Higher : Vectors

2018 P1 Q5

$A(-3, 4, -7)$, $B(5, t, 5)$ and $C(7, 9, 8)$ are collinear.

(a) State the ratio in which B divides AC. 1

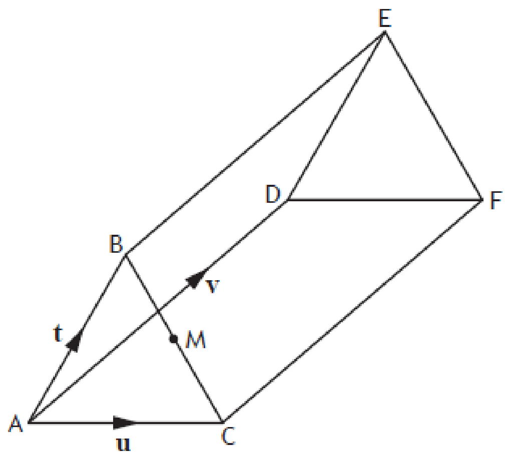
(b) State the value of t. 1

Ans (a) 4:1 (b) $t = 8$

2018 P1 Q9

The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. 1

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. 2

Ans (a) $-\mathbf{t} + \mathbf{u}$ (b) $-\frac{1}{2}\mathbf{t} - \frac{1}{2}\mathbf{u} + \mathbf{v}$

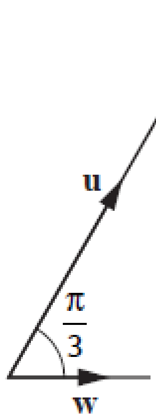
2018 P1 Q12

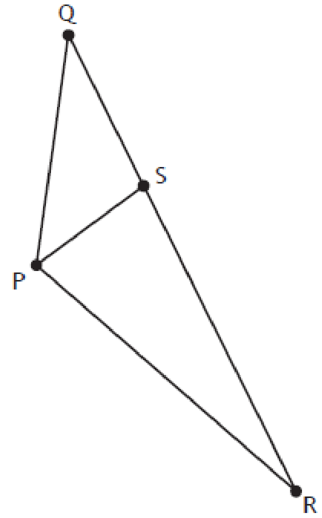
Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. 1

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. 3

Ans (a) $\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$ (b) $p = -2$, $p = -6$

2018 P2 Q2	Vectors **u** and **v** are defined by $\mathbf{u}=\begin{pmatrix}-1\\4\\-3\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}-7\\8\\5\end{pmatrix}$. (a) Find **u.v**. (b) Calculate the acute angle between **u** and **v**.	1 4
Ans	(a) 24 (b) 66·4° or 1·16 radians	
2017 P1 Q5	Vectors **u** and **v** are $\begin{pmatrix}5\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\-8\\6\end{pmatrix}$ respectively. (a) Evaluate **u.v**. (b)  Vector **w** makes an angle of $\frac{\pi}{3}$ with **u** and $\lvert\mathbf{w}\rvert=\sqrt{3}$. Calculate **u.w**.	1 3
Ans	*(a) 1 (b) 4·5*	

2017 P2 Q5	In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.  (a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. The point S divides QR in the ratio 1:2. (b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. (c) Hence, find the size of angle QPS.	2 2 5
Ans	*(a)* $-3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ *(b)* $\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ *(c)* $45{\cdot}6°$	
2016 P1 Q7	Three vectors can be expressed as follows: $\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$ $\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (a) Find $\overrightarrow{FH}$. (b) Hence, or otherwise, find $\overrightarrow{FE}$.	2 2
Ans	*(a)* $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ *(b)* $-\mathbf{i} - 5\mathbf{k}$	

2016 P1 Q11

(a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B. 2

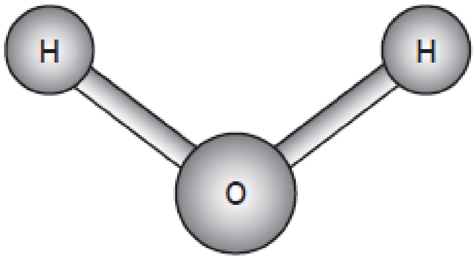
(b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k. 3

Ans (a) B(2, 1, 0) (b) $k = \frac{1}{9}$

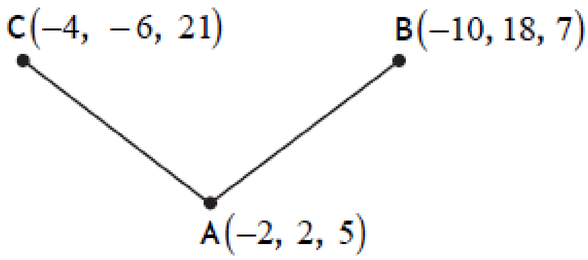
2016 P2 Q5

The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point A$(-2, 2, 5)$.

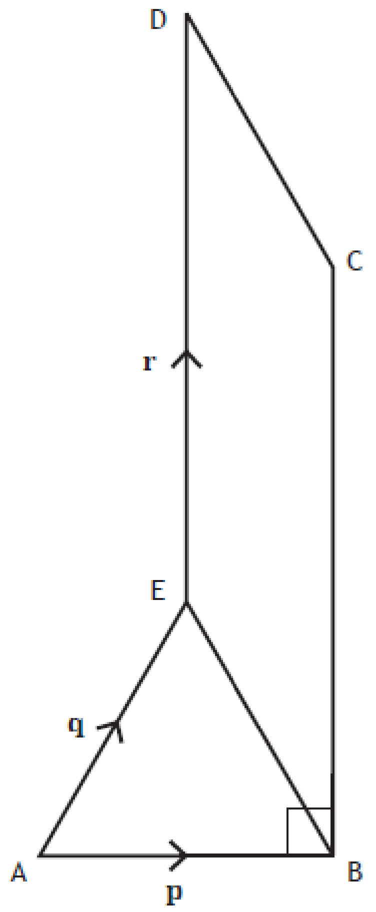
The two hydrogen atoms are positioned at points B$(-10, 18, 7)$ and C$(-4, -6, 21)$ as shown in the diagram below.



