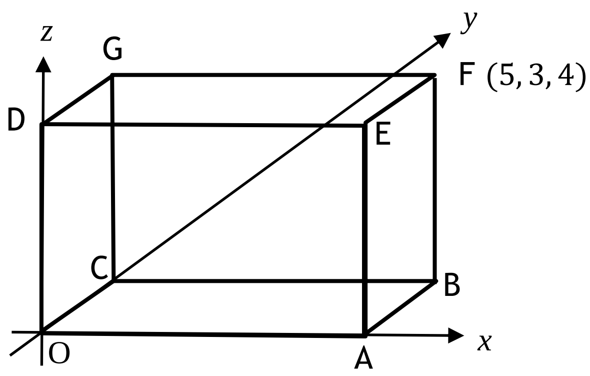
6. Three points A, B and C have the coordinates $(2, 5, 3)$, $(-1, 3, 0)$ and $(1, 4, 2)$ respectively. Find the vectors

(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$

(d) $\overrightarrow{AB}$ **(e)** $\overrightarrow{BC}$ **(f)** $\overrightarrow{AC}$

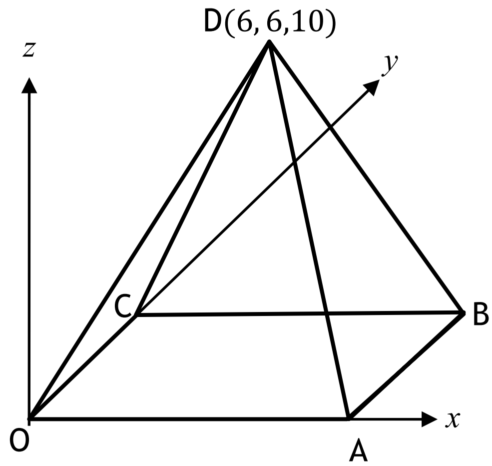
7. The diagram shows the cuboid OABCDEFG. O is the origin and OA, OC and OD are aligned with the x, y and z axes respectively. The point F has coordinates $(5, 3, 4)$.

List the coordinates of the other six vertices.



8. The diagram shows the square based pyramid DOABC. O is the origin with OA and OC aligned with the x and y axes respectively. The point D has coordinates $(6, 6, 10)$.

Write down the coordinates of the points A, B and C.



R2 I can express and manipulate vectors in the form $a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$.

1. Write the following vectors, given in unit vector form, in component form.

(a) $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ (b) $\boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ (c) $\boldsymbol{c} = 4\boldsymbol{i} + 2\boldsymbol{j}$

(d) $\boldsymbol{d} = 3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ (e) $\boldsymbol{e} = \boldsymbol{i} - 6\boldsymbol{j} - \mathbf{4}\boldsymbol{k}$ (f) $\boldsymbol{f} = -\boldsymbol{i} + \mathbf{3}\boldsymbol{k}$

2. **Write the following vectors, given in component form, in unit vector form.**

(a) $\boldsymbol{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (b) $\boldsymbol{q} = \begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix}$ (c) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$

(d) $\boldsymbol{s} = \begin{pmatrix} 9 \\ 0 \\ -3 \end{pmatrix}$ (e) $\boldsymbol{t} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (f) $\boldsymbol{u} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

3. Two vectors are defined, in unit vector form, as $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + \mathbf{5}\boldsymbol{k}$.

(a) Express $\boldsymbol{a} + \boldsymbol{b}$ **in unit vector form.**

(b) Express $2\boldsymbol{a} - \boldsymbol{b}$ **in unit vector form.**

(c) Find $|\boldsymbol{a} + \boldsymbol{b}|$**.**

(d) Find $|\mathbf{2}\boldsymbol{a} - \boldsymbol{b}|$**.**

4. Two vectors are defined, in unit vector form, as $\boldsymbol{p} = 3\boldsymbol{i} - \boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - \mathbf{2}\boldsymbol{j} + \mathbf{3}\boldsymbol{k}$.

