

Functions and Graphs

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have had experience of graphing linear and quadratic functions.

1. Sketch the graphs of the following straight lines:

(a) $y = 2x + 3$ (b) $y = -3x - 2$ (c) $y = \frac{1}{2}x + 1$
(d) $2x + y - 4 = 0$ (e) $x - 3y + 6 = 0$ (f) $3x + 4y - 8 = 0$

2. For the following Quadratic Functions:

- Calculate where the graph crosses the x-axis and the y-axis
- Find the Turning Point and state it's nature
- Sketch the graph

(a) $y = x^2 - 4x + 3$ (b) $y = x^2 + 10x + 24$ (c) $y = x^2 + 2x - 15$
(d) $y = x^2 - 4x - 12$ (e) $y = x^2 - x - 12$ (f) $y = 4x^2 - 8x + 3$

3. For the following Quadratic Functions:

- Express in the form $y = a(x + b)^2 + c$
- State the Turning Point and state it's nature
- Sketch the graph indicating the turning point and y-intercept

(a) $y = x^2 - 4x + 5$ (b) $y = x^2 + 6x - 1$ (c) $y = x^2 - 3x + 4$
(d) $y = 2x^2 - 12x + 5$ (e) $y = 2 + 8x - x^2$ (f) $y = 3x^2 - 12x - 4$
(g) $y = 3 + 6x - x^2$ (h) $y = 5 - 12x - 2x^2$ (i) $y = x^2 + 5x - 2$
(j) $y = 3x^2 - 18x + 5$ (k) $y = 2x^2 + 8x - 4$ (l) $y = 4x^2 - 8x + 1$

(m) $y = 2x^2 - 10x - 3$ (n) $y = 2 + 8x - 2x^2$ (o) $y = 4x^2 - 16x + 9$
 (p) $y = 7 + 12x - 3x^2$ (q) $y = 5 - 12x - 4x^2$ (r) $y = 6x^2 + 24x - 5$

R2 I have found x and y - intercepts for a range of graphs of functions.

1. Calculate where the graph of the following functions crosses the x -axis and the y -axis:

(a) $y = 4x + 8$ (b) $y = \frac{1}{4}x - 3$ (c) $3x + 5y - 15 = 0$
 (d) $y = x^2 - 3x$ (e) $y = x^2 + 9x$ (f) $y = 3x^2 - 12x$
 (g) $y = x^2 - 16$ (h) $y = 4x^2 - 9$ (i) $y = 2x^2 - 18$
 (j) $y = x^2 - 7x + 10$ (k) $y = x^2 + 6x - 27$ (l) $y = 6x^2 - 13x - 5$
 (m) $y = \sqrt{x + 4}$ (n) $y = \sqrt{2x + 9}$ (o) $y = \sqrt{x + 16}$

R3 I can solve linear and quadratic inequalities

1. Solve the following inequalities:

(a) $4x - 12 < 0$ (b) $15 - 3x \geq 0$ (c) $6x + 15 \leq 0$
 (d) $-2x - 7 > 0$ (e) $5x + 17 \leq 0$ (f) $18 - 4x \geq 0$
 (g) $x^2 - 5x = 0$ (h) $x^2 + 7x = 0$ (i) $4x^2 - 36x = 0$
 (j) $x^2 - 49 = 0$ (k) $9x^2 - 25 = 0$ (l) $3x^2 - 12 = 0$
 (m) $x^2 - 5x + 6 = 0$ (n) $x^2 + 8x - 20 = 0$ (o) $x^2 - 2x - 35 = 0$
 (p) $6x^2 - 11x + 3 = 0$ (q) $2x^2 + 7x + 6 = 0$ (r) $4x^2 - 17x - 15 = 0$

R4 I can understand and use basic set notation.

1. Using the $\{ \}$ brackets notation, list the following sets:
 - (a) The set of months ending in the letter y.
 - (b) The set of the first ten prime numbers.
 - (c) The set of letters of the alphabet between G and P.
 - (d) The set of odd numbers greater than 20 but less than 30.