(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. 2

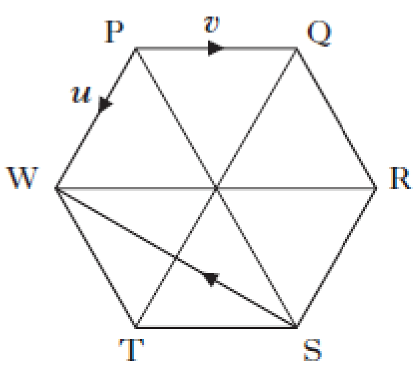
(b) Hence, or otherwise, find the size of angle BAC. 4

Ans (a) $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$ (b) $104{\cdot}3°$

2015 NH P1 Q1	Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular. Determine the value of t.	2
Ans	$t = 9$	
2015 NH P2 Q6	Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown. • BCDE is a parallelogram • ABE is an equilateral triangle • $\lvert\mathbf{p}\rvert = 3$ • Angle ABC = 90°  (a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$. (b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. (c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $\lvert\mathbf{r}\rvert$.	3 1 3
Ans	*(a)* $4\frac{1}{2}$ *(b)* $-q + p + r$ *(c)* $\lvert r\rvert = \frac{3\sqrt{3}}{\cos 30°}$	

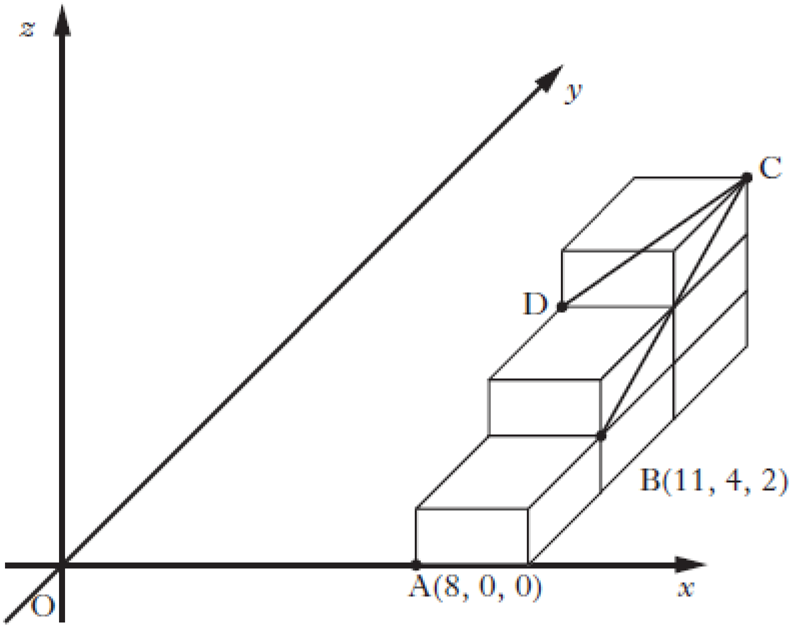
2015 P1 Q6	The points P, Q and R are collinear. P is the point (–1, 6, 4), Q is the point (2, 0, 13) and $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$. Calculate the ratio in which Q divides PR. A 2 : 3 B 3 : 2 C 3 : 5 D 5 : 2	2
Ans	*B*	
2015 P1 Q12	Given that the point R is (3, –1, 2), $\overrightarrow{RS} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\overrightarrow{RT} = 3\overrightarrow{RS}$, find the coordinates of T. A (3, 2, –11) B (3, 4, –11) C (9, 2, –7) D (9, 4, –7)	2
Ans	*C*	
2015 P1 Q17	Vectors **u** and **v** have components $\begin{pmatrix} \frac{3}{5} \\ 0 \\ t \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 0 \\ -10 \end{pmatrix}$ respectively. Here are two statements about **u** and **v**: (1) when $t = \frac{4}{5}$, **u** is a unit vector (2) when $t = 1$, **u** and **v** are parallel Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	2
Ans	*D*	

2014 P1 Q6	Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form. A $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$ B $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$ C $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$ D $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$	2
Ans	*A*	
2014 P1 Q14	The vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular. What is the value of k? A $\frac{-6}{7}$ B -1 C 1 D $\frac{6}{7}$	2
Ans	*D*	

2014 P1 Q16	The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$. A $\frac{2}{3}$ B $\frac{4}{3}$ C $\frac{7}{3}$ D 3	2
Ans	*C*	
2014 P1 Q19	The diagram shows a regular hexagon PQRSTW. $\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.  What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$? A $-\boldsymbol{u} - 2\boldsymbol{v}$ B $-\boldsymbol{u} - \boldsymbol{v}$ C $\boldsymbol{u} - \boldsymbol{v}$ D $\boldsymbol{u} + 2\boldsymbol{v}$	2
Ans	*A*	

2014 P2 Q4

Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. 2

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. 2

(*c*) Find the size of the angle BCD. 5

Ans *(a)* *C(11, 12, 16)* , *D(8, 8, 4)* *(b)* $\overrightarrow{CB} = \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$ *(c)* *33·9°*

