Trigonometry

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can convert radians to degrees and vice versa.

1. Convert the following angles from degrees to radians, giving your answer as an exact value.

(a)	$30°$	**(b)**	$45°$	**(c)**	$60°$
(d)	$90°$	**(e)**	$180°$	**(f)**	$360°$
(g)	$120°$	**(h)**	$135°$	**(i)**	$150°$
(j)	$210°$	**(k)**	$225°$	**(l)**	$240°$
(m)	$270°$	**(n)**	$300°$	**(o)**	$315°$
(n)	$330°$	**(o)**	$540°$	**(p)**	$720°$

2. Convert the following angles from degrees to radians, giving your answer to 3 significant figures.

(a)	$37°$	**(b)**	$142°$	**(c)**	$226°$
(d)	$281°$	**(e)**	$307°$	**(f)**	$453°$

3. Convert the following angles from radians to degrees.

(a) π radians **(b)** 2π radians **(c)** 3π radians

(d) $\frac{\pi}{2}$ radians **(e)** $\frac{3\pi}{2}$ radians **(f)** $\frac{5\pi}{2}$ radians

(g) $\frac{\pi}{3}$ radians **(h)** $\frac{2\pi}{3}$ radians **(i)** $\frac{4\pi}{3}$ radians

(j) $\frac{5\pi}{3}$ radians **(k)** $\frac{7\pi}{3}$ radians **(l)** $\frac{\pi}{4}$ radians

(m) $\frac{3\pi}{4}$ radians **(n)** $\frac{5\pi}{4}$ radians **(o)** $\frac{7\pi}{4}$ radians

(p) $\frac{\pi}{6}$ radians **(q)** $\frac{5\pi}{6}$ radians **(r)** $\frac{7\pi}{6}$ radians

(s) $\frac{11\pi}{6}$ radians **(t)** $\frac{\pi}{12}$ radians **(u)** $\frac{5\pi}{12}$ radians

4. Convert the following angles from radians to degrees, giving your answer to 3 significant figures.

(a) 1 radian **(b)** 2 radians **(c)** 3 radians

(d) 4 radians **(e)** $1 \cdot 4$ radians **(f)** $2 \cdot 7$ radians

R2 I can use and apply exact values.

1. Write down the exact value of

(a) $\sin 30°$ **(b)** $\sin 60°$ **(c)** $\sin 45°$

(d) $\sin 120°$ **(e)** $\sin 150°$ **(f)** $\sin 135°$

(g) $\sin 90°$ **(h)** $\sin 180°$ **(i)** $\sin 270°$

(j) $\sin 210°$ **(k)** $\sin 225°$ **(l)** $\sin 240°$

(m) $\sin 300°$ **(n)** $\sin 330°$ **(o)** $\sin 315°$

2. Write down the exact value of

(a)	$\cos 30°$	**(b)**	$\cos 60°$	**(c)**	$\cos 45°$
(d)	$\cos 120°$	**(e)**	$\cos 150°$	**(f)**	$\cos 135°$
(g)	$\cos 90°$	**(h)**	$\cos 180°$	**(i)**	$\cos 270°$
(j)	$\cos 210°$	**(k)**	$\cos 225°$	**(l)**	$\cos 240°$
(m)	$\cos 300°$	**(n)**	$\cos 330°$	**(o)**	$\cos 315°$

3. Write down the exact value of

(a)	$\tan 30°$	**(b)**	$\tan 60°$	**(c)**	$\tan 45°$
(d)	$\tan 120°$	**(e)**	$\tan 150°$	**(f)**	$\tan 135°$
(g)	$\tan 90°$	**(h)**	$\tan 180°$	**(i)**	$\tan 270°$
(j)	$\tan 210°$	**(k)**	$\tan 225°$	**(l)**	$\tan 240°$
(m)	$\tan 300°$	**(n)**	$\tan 330°$	**(o)**	$\tan 315°$

4. Write down the exact value of

(a)	$\sin\frac{\pi}{6}$	**(b)**	$\sin\frac{\pi}{4}$	**(c)**	$\sin\frac{\pi}{3}$
(d)	$\sin\pi$	**(e)**	$\sin 2\pi$	**(f)**	$\sin\frac{3\pi}{2}$
(g)	$\sin\frac{5\pi}{6}$	**(h)**	$\sin\frac{3\pi}{4}$	**(i)**	$\sin\frac{2\pi}{3}$
(j)	$\sin\frac{7\pi}{6}$	**(k)**	$\sin\frac{5\pi}{4}$	**(l)**	$\sin\frac{4\pi}{3}$
(m)	$\sin\frac{11\pi}{6}$	**(n)**	$\sin\frac{7\pi}{4}$	**(o)**	$\sin\frac{5\pi}{3}$