2. Describe the following sets in words:
 - (a) $\{ \text{January, June, July} \}$
 - (b) $\{ \text{Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto} \}$
 - (c) $\{ \text{Cone, Pyramid} \}$
 - (d) $\{ 1, 4, 9, 16, 25 \}$

3. Connect these elements with their sets, using \in :
Elements: a, green, 12, b, 13
Sets: $\{a,b,c\}$, $\{\text{even numbers}\}$, $\{\text{colours}\}$, $\{10,11,12,13\}$,
 $\{\text{vowels in the English alphabet}\}$, $\{\text{red,yellow,green}\}$

4. State which of the following are true and which are false:
 - (a) $2 \in \{ \text{prime numbers} \}$
 - (b) $\{ x,y,z \}$ and $\{ p,q,r \}$ are equal sets
 - (c) $\{ 0 \}$ is the empty set
 - (d) $\{ k,l,m,n \} = \{ m,l,k,n \}$
 - (e) If $A = \{ \text{whole numbers greater than 50} \}$, then $46 \notin A$

5. Using set notation, rewrite the following:
- 3 is a member of the set W.
 - The empty set.
 - x does not belong to the set A.
 - S is a subset of the set T.
 - The set P is equal to the set Q.
6. $S = \{ 1,2,3,4,5,6,7,8,9,10 \}$. List the following subsets of S:
- The set of prime numbers in S.
 - The set of odd numbers in S less than 8.
 - The set of elements in S which are factors of 70.
 - The set of numbers in S which are divisible by 3.
7. Find a set equal to each of the following:
- $\{ 1,2,3 \} \cap \{ 2,3,4,5 \}$
 - $\{ 1,2,3 \} \cap \{ 4,5,6 \}$
 - $\{ 1,2,3 \} \cap \{ 3,1,2 \}$
 - $\emptyset \cap \{ 2,3,4,5 \}$
8. $E = \{ 1,2,3,4,5,6,8,10 \}$ $A = \{ 1,2,3,4 \}$ $B = \{ 3,4,5 \}$ and $C = \{ 2,4,6,8,10 \}$
- Find $A \cap B$, $B \cap C$ and $A \cap C$.
 - The set of elements common to A,B and C is denoted by $A \cap B \cap C$.
Find $A \cap B \cap C$.
9. Given that $A = \{ 0,1,2 \}$, which of the following are true?
- $2 \in A$
 - $1 \subset A$
 - $\{1\} \subset A$
 - $0 \in \emptyset$
 - $A \subset A$
 - $1 \notin A$

10. $P = \{ 1,2,3,4,5,6,7 \}$ $Q = \{ 5,6,7,8,9,10 \}$ are subsets of $E = \{ 1,2,3,\dots,12 \}$.

List the members of the following sets:

- | | | |
|-----------------|-------------------|------------------------|
| (a) $P \cap Q$ | (b) $P \cup Q$ | (c) P' |
| (d) Q' | (e) $(P \cap Q)'$ | (f) $(P \cup Q)'$ |
| (g) $P \cap Q'$ | (h) $P' \cap Q$ | (i) $P \cap \emptyset$ |

R5 I have investigated domains and ranges.

1. State a suitable domain for the following functions:

- | | | |
|----------------------------------|----------------------------------|---|
| (a) $f(x) = \frac{x^2}{x-1}$ | (b) $f(x) = \frac{4x-2}{2x-3}$ | (c) $f(x) = \frac{x^2+5}{(x-1)(x+4)}$ |
| (d) $f(x) = \frac{4x^2}{x^2-3x}$ | (e) $f(x) = \frac{2x+7}{x^2-16}$ | (f) $f(x) = \frac{x^2-5x+4}{x^2+8x+12}$ |
| (g) $f(x) = \sqrt{x-7}$ | (h) $f(x) = \sqrt{10-x}$ | (i) $f(x) = \sqrt{x^2-9}$ |
| (j) $f(x) = \sqrt{36-x^2}$ | (k) $f(x) = \sqrt{x^2-5x-6}$ | (l) $f(x) = \sqrt{x^2+3x}$ |

2. State the range of each function given its domain:

- | | |
|-----------------------------------|-------------------------------|
| (a) $f(x) = 3x - 4$; | $x \in \{ 2, 3, 4, 5 \}$ |
| (b) $f(x) = x^2 - 3x + 4$; | $x \in \{ -2, -1, 0, 1, 2 \}$ |
| (c) $f(x) = 3x^2 - 7$; | $x \in \{ -3, -2, 0, 2, 3 \}$ |
| (d) $f(x) = \frac{x^2+3}{2x-1}$; | $x \in \{ 1, 3, 5, 7 \}$ |

R6 I can calculate a basic composite function.