(a) Express $\boldsymbol{p} + 2\boldsymbol{q}$ **in unit vector form.**

(b) Express $3\boldsymbol{p} - 4\boldsymbol{q}$ **in unit vector form.**

(c) Find $|\boldsymbol{p} + 2\boldsymbol{q}\ |$**.**

(d) Find $|3\boldsymbol{p} - 4\boldsymbol{q}|$**.**

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	**I can determine whether or not coordinates are collinear, using the appropriate language, and can apply my knowledge of vectors to divide lines in a given ratio.**

1. The point Q divides the line joining P$(-1,-1,3)$ and R$(5,-1,-3)$ in the ratio 5:1. Find the coordinates of Q.

2. John is producing a 3D design on his computer.

Relative to suitable axes 3 points in his design have coordinates P$(-3,4,7)$, Q$(-1,8,3)$ and R$(0,10,1)$.

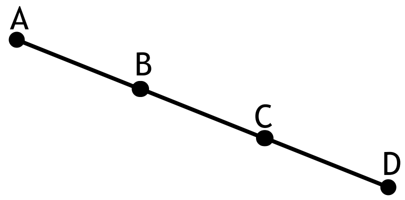
(a) Show that P, Q and R are collinear.

(b) Find the coordinates of S such that $\overrightarrow{PS} = 4\overrightarrow{PQ}$.

3. A and B are the points $(0,-2,3)$ and $(3,0,2)$ respectively.

B and C are the points of trisection of AD, that is AB = BC = CD.

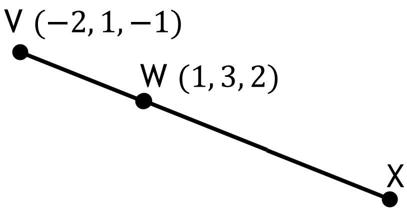
Find the coordinates of D.



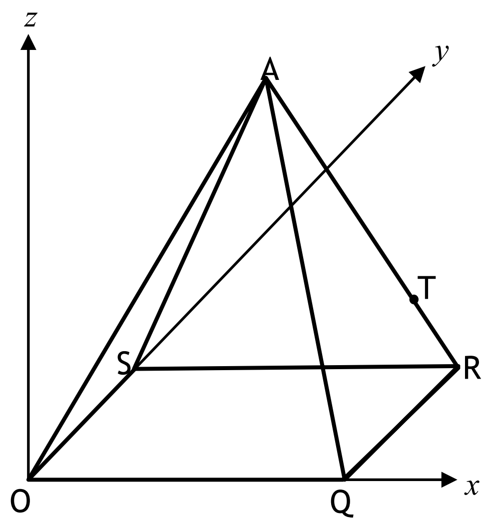
4. The points V, W and X are shown on the line opposite.

V, W and X are collinear points such that WX = 2VW.

Find the coordinates of X.



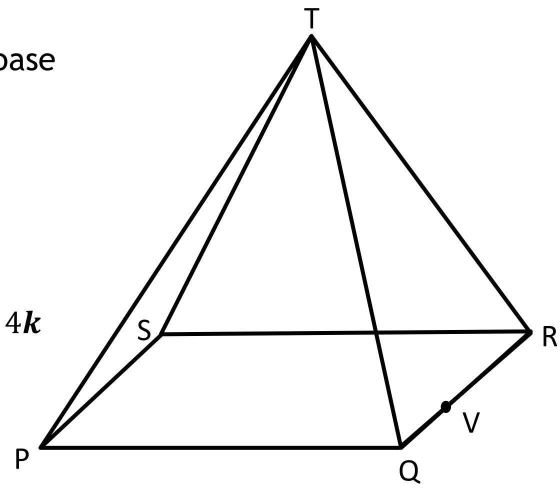
5. AOQRS is a pyramid. Q is the point $(16, 0, 0)$, R is $(16, 8, 0)$ and A is $(8, 4, 12)$. T divides RA in the ratio 1:3.



(a) Find the coordinates of the point T.

(b) Express $\overrightarrow{QT}$ in component form.