5. Write down the exact value of

(a)	$\cos\frac{\pi}{6}$	**(b)**	$\cos\frac{\pi}{4}$	**(c)**	$\cos\frac{\pi}{3}$
(d)	$\cos\pi$	**(e)**	$\cos 2\pi$	**(f)**	$\cos\frac{3\pi}{2}$
(g)	$\cos\frac{5\pi}{6}$	**(h)**	$\cos\frac{3\pi}{4}$	**(i)**	$\cos\frac{2\pi}{3}$

(j) $\cos\frac{7\pi}{6}$ **(k)** $\cos\frac{5\pi}{4}$ **(l)** $\cos\frac{4\pi}{3}$

(m) $\cos\frac{11\pi}{6}$ **(n)** $\cos\frac{7\pi}{4}$ **(o)** $\cos\frac{5\pi}{3}$

6. Write down the exact value of

(a) $\tan\frac{\pi}{6}$ (b) $\tan\frac{\pi}{4}$ (c) $\tan\frac{\pi}{3}$

(d) $\tan\pi$ **(e)** $\tan 2\pi$ **(f)** $\tan\frac{3\pi}{2}$

(g) $\tan\frac{5\pi}{6}$ **(h)** $\tan\frac{3\pi}{4}$ **(i)** $\tan\frac{2\pi}{3}$

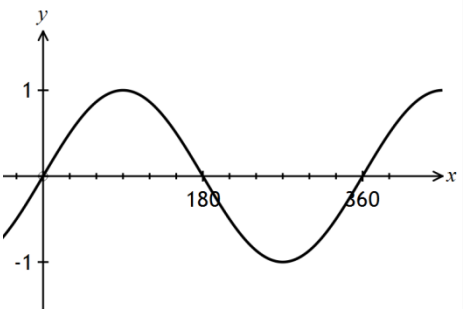
(j) $\tan\frac{7\pi}{6}$ **(k)** $\tan\frac{5\pi}{4}$ **(l)** $\tan\frac{4\pi}{3}$

(m) $\tan\frac{11\pi}{6}$ **(n)** $\tan\frac{7\pi}{4}$ **(o)** $\tan\frac{5\pi}{3}$

R3 I can sketch or identify a basic trig graph under a single transformation.

1. Write down the equation of each of the graphs

(a)



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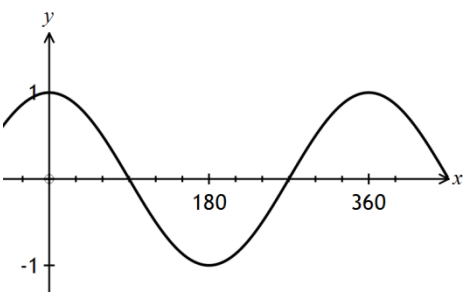

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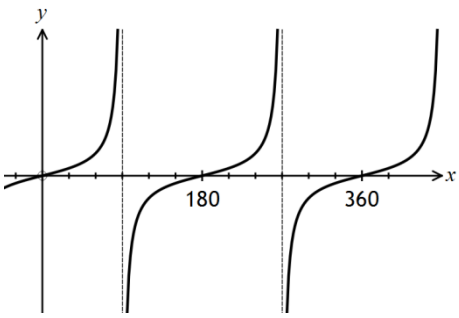
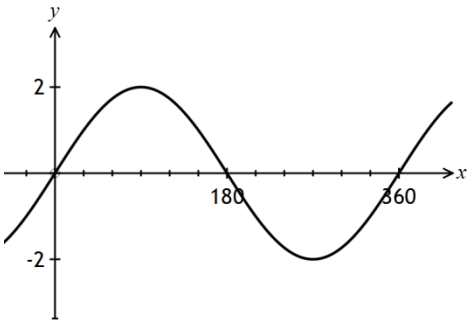
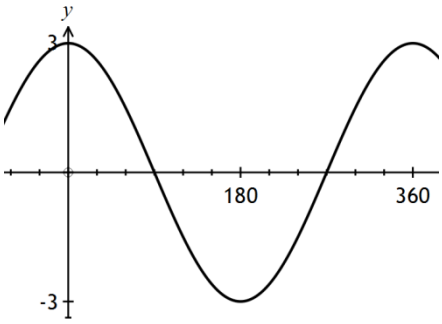