1. Given $f(x) = x + 1$, $g(x) = x^2$ and $h(x) = x^2 - 2$, find the following functions:

- | | | |
|---------------|---------------|---------------|
| (a) $f(g(x))$ | (b) $f(h(x))$ | (c) $f(f(x))$ |
| (d) $g(f(x))$ | (e) $g(h(x))$ | (f) $g(g(x))$ |
| (g) $h(f(x))$ | (h) $h(g(x))$ | (i) $h(h(x))$ |

2. Given $f(x) = 2x - 3$, $g(x) = x^2$ and $h(x) = x^2 + 4$, find the following functions:

- | | | |
|---------------|---------------|---------------|
| (a) $f(g(x))$ | (b) $f(h(x))$ | (c) $f(f(x))$ |
| (d) $g(f(x))$ | (e) $g(h(x))$ | (f) $g(g(x))$ |
| (g) $h(f(x))$ | (h) $h(g(x))$ | (i) $h(h(x))$ |

3. Given $f(x) = x^2$, $g(x) = 3x + 1$ and $h(x) = 4 - 2x$, find the following functions:

- | | | |
|---------------|---------------|---------------|
| (a) $f(g(x))$ | (b) $f(h(x))$ | (c) $f(f(x))$ |
| (d) $g(f(x))$ | (e) $g(h(x))$ | (f) $g(g(x))$ |
| (g) $h(f(x))$ | (h) $h(g(x))$ | (i) $h(h(x))$ |

4. Given $f(x) = x - 2$, $g(x) = \frac{2}{x^2}$ and $h(x) = \frac{4}{x+1}$, find the following functions:

- | | | |
|---------------|---------------|---------------|
| (a) $f(g(x))$ | (b) $f(h(x))$ | (c) $f(f(x))$ |
| (d) $g(f(x))$ | (e) $g(h(x))$ | (f) $g(g(x))$ |
| (g) $h(f(x))$ | (h) $h(g(x))$ | (i) $h(h(x))$ |

5. Given $f(x) = x - 2$, $g(x) = \sin x$ and $h(x) = \log_a x$, find the following functions:

- (a) $f(g(x))$ (b) $f(h(x))$ (c) $f(f(x))$
(d) $g(f(x))$ (e) $g(g(x))$ (f) $h(f(x))$

6. Given $f(x) = 2x$, $g(x) = \cos x$ and $h(x) = e^x$, find the following functions:

- (a) $f(g(x))$ (b) $f(h(x))$ (c) $f(f(x))$
(d) $g(f(x))$ (e) $g(g(x))$ (f) $h(f(x))$

7. Given $f(x) = 3x^2 + 2x - 1$, $g(x) = \sin x$ and $h(x) = \log_4 x$, find the following functions:

- (a) $f(g(x))$ (b) $f(h(x))$ (c) $f(f(x))$
(d) $g(f(x))$ (e) $g(g(x))$ (f) $h(f(x))$

8. Given $f(x) = x + 2$, $g(x) = e^x$ and $h(x) = \tan x$, find the following functions:

- (a) $f(g(x))$ (b) $f(h(x))$ (c) $f(f(x))$
(d) $g(f(x))$ (e) $g(g(x))$ (f) $h(f(x))$

R7 I understand that $f(g(x)) = x$ implies that $g(x)$ is the inverse of $f(x)$.

1. If $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$
 - (a) Find $f(g(x))$ and $g(f(x))$.
 - (b) State a relationship between $f(x)$ and $g(x)$.

2. If $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$
 - (a) Find $f(g(x))$ and $g(f(x))$.
 - (b) State a relationship between $f(x)$ and $g(x)$.

3. If $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$
 - (a) Find $f(g(x))$ and $g(f(x))$.
 - (b) State a relationship between $f(x)$ and $g(x)$.

4. If $f(x) = 6x - 3$ and $g(x) = \frac{x+3}{6}$
 - (a) Find $f(g(x))$ and $g(f(x))$.
 - (b) State a relationship between $f(x)$ and $g(x)$.

5. If $f(x) = 5x + 1$ and $g(x) = \frac{x-1}{5}$
 - (a) Find $f(g(x))$ and $g(f(x))$.
 - (b) State a relationship between $f(x)$ and $g(x)$.