6. PQRST is a pyramid with a rectangular base PQRS.



V divides QR in the ratio 1:3 and

$\overrightarrow{TP} = -7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k},$

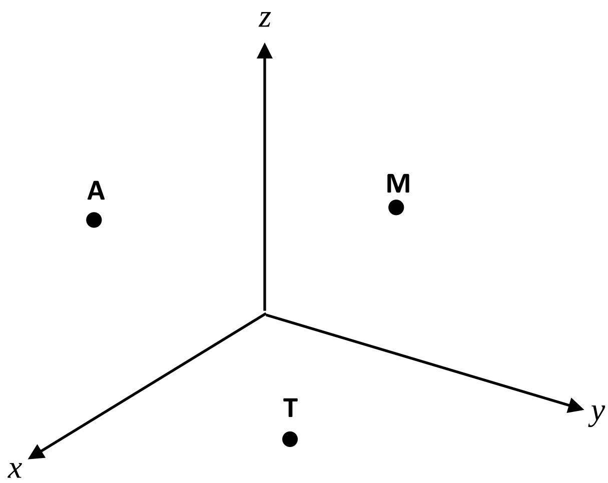
$\overrightarrow{PQ} = 6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$ and $\overrightarrow{PS} = 8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$

Find $\overrightarrow{TV}$ in component form.

NR2 I can apply knowledge of vectors to find an angle in three dimensions.

1. A surveyor is checking a room for movement in the walls due to subsidence. She sets up 3 points. Two of the points, A and M, are on two different walls which meet perpendicularly along the z axis, relative to the axes shown. The other point T is on the floor.

The three points have coordinates $(6, 0, 7)$, $(0, 5, 6)$ and $(4, 5, 0)$.



(a) Match the three points to the correct coordinates.

(b) Write $\overrightarrow{TA}$ and $\overrightarrow{TM}$ in component form.

(c) Find the size of angle ATM.

2. V, W and X have coordinates $(1, 3, -1)$, $(2, 0, 1)$ and $(-3, 1, 2)$ respectively.

(a) Find $\overrightarrow{VW}$ and $\overrightarrow{VX}$ in component form.

(b) Hence find the size of angle WVX.

3. Three planes, Tango (T), Delta (D) and Bravo (B) are being tracked by radar. Relative to a suitable origin, the positions of the three planes are T$(23, 0, 8)$, D$(-12, 0, 9)$ and B$(28, -15, 7)$

(a) Express the vectors $\overrightarrow{BT}$ and $\overrightarrow{BD}$ in component form.

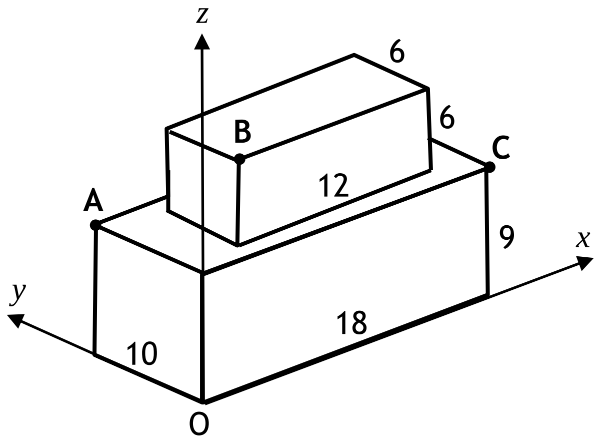
(b) Find the size of angle TBD.

4. A cuboid measuring 12cm by 6cm by 6cm is placed centrally on top of another cuboid measuring 18cm by 10cm by 9cm.

Coordinate axes are taken as shown.

(a) The point A has coordinates $(0, 10, 9)$ and the point C has coordinates $(18, 0, 9)$. Write down the coordinates of B.

(b) Find the size of angle ABC.



5. A square-based pyramid, OABCD, has a height of 10 units and the square base has a length of 8 units.

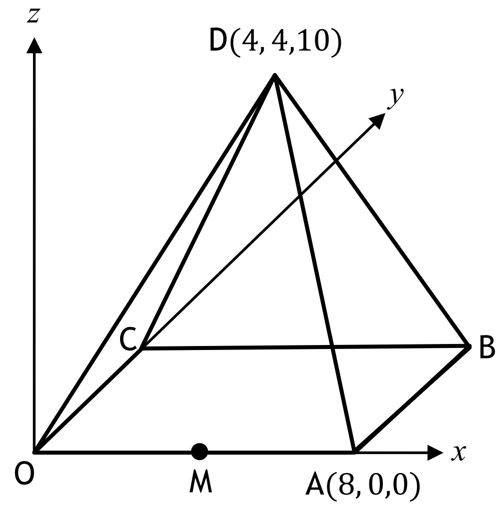
The coordinates of two points, A and D are shown on the diagram.