(d)

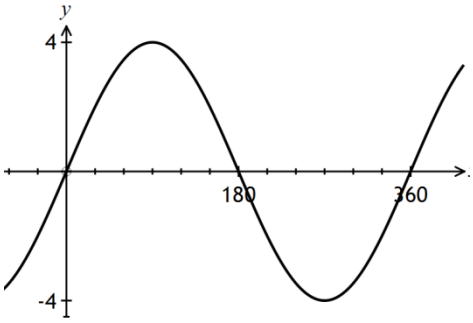


2. Write down the equation of each of the graphs

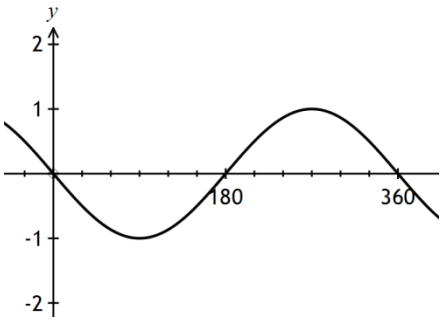
(a)



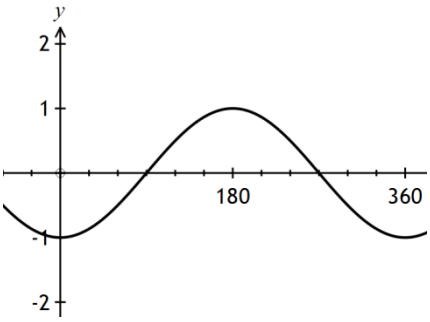
(b)



(c)

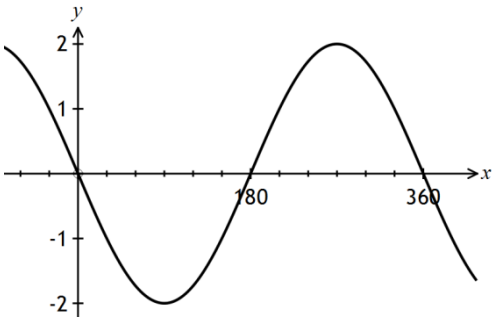


(d)

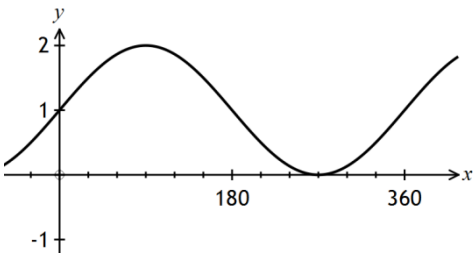


3. Write down the equation of each of the graphs

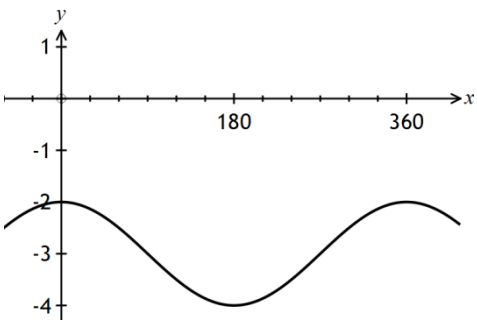
(a)



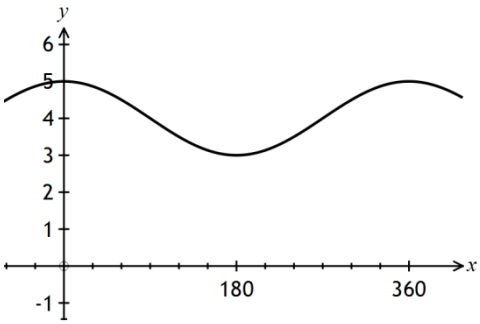
(b)



(c)

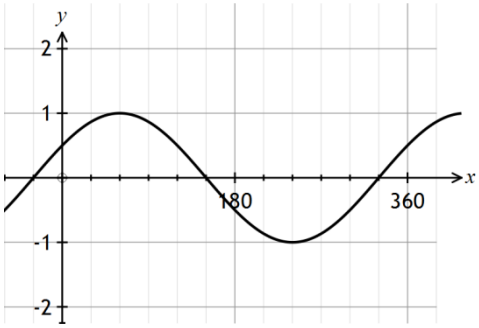


(d)