Section B

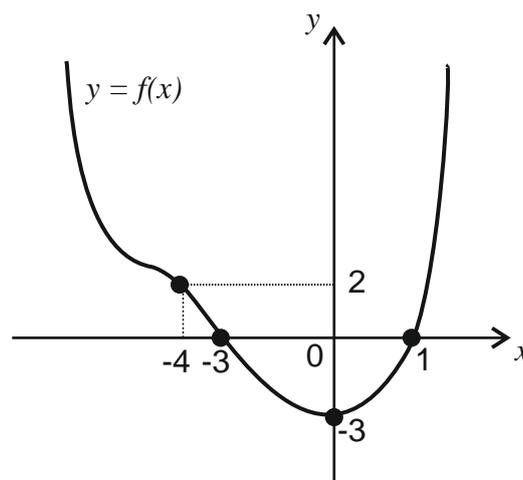
This section is designed to provide examples which develop Course Assessment level skills

NR1 I can identify and sketch a function after a transformation of the form $kf(x)$, $f(x) + k$, $f(kx)$, $f(x + k)$, $-f(x)$, $f(-x)$, or a combination of these.

1. The diagram shows the graph of a function f .

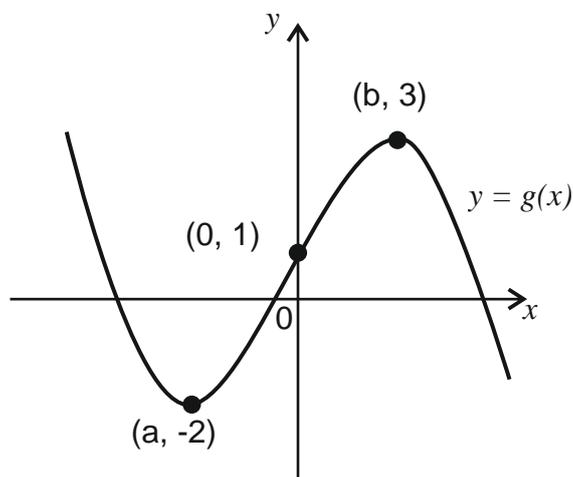
f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

- (a) Sketch the graph $y = f(-x)$.
 (b) On the same diagram, sketch the graph $y = 2f(-x)$.



2. The diagram shows the graph of $y = g(x)$.

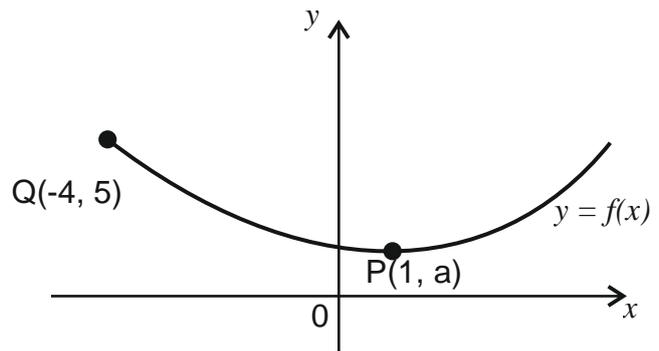
- (a) Sketch the graph of $y = -g(x)$.
 (b) On the same diagram, sketch the graph $y = 3 - g(x)$.



3. The diagram shows the graph of a function $y = f(x)$.

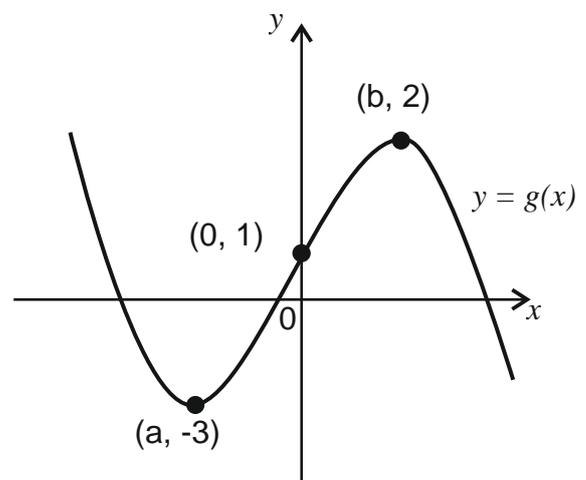
Copy the diagram and on it sketch the graphs of:

- (a) $y = f(x-4)$.
 (b) $y = 2 + f(x-4)$.