2013 P1 Q12

If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$. 2

A $\sqrt{14}$ units

B $\sqrt{42}$ units

C $\sqrt{66}$ units

D $\sqrt{70}$ units

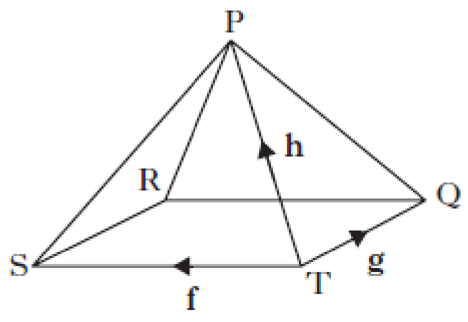
Ans *C*

2013 P1 Q14	Given that $\lvert\mathbf{a}\rvert = 3$, $\lvert\mathbf{b}\rvert = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$? A 11 B 14 C 15 D 21	2
Ans	*B*	
2013 P1 Q24	(*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear. (ii) Find the ratio in which T divides AB. (*b*) The point C lies on the x-axis. If TB and TC are perpendicular, find the coordinates of C.	4 5
Ans	*(a) (i) Proof (ii) 2:3 (b) C(7,0,0)*	
2012 P1 Q7	If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t? A -3 B -2 C $\frac{2}{3}$ D 1	2
Ans	*A*	

<table>
<tr><td>2012 P1 Q10</td><td>

The diagram shows a square-based pyramid P,QRST.

$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.



Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.

A $-\mathbf{f}+\mathbf{g}-\mathbf{h}$

B $-\mathbf{f}-\mathbf{g}+\mathbf{h}$

C $\mathbf{f}-\mathbf{g}-\mathbf{h}$

D $\mathbf{f}+\mathbf{g}+\mathbf{h}$

</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">B</td></tr>
<tr><td>2012 P1 Q15</td><td>

If $\mathbf{u}=k\begin{pmatrix}3\\-1\\0\end{pmatrix}$, where $k>0$ and **u** is a unit vector, determine the value of k.

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{10}}$

</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">D</td></tr>
</table>

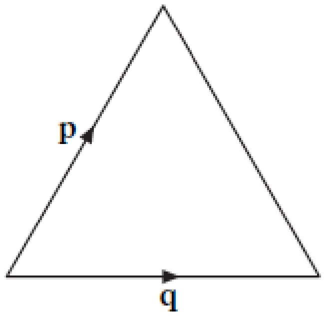
2012 P1 Q17	Given that $\mathbf{a}=\begin{pmatrix}3\\4\\0\end{pmatrix}$ and $\mathbf{a}.(\mathbf{a}+\mathbf{b})=7$, what is the value of $\mathbf{a}.\mathbf{b}$? A $\frac{7}{25}$ B $-\frac{18}{5}$ C -6 D -18	2
Ans	*D*	
2012 P2 Q5(a)	A is the point (3, −3, 0), B is (2, −3, 1) and C is (4, *k*, 0). (*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form. (ii) Show that $\cos A\hat{B}C = \frac{3}{\sqrt{2(k^2+6k+14)}}$.	7
Ans	*(a)(i)* æ ç ç ç è 1 0 - 1 ö ÷ ÷ ÷ ø *(ii) Proof*	
2011 P1 Q1	Given that $\mathbf{p}=\begin{pmatrix}2\\5\\-7\end{pmatrix}$, $\mathbf{q}=\begin{pmatrix}1\\0\\-1\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}-4\\2\\0\end{pmatrix}$, express $2\mathbf{p}-\mathbf{q}-\frac{1}{2}\mathbf{r}$ in component form. A $\begin{pmatrix}1\\9\\-15\end{pmatrix}$ B $\begin{pmatrix}1\\11\\-13\end{pmatrix}$ C $\begin{pmatrix}5\\9\\-13\end{pmatrix}$ D $\begin{pmatrix}5\\11\\-15\end{pmatrix}$	2
Ans	*C*	

2011 P1 Q14

An equilateral triangle of side 3 units is shown.

The vectors **p** and **q** are as represented in the diagram.

What is the value of $\mathbf{p}.\mathbf{q}$?



A $\quad 9$

B $\quad \frac{9}{2}$

C $\quad \frac{9}{\sqrt{2}}$

D $\quad 0$

2

Ans *B*

2011 P1 Q15

Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.

A $\quad 2 : 3$

B $\quad 3 : 2$

C $\quad 2 : 5$

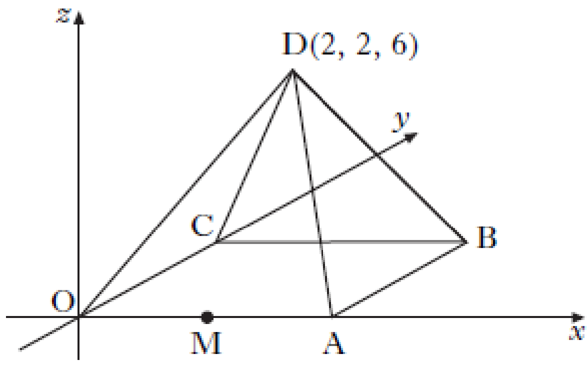
D $\quad 3 : 5$

2

Ans *B*

2011 P2 Q1

D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(*a*) State the coordinates of B. 1

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. 3

(*c*) Find the size of angle BDM. 5

Ans	*(a) B(4,4,0)* *(b)* $\overrightarrow{DB}=\begin{pmatrix}2\\2\\-6\end{pmatrix}$, $\overrightarrow{DM}=\begin{pmatrix}0\\-2\\-6\end{pmatrix}$ *(c) 40·3°*	
2010 P1 Q3	Given that $\mathbf{u}=\begin{pmatrix}2\\0\\1\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}-1\\2\\4\end{pmatrix}$, find $3\mathbf{u}-2\mathbf{v}$ in component form. A $\begin{pmatrix}4\\-1\\-5\end{pmatrix}$ B $\begin{pmatrix}4\\-4\\11\end{pmatrix}$ C $\begin{pmatrix}8\\-1\\5\end{pmatrix}$ D $\begin{pmatrix}8\\-4\\-5\end{pmatrix}$	2
Ans	*D*	
2010 P1 Q10	The vectors $x\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ and $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ are perpendicular. What is the value of x? A 0 B 1 C $\frac{4}{3}$ D $\frac{10}{3}$	2
Ans	*B*	