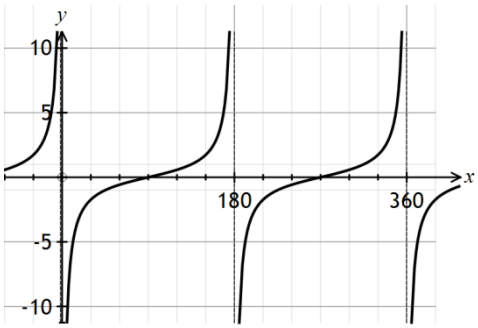


4. Write down the equation of each of the graphs

(a)



(b)



5. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

(a) $y = 3\cos x°$ for $0 \leq x \leq 360$

(b) $y = \sin x° + 1$ for $0 \leq x \leq 360$

(c) $y = \cos x - 1$ for $0 \leq x \leq 2\pi$

(d) $y = 2\sin x$ for $0 \leq x \leq 2\pi$

(e) $y = -2\cos x°$ for $0 \leq x \leq 360$

(f) $y = \tan(x - 45)°$ for $0 \leq x \leq 360$

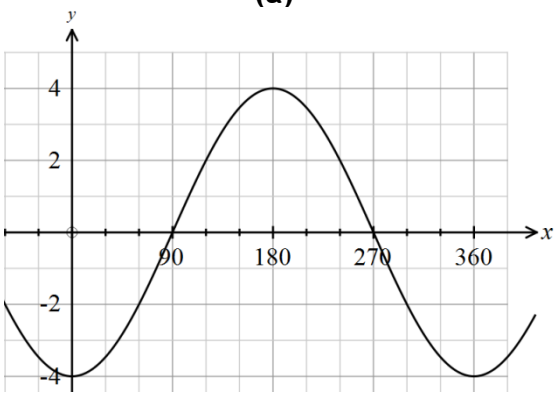
(g) $y = \cos\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$

(h) $y = -3\sin x$ for $0 \leq x \leq 2\pi$

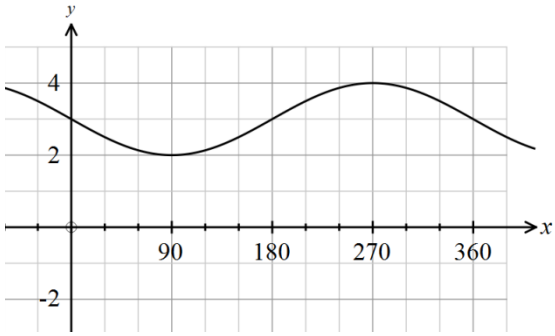
R4 **I can sketch or identify a basic trig graph under combined transformations.**

1. Write down the equation of each of the graphs

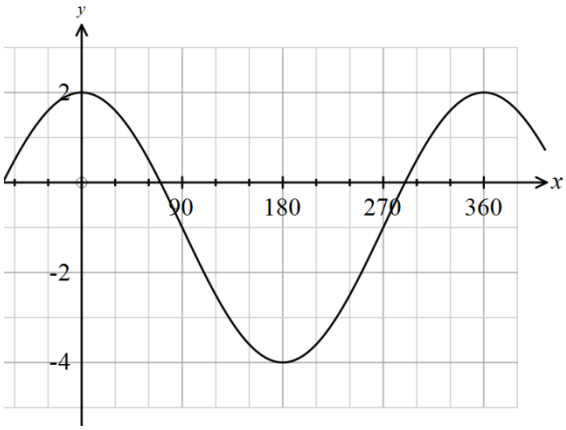
(a)



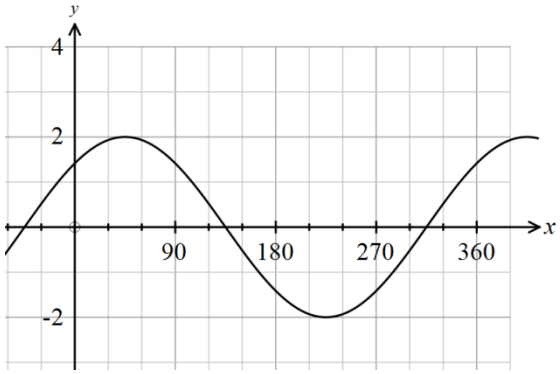
(b)