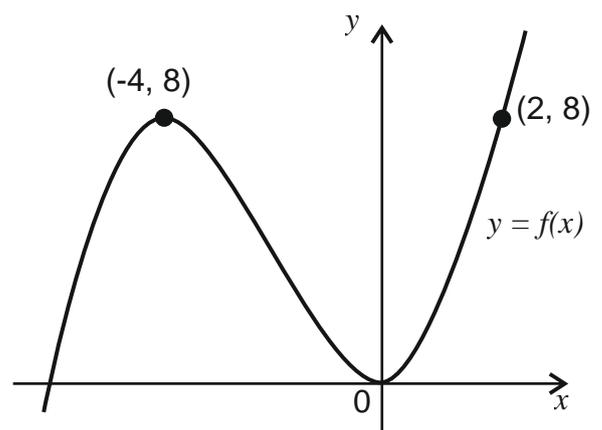
4. The diagram shows the graph of $y = g(x)$.

- (a) Sketch the graph of $y = -g(x)$.
 (b) On the same diagram, sketch the graph $y = 4 - g(x)$.

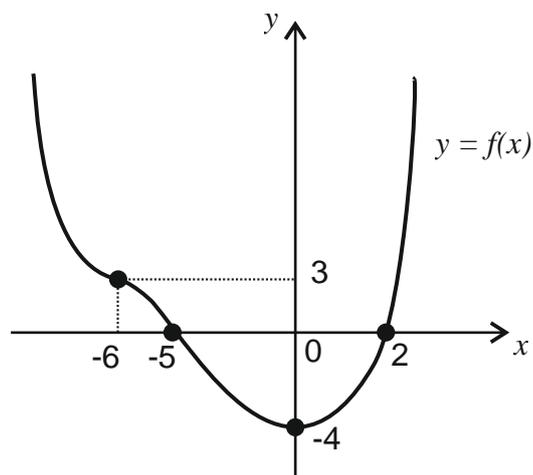


5. The diagram shows a sketch of the function $y = f(x)$.

- (a) Copy the diagram and on it sketch the graph of $y = f(2x)$.
 (b) On a separate diagram sketch the graph of $y = 1 - f(2x)$.

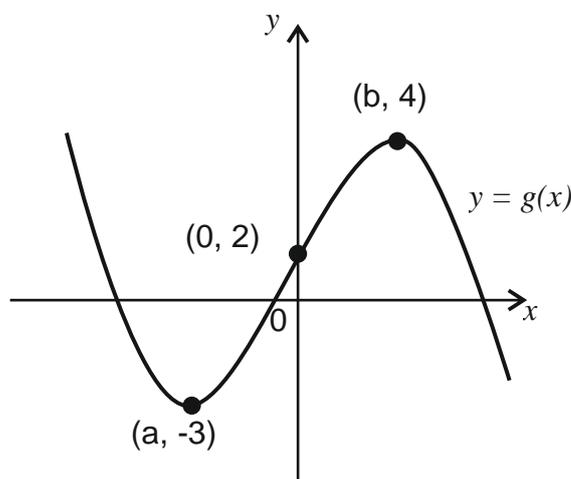


6. The diagram shows the graph of a function f .
 f has a minimum turning point at $(0, -4)$ and a point of inflexion at $(-6, 3)$.



- (a) Sketch the graph $y = f(-x)$.
 (b) On the same diagram, sketch the graph $y = 3f(-x)$.

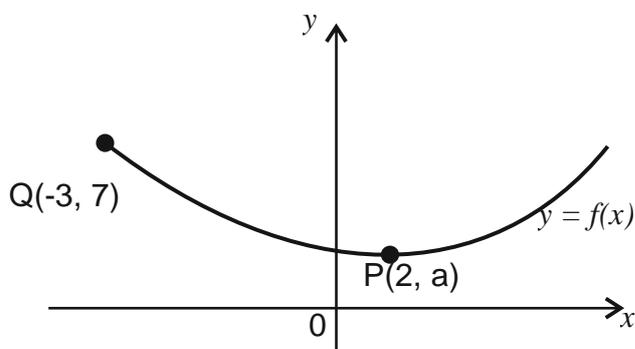
7. The diagram shows the graph of $y = g(x)$.



- (a) Sketch the graph of $y = -g(x)$.
 (b) On the same diagram, sketch the graph $y = 4 - g(x)$.