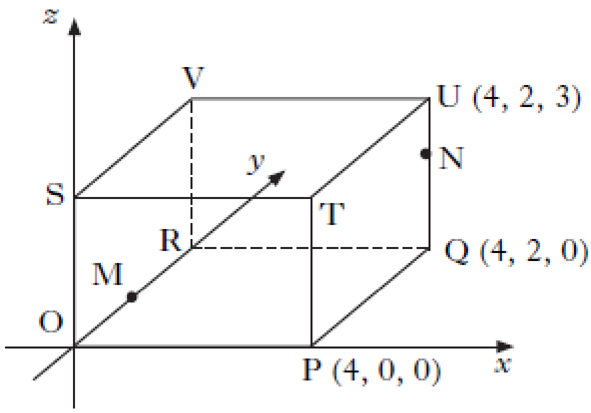
2010 P2 Q1

The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. 2

(*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. 2

(*c*) Calculate the size of angle MVN. 5

Ans *(a) M(0,1,0) N(4,2,2)* *(b)* $\begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ *(c) 76·7º*

2009 P1 Q17

The vector $\boldsymbol{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$.

Which of the following is a unit vector parallel to $\boldsymbol{u}$? 2

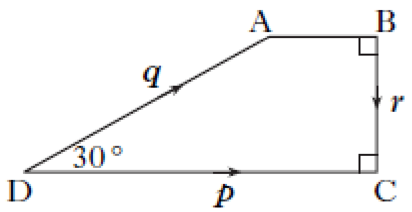
A $-\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{k}$

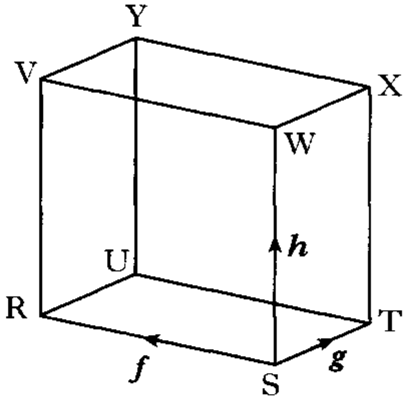
B $-3\boldsymbol{i} + 4\boldsymbol{k}$

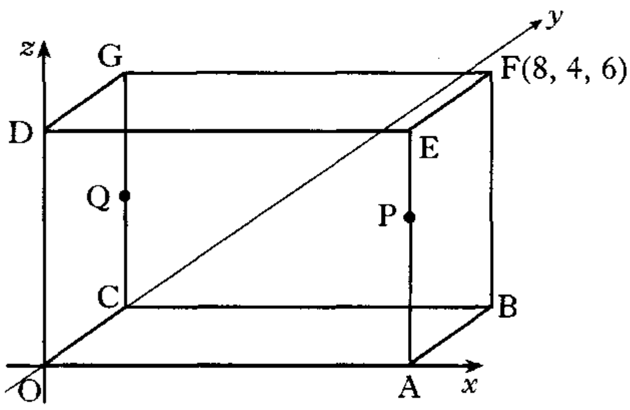
C $-\frac{3}{\sqrt{7}}\boldsymbol{i} + \frac{4}{\sqrt{7}}\boldsymbol{k}$

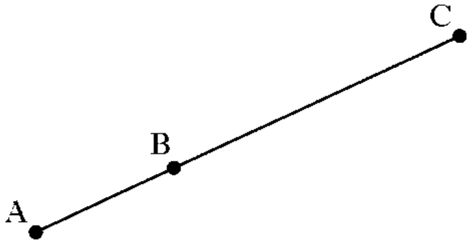
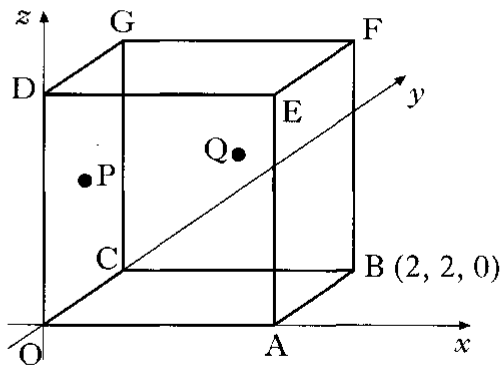
D $-\frac{1}{3}\boldsymbol{i} + \frac{1}{4}\boldsymbol{k}$

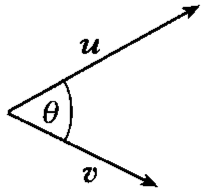
Ans *A*

2009 P1 Q22	D, E and F have coordinates (10, −8, −15), (1, −2, −3) and (−2, 0, 1) respectively. (*a*) (i) Show that D, E and F are collinear. (ii) Find the ratio in which E divides DF. (*b*) G has coordinates (k, 1, 0). Given that DE is perpendicular to GE, find the value of k.	4 4
Ans	*(a))i) Proof (ii) 3:1 (b) k = 7*	
2009 P2 Q7	Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°. It is also given that $\lvert \boldsymbol{p} \rvert = 4$ and $\lvert \boldsymbol{q} \rvert = 3$. (*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})$. (*b*) Find $\lvert \boldsymbol{q}+\boldsymbol{r} \rvert$ and $\lvert \boldsymbol{p}-\boldsymbol{q} \rvert$. 	6 4
Ans	*(a)* $6\sqrt{3}$ *and* $\frac{9}{4}$ *(b)* $\lvert q+r \rvert = \frac{3\sqrt{3}}{2}$, $\lvert p-q \rvert = 2\times05$	
2008 P1 Q3	The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular. What is the value of k? A 0 B 3 C 4 D 5	2
Ans	*C*	
2008 P1 Q11	E(−2, −1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points. P lies between E and F. What is the ratio in which P divides EF? A 1:1 B 1:2 C 1:4 D 1:6	2
Ans	*B*	

<table>
<tr><td>2008 P1 Q12</td><td>In the diagram RSTU, VWXY represents a cuboid.
$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.
Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.