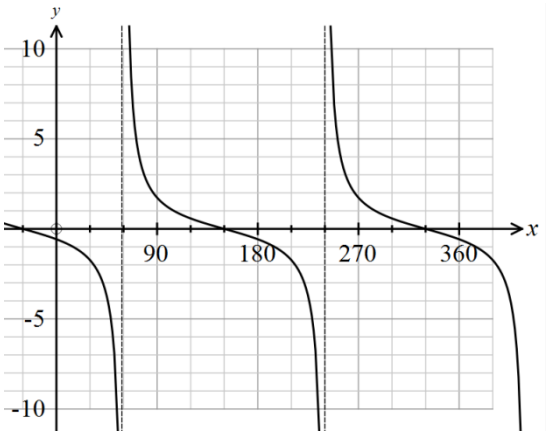
(c)


y
2
-2
-4
90
180
270
360
x

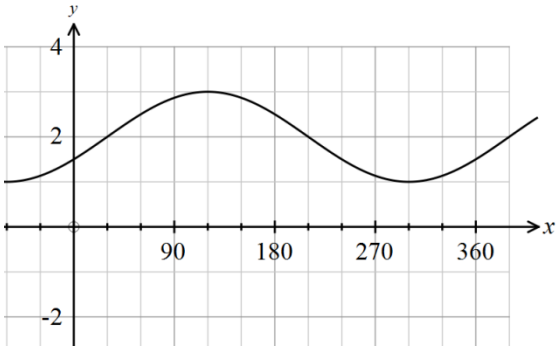
(d)


y
4
2
-2
90
180
270
360
x

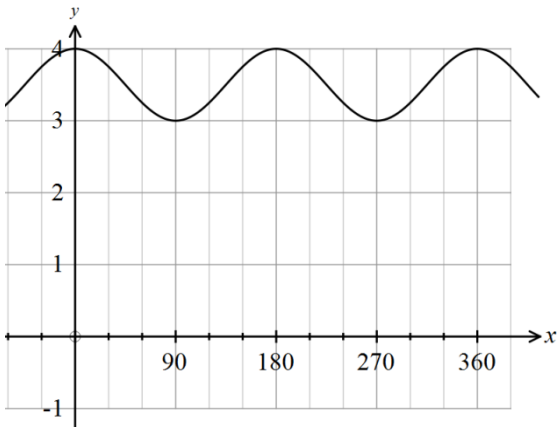
(e)


y
10
5
-5
-10
90
180
270
360
x

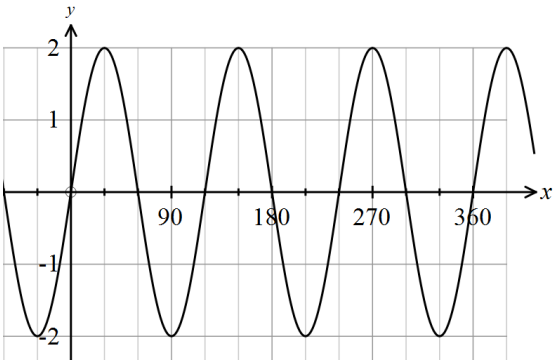
(f)


y
4
2
-2
90
180
270
360
x

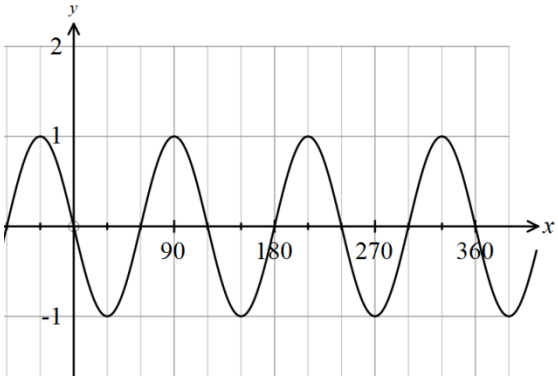
(g)


y
4
3
2
1
-1
90
180
270
360
x

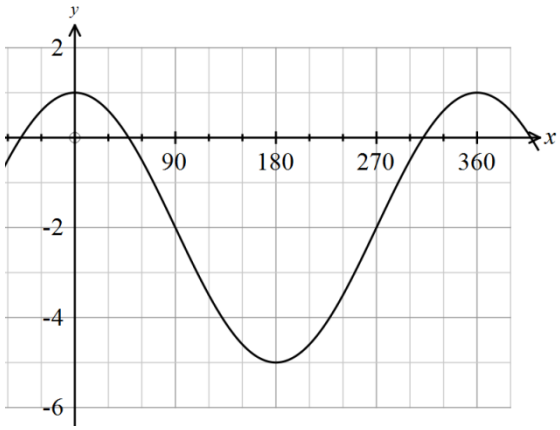
(h)


y
2
1
-1
-2
90
180
270
360
x

(i)