8. The diagram shows the graph of a function $y = f(x)$.

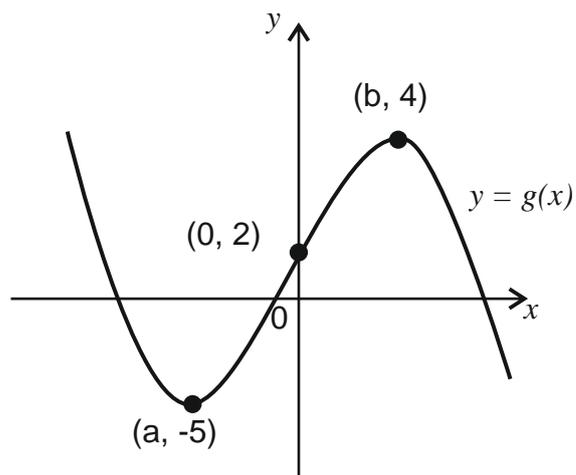
Copy the diagram and on it sketch the graphs of:



- (a) $y = f(x+2)$.
 (b) $y = 3 + f(x+2)$.

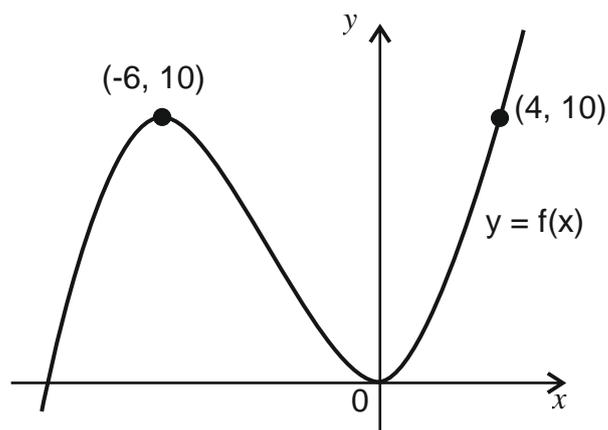
9. The diagram shows the graph of $y = g(x)$.

- (a) Sketch the graph of $y = -g(x)$.
(b) On the same diagram, sketch the graph $y = 5 - g(x)$.



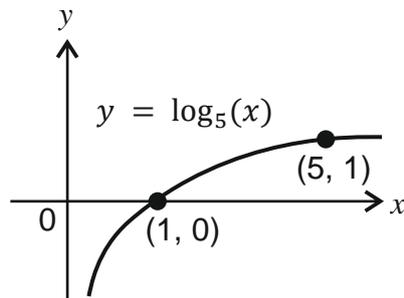
10. The diagram shows a sketch of the function $y = f(x)$.

- (a) Copy the diagram and on it sketch the graph of $y = f(2x)$.
(b) On a separate diagram sketch the graph of $y = 3 - f(2x)$.



NR2 I can sketch logarithmic and exponential functions and determine a suitable domain or range for a given function/composite function.

1.



The diagram shows a sketch of part of the graph of $y = \log_5 x$.

a) Make a copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 x + 1$.

Find the coordinates of the point where it crosses the x-axis.

b) Make a second copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$.

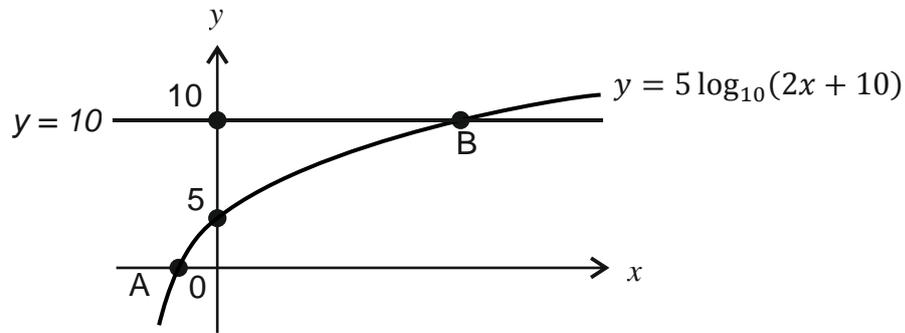
2. The functions f and g , defined on suitable domains, are given by

$$f(x) = \frac{1}{x^2 - 4} \quad \text{and} \quad g(x) = 2x + 1.$$

a) Find an expression for $h(x)$ where $h(x) = g(f(x))$.
Give your answer as a single fraction.

b) State a suitable domain for h .