A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$
B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$
C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$
D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">C</td></tr>
<tr><td>2008 P1 Q18</td><td>Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.
Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.
A 0
B 14
C 26
D 28</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">C</td></tr>
<tr><td>2008 P2 Q2</td><td>The diagram shows a cuboid OABC, DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2:1.
Q is the midpoint of CG.
(a) State the coordinates of P and Q.
(b) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$.
(c) Find the size of angle QPA.

</td><td>2
2
5</td></tr>
</table>

Ans	*(a) P(8,0,4), Q(0,4,3)* *(b)* $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ 4 \\ -1 \end{pmatrix}$ $\overrightarrow{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ *(c) 83·6°*	
2007 P1 Q2	Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively. A, B and C are collinear points and C is positioned such that BC = 2AB. Find the coordinates of C. 	4
Ans	(7,7,8)	
2007 P2 Q1	OABCDEFG is a cube with side 2 units, as shown in the diagram. B has coordinates (2, 2, 0). P is the centre of face OCGD and Q is the centre of face CBFG.  (*a*) Write down the coordinates of G. (*b*) Find ***p*** and ***q***, the position vectors of points P and Q. (*c*) Find the size of angle POQ.	1 2 5
Ans	(*a*) (0,2,2) (*b*) $P = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $Q = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ (*c*) 30°/$\frac{\pi}{6}$ radians	

2006 P1 Q9	$\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.  (*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. (*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. (*c*) Deduce the only possible value of k. (*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$.	2 5 1 3
Ans	(*a*) $k^3 + 3k^2 - k - 2 = 1$ *and complete* (*b*) $(k + 3)(k + 1)(k - 1)$ stated explicitly (*c*) $k = 1$ (*d*) $\cos\theta = \frac{1}{11}$	
2006 P2 Q6	P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$. (*a*) Write down $\overrightarrow{PQ}$ in component form. (*b*) Calculate the length of $\overrightarrow{PQ}$. (*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$.	1 1 1
Ans	(*a*) $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ (*b*) $\lvert\overrightarrow{PQ}\rvert = 5$ (*c*) $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ or $\begin{pmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$	

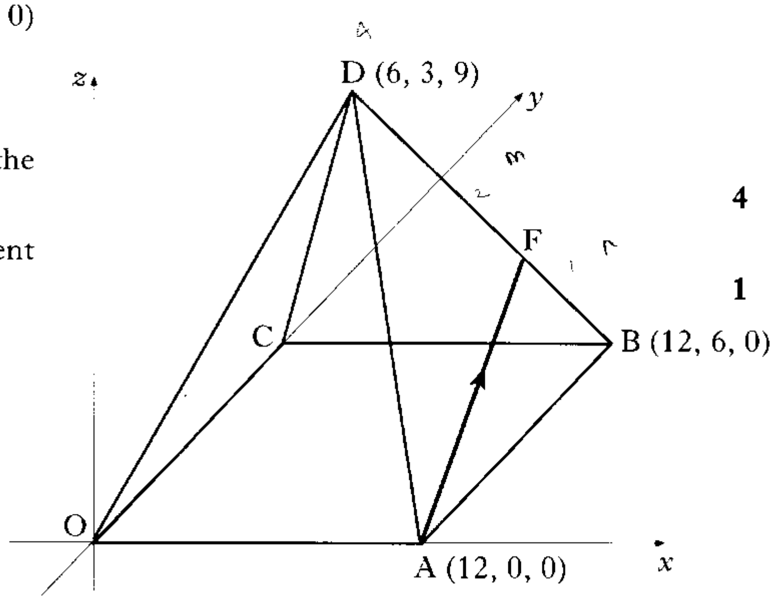
2005 P1 Q3

D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

(*a*) Find the coordinates of the point F. **4** 4

(*b*) Express $\overrightarrow{AF}$ in component form. **1** 1



Ans

(*a*) F = (10,5,3)

(*b*) $AF = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$

2005 P2 Q4

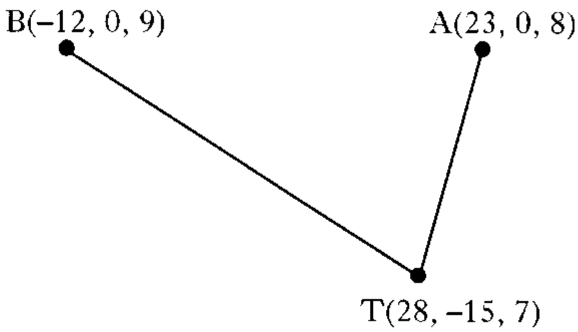
The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(−12, 0, 9) and T(28, −15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. 2

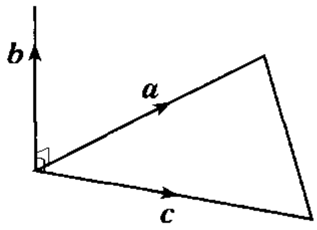
(*b*) Calculate the angle between these two beams. 5