(j)



2. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

(a) $y = 2\cos x° - 3$ for $0 \leq x \leq 360$ **(b)** $y = -\cos 2x°$ for $0 \leq x \leq 360$

(c) $y = 2\cos\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$

(d) $y = \sin\left(x - \frac{\pi}{6}\right) + 2$ for $0 \leq x \leq 2\pi$

(e) $y = 4\cos 2x°$ for $0 \leq x \leq 360$

(f) $y = -\sin(x - 30)°$ for $0 \leq x \leq 360$

(g) $y = \cos 2x - 1$ for $0 \leq x \leq 2\pi$ **(h)** $y = 2 - \sin x$ for $0 \leq x \leq 2\pi$

R5 I can use the addition and double angle formulae.

1. Expand and use exact values to simplify

(a) $\sin\left(x + \frac{\pi}{6}\right)$ **(b)** $\sin(x - 60)°$ **(c)** $\cos\left(x - \frac{\pi}{4}\right)$

(d) $\cos(x + 45)°$ **(e)** $\cos\left(x + \frac{\pi}{3}\right)$ **(f)** $\sin(x + 60)°$

(g) $\sin(x - 90)°$ **(h)** $\sin(x + \pi)$ **(i)** $\cos(x + 180)°$

2. Use an appropriate substitution (such as 45 – 30 = 15) then expand to find the exact values of

(a) $\sin 15°$ **(b)** $\sin 75°$ **(c)** $\cos 105°$

3. Given that $\sin x° = \frac{3}{5}$ and $\cos x° = \frac{4}{5}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\sin 3x°$ (Hint $3x = 2x + x$)

4. Given that $\sin x° = \frac{5}{13}$ and $\cos x° = \frac{12}{13}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\sin 4x°$ (Hint $4x = 2(2x)$)

5. Given that $\sin x° = \frac{1}{\sqrt{5}}$ and $\cos x° = \frac{2}{\sqrt{5}}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\cos 3x°$

6. Given that $\sin x° = \frac{2}{\sqrt{13}}$ and $\cos x° = \frac{3}{\sqrt{13}}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\cos 4x°$

R6	**I can convert $\boldsymbol{acosx + bsinx}$ to $\boldsymbol{kcos(x \pm \alpha)}$ or $\boldsymbol{ksin(x \pm \alpha)}$, where α is in any quadrant $\boldsymbol{k > 0}$.**

1. A function f is defined as $f(x) = 5\cos x° - 2\sin x°$.

Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

2. Express $\sin x - \cos x$ in the form $k\sin(x - \alpha)$ where $k > 0$ and $0 \leq \alpha < 2\pi$.

3. A function g is defined as $g(x) = 3\cos x° + \sin x°$.

Express $g(x)$ in the form $k\sin(x + \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$.

4. Express $\sin x + 2\cos x$ in the form $r\cos(x - a)$ where $r > 0$ and $0 \leq a < 2\pi$.

5. A function Q is defined as $Q(x) = 2\cos x° - 3\sin x°$.

Express $Q(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

6. Express $3\sin x - 4\cos x$ in the form $a\sin(x - b)$ where $a > 0$ and $0 \leq b < 2\pi$.

7. A function f is defined as $f(x) = 2\cos x° - \sin x°$.

Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

8. Express $\sin x - \sqrt{3}\cos x$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

9. A function f is defined as $f(x) = \sqrt{5}\cos x° + 3\sin x°$.

Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

R7 **I have revised solving basic trigonometric equations in degrees and radians.**

1. Solve the equations:

(a) $5\tan x° - 6 = 2, \quad 0 \leq x \leq 360.$

(b) $7\sin x° + 1 = -5, \quad 0 \leq x \leq 360.$

(c) $4\cos x° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $3\tan x + 3 = 7, \quad 0 \leq x \leq 2\pi.$