(a) Write down the coordinates of the point B.

(b) Determine the components of the vectors $\overrightarrow{DA}$ and $\overrightarrow{DB}$.

(c) Find the size of angle ADB.

M is the midpoint of OA.



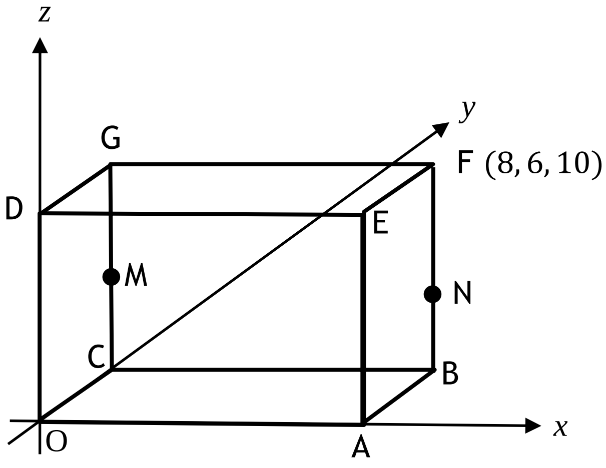
(d) Write down the coordinates of C and M.

(e) Determine the components of the vectors $\overrightarrow{DC}$ and $\overrightarrow{DM}$.

(f) Find the size of angle CDM.

6. The diagram shows a cuboid OABCDEFG with the lines OA, OC and OD lying on the axes.

The point F has coordinates $(8, 6, 10)$, M is the midpoint of CG and N divides BF in the ratio 2:3.



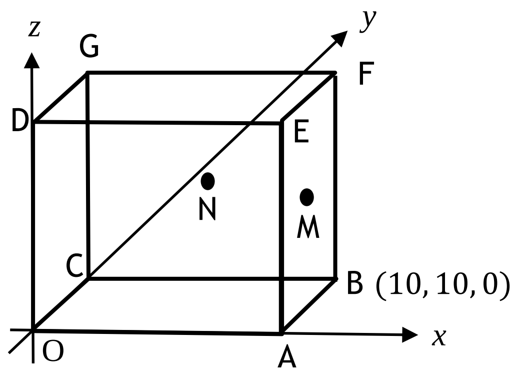
(a) State the coordinates of A, M and N.

(b) Determine the components of the vectors $\overrightarrow{MA}$ and $\overrightarrow{MN}$.

(c) Find the size of angle AMN.

7. The diagram shows a cube OABCDEFG. B has coordinates $(10, 10, 0)$

M is the centre of AEFB and N is the centre of face GFBC.

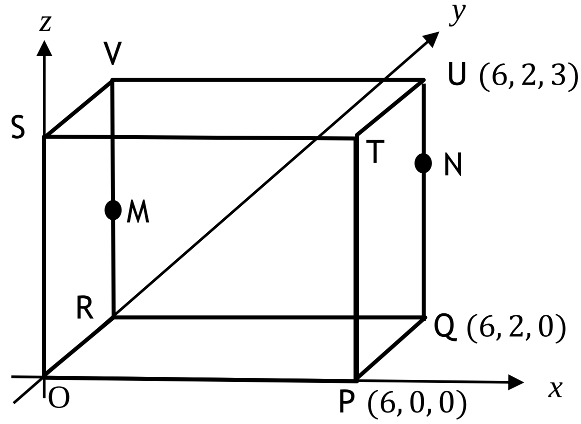


(a) Write down the coordinates of G.

(b) Find $\boldsymbol{m}$ and $\boldsymbol{n}$, the position vectors of M and N.

(c) Find the size of angle MON.

8. In the diagram OPQRSTUV is a cuboid. M is the midpoint of VR and N is the point on UQ such that $\text{UN} = \frac{1}{3}\text{UQ}$.



(a) State the coordinates of T, M and N.

(b) Determine the components of the vectors $\overrightarrow{TM}$ and $\overrightarrow{TN}$.

(c) Find the size of angle MTN.

NR3 I know the properties of the scalar product and their uses.

1. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are defined by $\boldsymbol{p} = -3\boldsymbol{i} - 12\boldsymbol{k}$ and $\boldsymbol{q} = 8\boldsymbol{i} + 7\boldsymbol{j} - 2\boldsymbol{k}$.