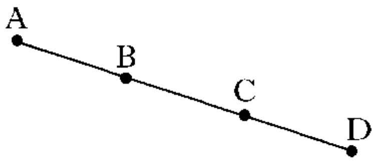
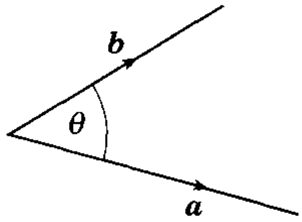
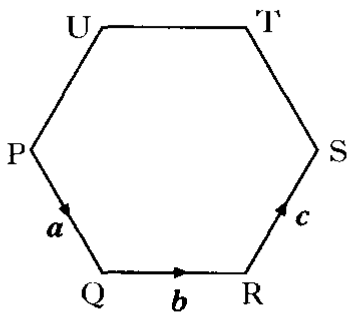


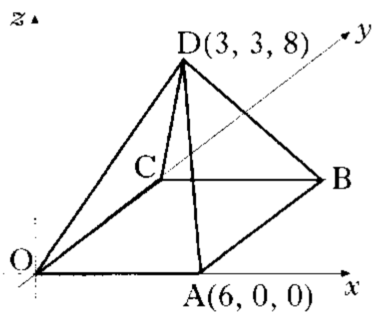
Ans

(*a*) $\overrightarrow{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$ $\overrightarrow{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$

(*b*) 50·9° **or** 0·889^c **or** 56·6 grads

2005 P2 Q10	Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$. Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.  	4
Ans	$13\frac{1}{2}$	
2004 P1 Q5	A, B and C have coordinates (−3, 4, 7), (−1, 8, 3) and (0, 10, 1) respectively. (*a*) Show that A, B and C are collinear. (*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$.	3 2
Ans	**5.** (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = \frac{3}{2} \times \overrightarrow{AB}$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have common direction and common point, hence A, B and C are collinear. (*b*) D = (5, 20, −9)	
2004 P2 Q2	P, Q and R have coordinates (1, 3, −1), (2, 0, 1) and (−3, 1, 2) respectively. (*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. (*b*) Hence or otherwise find the size of angle PQR.	2 5
Ans	(*a*) $\overrightarrow{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ $\overrightarrow{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ (*b*) $P\hat{Q}R = 72{\cdot}0°$	
2003 P1 Q3	Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$. Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other.	2
Ans	*Vectors are perpendicular.*	

2003 P1 Q6	A and B are the points (–1, –3, 2) and (2, –1, 1) respectively. B and C are the points of trisection of AD, that is AB = BC = CD. Find the coordinates of D.  	3
Ans	(8, 3, –1)	
2003 P2 Q9	The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. If $\lvert\boldsymbol{a}\rvert = 5$, $\lvert\boldsymbol{b}\rvert = 4$ and $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$.  	4
Ans	56·6°	
2002W P1 Q2	(*a*) If $\boldsymbol{u} = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$.	2
	(*b*) Hence, or otherwise, show that $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$ are perpendicular.	2
Ans	(*a*) • $u + 3v = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ • $u - 3v = \begin{pmatrix} -2 \\ 13 \\ -5 \end{pmatrix}$ (*b*) proof • $\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 13 \\ 5 \end{pmatrix}$ $= -8 + 13 - 5 = 0$ **Alternative** • $(u+3v).(u-3v)$ $= \lvert u\rvert^2 - 9\lvert v\rvert^2$ $= 54 - 9 \times 6 = 0$	
2002W P1 Q11	PQRSTU is a regular hexagon of side 2 units. $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively. Find the value of $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.  	3
Ans	0	

2002W P2 Q2	With reference to a suitable set of coordinate axes, A, B and C are the points (−8, 10, 20), (−2, 1, 8) and (0, −2, 4) respectively. Show that A, B and C are collinear and find the ratio AB : BC.	4
Ans	$\vec{AB} = \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix}$ **or** $\vec{BC} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$ **or** $\vec{AC} = \begin{pmatrix} 8 \\ -12 \\ -16 \end{pmatrix}$ e.g. $\vec{AB} = 3\vec{BC}$ $\left(\textbf{or} \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} \right)$ e.g. $\vec{AB}$, $3\vec{BC}$ have common direction, B common pt. so A, B, C collinear AB : BC = 3 : 1	
2002 P1 Q2	The point Q divides the line joining P(−1, −1, 0) to R(5, 2, −3) in the ratio 2 : 1. Find the coordinates of Q.	3
Ans	(3, 1, −2)	
2002 P2 Q2	The diagram shows a square-based pyramid of height 8 units. Square OABC has a side length of 6 units. The coordinates of A and D are (6, 0, 0) and (3, 3, 8). C lies on the y-axis.  (*a*) Write down the coordinates of B. (*b*) Determine the components of $\vec{DA}$ and $\vec{DB}$. (*c*) Calculate the size of angle ADB.	 1 2 4

Ans

(*a*) (6, 6, 0)

(*b*) $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

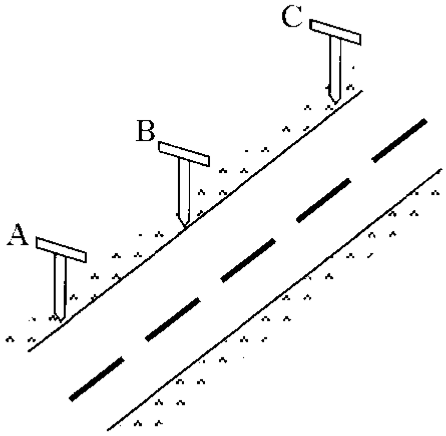
(*c*) $\cos A\hat{D}B = \dfrac{\overrightarrow{DA}.\overrightarrow{DB}}{|\overrightarrow{DA}||\overrightarrow{DB}|}$

38·7 °

2001 P1 Q3

(*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(−8, −10, −2), B(−2, −1, 1) and C(6, 11, 5).