(e) $4\sin x - 2 = -3, \quad 0 \leq x \leq 2\pi.$

(f) $9\cos x - 5 = 0, \quad 0 \leq x \leq 2\pi.$

2. Solve the equations:

(a) $9\tan 2x° - 5 = 3, \quad 0 \leq x \leq 180.$

(b) $4\sin 3x° + 1 = -2, \quad 0 \leq x \leq 360.$

(c) $3\cos 2x° + 2 = 0, \quad 0 \leq x \leq 360.$

3. Solve the equations:

(a) $\tan(x + 30)° = 3, \quad 0 \leq x \leq 360.$

(b) $5\sin(x + 10)° + 3 = -1, \quad 0 \leq x \leq 360.$

(c) $4\cos(x + 26)° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $\sqrt{3}\tan\left(x + \frac{\pi}{5}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

(e) $6\sin(x + 2) - 2 = 1, \quad 0 \leq x \leq 2\pi.$

(f) $\sqrt{2}\cos\left(x + \frac{\pi}{6}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

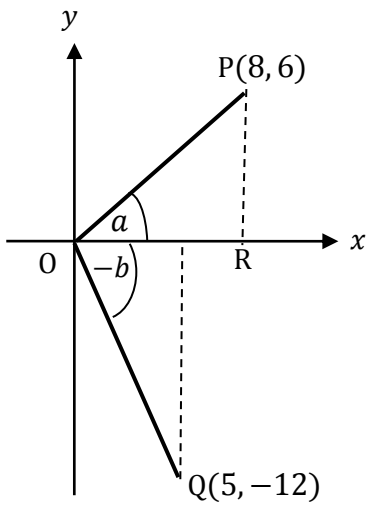
Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can apply Trig Formulae to Mathematical Problems (excluding where trig equations have to be solved but including exact values).

1. On the coordinate diagram shown, P is the point $(8, 6)$ and Q is the point $(5, -12)$.

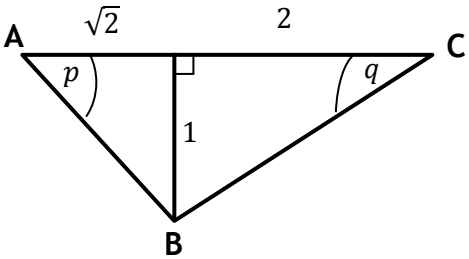
Angle POR = a and angle ROQ = $-b$.



(a) Find the exact value of $\sin(a - b)$.

(b) Find the exact value of $\cos 2a$.

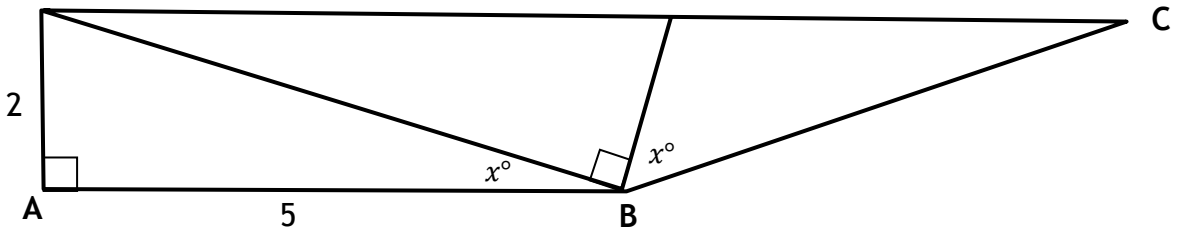
2. In triangle ABC, show that:



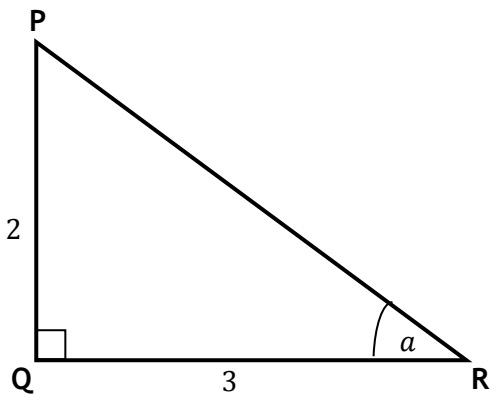
(a) The exact value of $\sin 2p = \frac{2\sqrt{2}}{3}$

(b) The exact value of $\cos(p + q) = \frac{2\sqrt{2}-1}{\sqrt{15}}$

3. For the shape shown, find the exact value of $\cos(A\hat{B}C)°$



4. The diagram shows the right angled triangle PQR, with dimensions given.



(a) Find the exact value of $\sin 2a°$.

(b) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$.