3.



Part of the graph of $y = 5 \log_{10}(2x + 10)$ is shown in the diagram (not to scale).

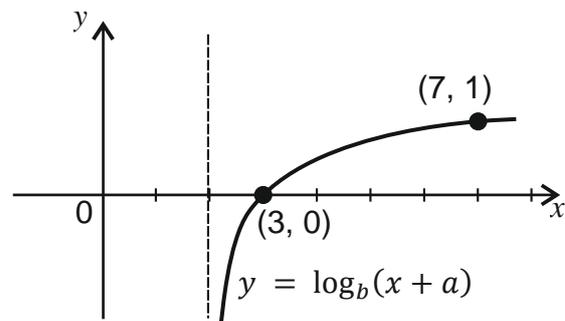
This graph crosses the x-axis at the point A and the straight line $y = 10$ at the point B.

Find algebraically the x-coordinates of A and B.

4.

The diagram shows part of the graph of $y = \log_b(x + a)$.

Determine the values of a and b .



5.

The diagram shows part of the graph of $y = 2^x$.

a) Sketch the graph of $y = 2^{-x} - 8$.

b) Find the coordinates of the points where it crosses the x and y axes.

6.

a) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$

b) Prove that the graphs intersect at a point where the x-coordinate is

$$\log_a\left(\frac{1}{a-1}\right)$$

7. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

a) Find $h(x)$ where $h(x) = g(f(x))$.

b) (i) Write down the coordinates of the minimum turning point $y = h(x)$

(ii) Hence state the range of the function h .

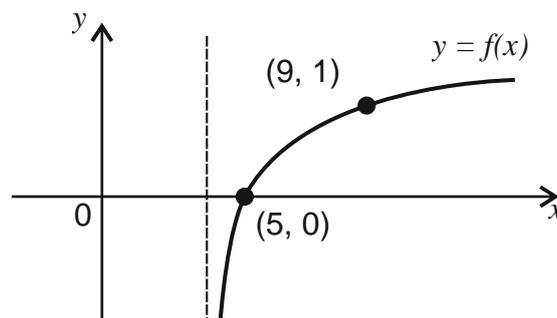
8. The function f is of the form

$$f(x) = \log_b(x - a)$$

The graph of $y = f(x)$ is shown in the diagram.

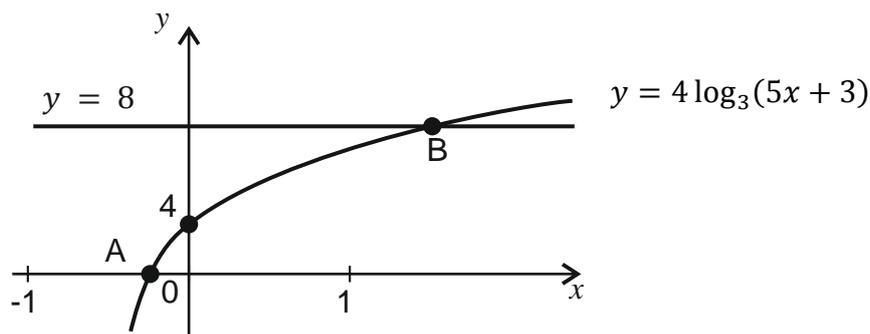
a) Write down the values of a and b .

b) State the domain of f .



9. Part of the graph of $y = 4\log_3(5x + 3)$ is shown in the diagram. This graph crosses the x -axis at the point A and the straight line $y = 8$ at the point B.

Find the x -coordinate of B.



10. Sketch the following pairs of graphs on the same set of axes:

(a) $y = 2^x$ and $y = 2^x + 1$

(b) $y = a^x$ and $y = 3(a^x)$

(c) $y = 3^x$ and $y = 3^{(x+1)}$

(d) $y = \log_2 x$ and $y = \log_2 x(x - 3)$

(e) $y = \log_3 x$ and $y = \log_3 x + 4$

(f) $y = \log_5 x$ and $y = \log_5 25$

(g) $y = \log_a x$ and $y = \log_a \frac{1}{x}$

(h) $y = \log_4 x$ and $y = 3\log_4(x + 1)$

NR3 I can determine a composite function.