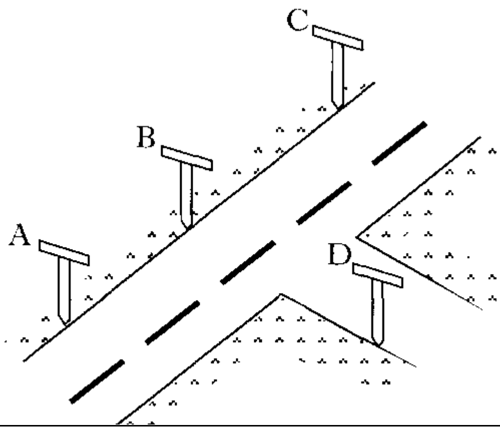
Determine whether or not the section of road ABC has been built in a straight line.



3

(*b*) A further T-rod is placed such that D has coordinates (1, −4, 4).

Show that DB is perpendicular to AB.



3

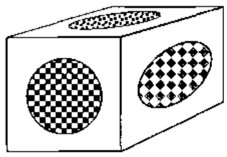
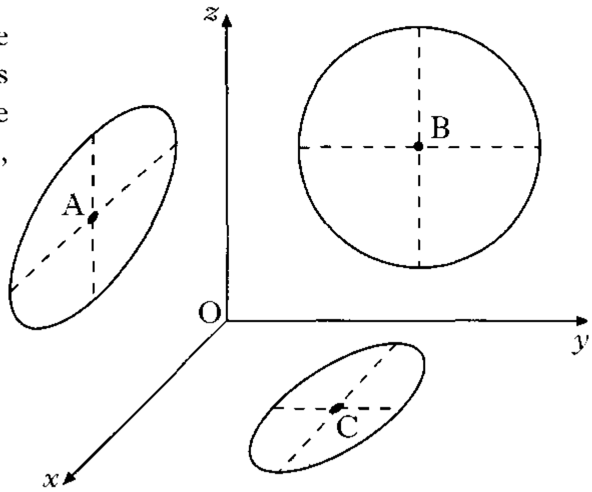
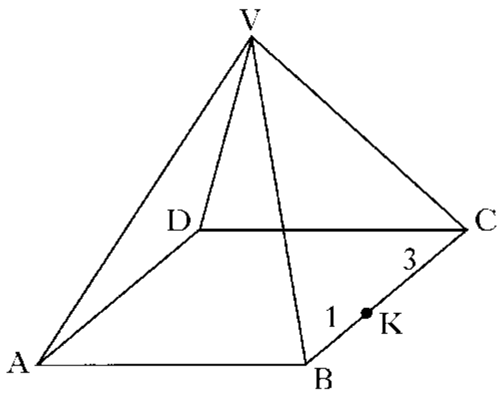
Ans

(*a*) the road ABC is straight

(*b*) $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\overrightarrow{AB}.\overrightarrow{BD} = 0$

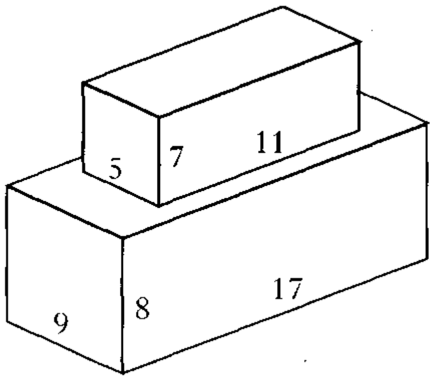
$\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$

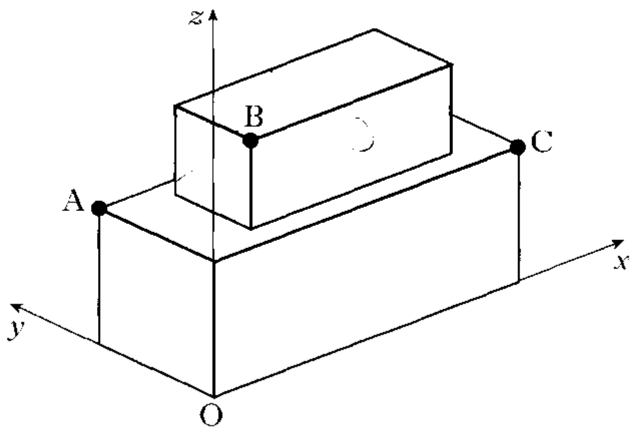
2001 P2 Q4	A box in the shape of a cuboid is designed with **circles** of different sizes on each face.  The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0). Find the size of angle ABC. 	7
Ans	71·5°	
2000 P1 Q7	VABCD is a pyramid with a rectangular base ABCD. Relative to some appropriate axes, $\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$; $\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$; $\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$. K divides BC in the ratio 1:3. Find $\overrightarrow{VK}$ in component form. 	3
Ans	$\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$	
2000 P2 Q7	For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?	2
Ans	*t* = 4	

2000 P2 Q9

A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).
Write down the coordinates of B. 1

(*b*) Calculate the size of angle ABC. 6

Ans

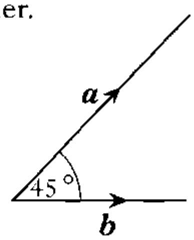
(*a*) B = (3, 2, 15)

(*b*) 92·5°

Specimen 2 P1 Q10

The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$.

These vectors are inclined at an angle of 45 ° to each other.



(*a*) Evaluate (i) $\boldsymbol{a.a}$

(ii) $\boldsymbol{b.b}$

(iii) $\boldsymbol{a.b}$ 2

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.