5. If $\cos 2x = -\frac{31}{49}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.

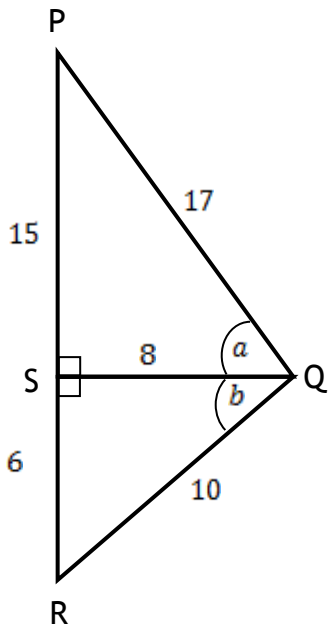
6. Using the fact that $\frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$, find the exact value of $\cos\left(\frac{5\pi}{12}\right)$.

7. It is given that $\cos a = \frac{3}{5}$ and $\sin b = \frac{2}{3}$.

(a) Find the exact value of $\sin(a + b)$ and $\cos(a + b)$.

(b) Hence find the exact value of $\tan(a + b)$.

8. Triangles PSQ and RSQ are right angled with dimensions as shown in the diagram.



(a) Show that $\cos(a + b)$ is $-\frac{13}{85}$.

(b) Calculate the value of $\sin(a + b)$.

(c) Hence calculate the value of $\tan(a + b)$.

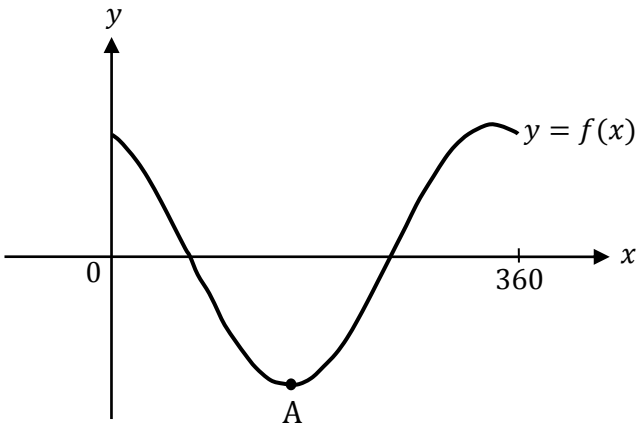
NR2 I have experience of using wave functions to find the maximum and minimum values.

1. A function f is defined as $f(x) = 5\cos x° - 2\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Part of the graph of $y = f(x)$ is shown in the diagram.

Find the coordinates of the minimum turning point A.



2. **(a)** Express $\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a < 2\pi$.

(b) Hence state the maximum and minimum values of $\sin x - \cos x$ and determine the values of x, in the interval $0 \leq x < 2\pi$, at which these maximum and minimum values occur.

3. **(a)** Express $12\sin x + 5\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

(b) Hence state the maximum value of $4 + 12\sin x + 5\cos x$ and determine the value of x, in the interval $0 \leq x < 2\pi$, at which the maximum occurs.

4. A function f is defined as $f(x) = 4\cos x° + 3\sin x°$.

Find the maximum and minimum values of $f(x)$ and the values of, in the range $0 \leq x < 360$, at which the maximum and minimum values occur.

5. A function g is defined as $g(x) = 3\cos x - 2\sin x$.

Find the maximum and minimum values of $g(x)$ and the values of, in the range $0 \leq x < 2\pi$, at which the maximum and minimum values occur.

Trigonometry, functions and graphs

1. A function f is defined as $f(x) = \sqrt{3}\cos x° + \sin x°$.

(a) Express $f(x)$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = f(x)$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points.

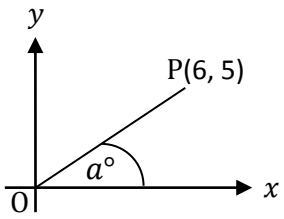
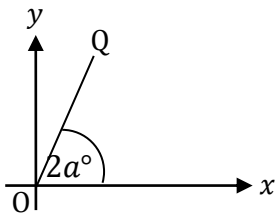
2. **(a)** Express $3\sin x° + 4\cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = 3\sin x° + 4\cos x° + 1$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points and where the curve cuts the axes.

Trigonometry and straight line

1. P is the point $(6, 5)$. The line OP is inclined at an angle of $a°$ to the x-axis.

(a) Find the exact values of $\sin 2a°$ and $\cos 2a°$.



(b) The line OQ is inclined at an angle of $2a°$ to the x-axis. Write down the exact value of the gradient of OQ.