Determine whether or not $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular to each other.

2. For what value of p are the vectors $\boldsymbol{a} = \begin{pmatrix} p \\ -2 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 14 \\ 2p \end{pmatrix}$ perpendicular?

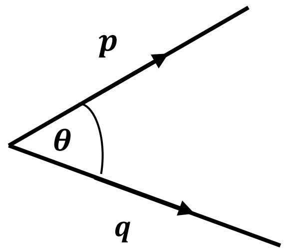
3. A and B have coordinates $(9, -7, -14)$, $(0, -1, -3)$ respectively.

C has coordinates $(k, 0, -1)$.

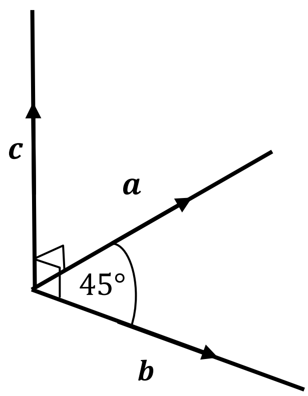
Given that AB is perpendicular to CB, find the value of k.

4. The diagram shows vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

If $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.(\boldsymbol{p} + \boldsymbol{q}) = 15$, find the size of the acute angle θ between $\boldsymbol{p}$ and $\boldsymbol{q}$.



5.



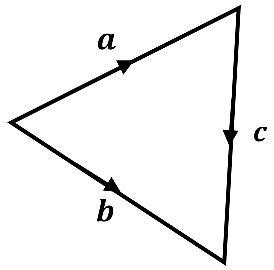
The diagram shows vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

$|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = \sqrt{2}$ and the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is $45°$.

$\boldsymbol{c}$ is perpendicular to $\boldsymbol{a}$ and to $\boldsymbol{b}$.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.

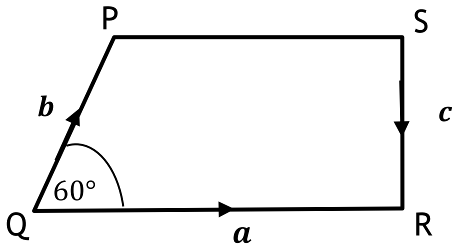
6. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ form an equilateral triangle of length 3 units.



(a) Find the scalar product $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.

(b) What does this tells us about the vectors $\boldsymbol{a}$ and $\boldsymbol{b}+\boldsymbol{c}$.

7. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are shown on the diagram. Angle PQR = $60°$.



It is also given that $|\boldsymbol{a}| = 3$ **and** $|\boldsymbol{b}| = 2$.

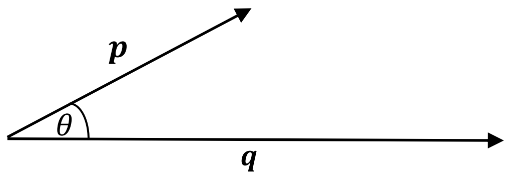
(a) **Evaluate** $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ **and** $\boldsymbol{c}.(\boldsymbol{a}-\boldsymbol{b})$.

(b) **Find** $|\boldsymbol{b}+\boldsymbol{c}|$ **and** $|\boldsymbol{a}-\boldsymbol{b}|$.

NR4 I have experience of cross topic exam standard questions.

Vectors and Polynomials

1. $\boldsymbol{p}$ and $\boldsymbol{q}$ are vectors given by $\boldsymbol{p} = \begin{pmatrix} k^2 \\ 3 \\ k+1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} k \\ k^2 \\ -2 \end{pmatrix}$, where $k > 0$.



(a) If $\boldsymbol{p} \cdot \boldsymbol{q} = 1 - k$, show that $k^3 + 3k^2 - k - 3 = 0$.

(b) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise fully.

(c) Deduce the only possible value of k.

Vectors and Quadratics

1. P is the point $(1, -3, 0)$, Q$(1, -1, 2)$ and R$(k, -2, 0)$

(a) Express $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form.

(b) Show that $\cos P\hat{Q}R = \frac{3}{\sqrt{2(k^2-2k+6)}}$

(c) If angle $PQR = 30°$, find the possible values of k.