1. Functions f and g , determined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.
Find:

(a) $g(f(x))$ (b) $f(g(x))$ (c) $g(g(x))$

2. Two functions f and g , are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

3. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$, are defined on a set of real numbers..

(a) Find $h(x)$ where $h(x) = g(f(x))$.

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$

(ii) Hence state the range of the function h .

4. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.

(b) Write down any restriction on the domain of h .

5. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$

(a) Find $p(x)$ where $p(x) = f(g(x))$.

(b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.

6. Functions f and g , determined on suitable domains, are given by $f(x) = x^2 - 1$ and $g(x) = 2 - 3x$.
- Find:
- (a) $g(f(x))$ (b) $f(g(x))$ (c) $g(g(x))$
7. Two functions f and g , are defined by $f(x) = 2x + 1$ and $g(x) = 2x - 1$, where x is a real number.
- (a) Find expressions for $f(g(x))$ and $g(f(x))$.
- (b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.
8. Functions $f(x) = x - 3$ and $g(x) = x^2 + 2$, are defined on a set of real numbers.
- (a) Find $h(x)$ where $h(x) = g(f(x))$.
- (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$
- (ii) Hence state the range of the function h .
9. Functions $f(x) = \frac{1}{x+2}$ and $g(x) = 3x - 1$ are defined on suitable domains.
- (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.
- (b) Write down any restriction on the domain of h .
10. $f(x) = 4 - x$ and $g(x) = \frac{4}{x}, x \neq 0$
- (a) Find $p(x)$ where $p(x) = f(g(x))$.
- (b) If $q(x) = \frac{4}{4-x}, x \neq 4$, find $p(q(x))$ in it's simplest form.

NR4 I can determine the inverse of a linear function.

1. Given $f(x) = 3x - 4$, find an expression for $f^{-1}(x)$.
2. Given $g(x) = 5x + 2$, find an expression for $g^{-1}(x)$.
3. Given $h(x) = 2x - 6$, find an expression for $h^{-1}(x)$.
4. Given $f(x) = \frac{1}{2}x + 5$, find an expression for $f^{-1}(x)$.
5. Given $g(x) = \frac{1}{4}x - 3$, find an expression for $g^{-1}(x)$.
6. Given $h(x) = 7 - 3x$, find an expression for $h^{-1}(x)$.
7. Given $f(x) = 2 - 4x$, find an expression for $f^{-1}(x)$.
8. Given $g(x) = \frac{2x-4}{5}$, find an expression $g^{-1}(x)$.
9. Given $h(x) = \frac{3x+2}{4}$, find an expression for $h^{-1}(x)$.
10. Given $f(x) = \frac{6-3x}{2}$, find an expression for $f^{-1}(x)$.

1. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$

(a) Find $g(f(x))$

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$

(c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.

(d) Hence solve $g(f(x)) + xh(x) = 0$

2. Functions f and g are defined on a suitable domain by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$

(a) Find expressions for

(i) $f(g(x))$

(ii) $g(f(x))$

(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$

3. Functions f, g and h are defined on suitable domains by

$$f(x) = x^2 - x + 10 \quad g(x) = 5 - x \quad \text{and} \quad h(x) = \log_2 x$$

(a) Find expressions for $h(f(x))$ and $h(g(x))$.

(b) Hence solve $h(f(x)) - h(g(x)) = 3$.

4. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

(i) $f(h(x))$

(ii) $g(h(x))$

(b) (i) $h(f(x))$

(ii) $h(g(x))$

5. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$

(a) (i) Find $p(x)$ where $p(x) = f(g(x))$

(ii) Find $q(x)$ where $q(x) = g(f(x))$

(b) Solve $p'(x) = q'(x)$

6. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$

(a) Find expressions for

(i) $f(g(x))$

(ii) $g(f(x))$

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.

7. Functions $a(x) = \sin x$, $b(x) = \cos x$ and $c(x) = x - \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for;

(i) $a(c(x))$;

(ii) $b(c(x))$.

(b) (i) Show that $a(c(x)) = \frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x$.

(ii) Find a similar expression for $b(c(x))$ and hence solve the equation $a(c(x)) + b(c(x)) = 1$ for $0 \leq x \leq 2\pi$.

8. Functions f and g are defined on suitable domains by $f(x) = \sin x^\circ$ and $g(x) = 2x$.

(a) Find expressions for;

(i) $f(g(x))$;

(ii) $g(f(x))$.

(b) Solve $3f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$.