Evaluate $\boldsymbol{p.p}$ and hence write down $|\boldsymbol{p}|$. 4

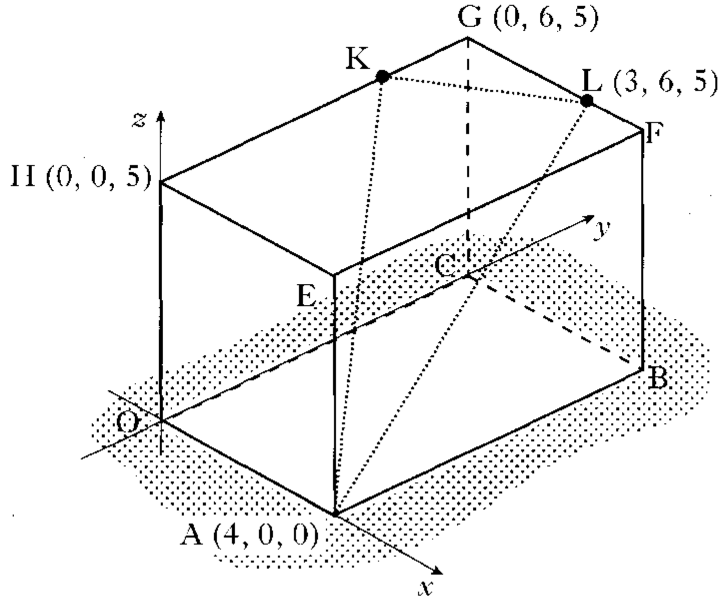
Ans

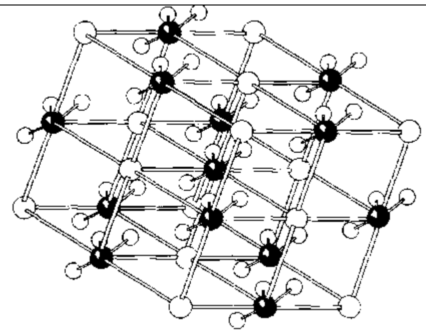
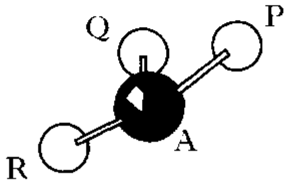
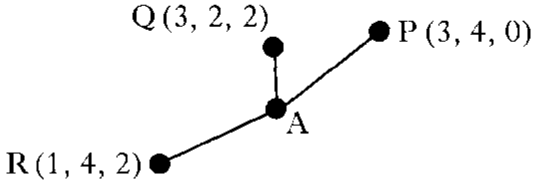
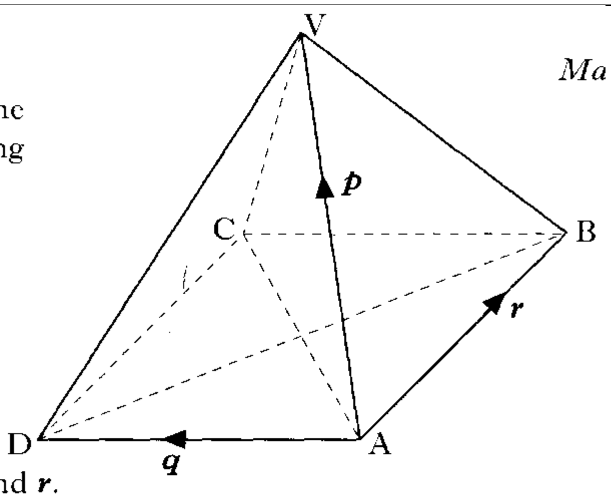
(*a*) $\boldsymbol{a.a} = 9$, $\boldsymbol{b.b} = 8$, $\boldsymbol{a.b} = 6$

(*b*) $\boldsymbol{p.p} = (2\boldsymbol{a} + 3\boldsymbol{b}).(2\boldsymbol{a} + 3\boldsymbol{b})$

$|\boldsymbol{p}|^2 = 4\boldsymbol{a.a} + 9\boldsymbol{b.b} + 12\boldsymbol{a.b}$

$|\boldsymbol{p}| = \sqrt{180}$

Specimen 2 P2 Q2	OABCEFGH is a cuboid. With axes as shown, O is the origin and the coordinates of A, H, G and L are (4, 0, 0), (0, 0, 5), (0, 6, 5) and (3, 6, 5) respectively. K lies two thirds of the way along HG, (ie HK:KG = 2:1).  (*a*) Determine the coordinates of K. (*b*) Write down the components of $\overrightarrow{AK}$ and $\overrightarrow{AL}$. (*c*) Calculate the size of angle KAL.	 2 2 5
Ans	(*a*) $\overrightarrow{HG} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \overrightarrow{HK} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \Rightarrow K = (0,4,5)$ (*b*) $\overrightarrow{AK} = \begin{pmatrix} -4 \\ 4 \\ 5 \end{pmatrix}, \quad \overrightarrow{AL} = \begin{pmatrix} -1 \\ 6 \\ 5 \end{pmatrix}$ (*c*) $\cos K\hat{A}L = \dfrac{4+24+25}{\sqrt{57}\sqrt{62}}$ $K\hat{A}L = 26 \cdot 9°$	

Specimen 1 P1 Q5	The diagram shows the rhombohedral crystal lattice of calcium carbonate.  The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.   (*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. (*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2 : 1. (i) Find the coordinates of T. (ii) Show that P, Q and R are equidistant from T.	5 6
Ans	(*a*) $\cos PQR = \frac{\overrightarrow{PQ}.\overrightarrow{RQ}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{RQ}\rvert} = \frac{4}{\sqrt{8}\sqrt{8}} = \frac{4}{8}$ (*b*) (i) M(2,3,2) T$\left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$ (ii) substitution into $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ gives $PT = QT = RT = 2\sqrt{\frac{2}{3}}$	
Specimen 1 P2 Q5	VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.  $\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$. (*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$. (ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. (*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$. (ii) Hence show that angle CVA is 90°.	3 4

Ans	(*a*) (i) 4·5 (ii) 9	(*b*) (i) $\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r}$ (ii) $\overrightarrow{CV}.\overrightarrow{AV} = \boldsymbol{p}.(\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r})$ $= \boldsymbol{p}.\boldsymbol{p} - \boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ $= 0$