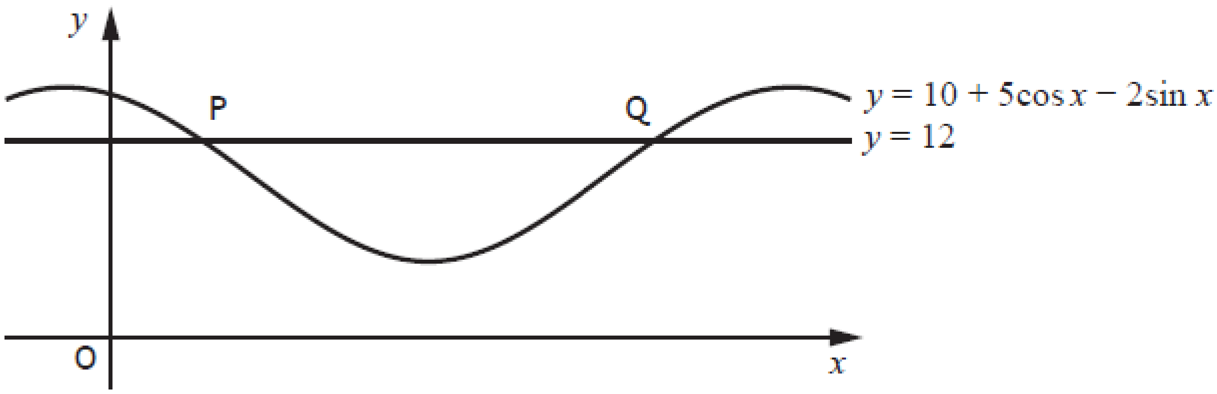
Higher : Wave Function

2018 P2 Q8	(a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. (b) Hence, or otherwise, find (i) the minimum value of $6\cos x° - 3\sin x°$ and (ii) the value of x for which it occurs where $0 \le x < 360$.	4 1 2
Ans	(a) $\sqrt{5}\cos(x-333\cdot4)°$ (b)(i) $-3\sqrt{5}$ *or* $-\sqrt{45}$ (ii) $x = 153\cdot4°$	
2017 P1 Q14	(a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k>0$ and $0<a<360$. (b) Hence, or otherwise, sketch the graph with equation $y=\sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. Use the diagram provided in the answer booklet.	4 3
Ans	(a) *2sin(x – 30)°* *(b) Correct graph showing roots 30 and 210, turning points of (120 , 2) and (300 , -2) and start/end points of (0 , -1) and (360 , -1)*	

2016 P2 Q8	(a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. (b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$. The line cuts the curve at the points P and Q.  Find the x-coordinates of P and Q.	4 4
Ans	(a) $\sqrt{29}Cos(x + 0{\cdot}38)$ (b) $0{\cdot}8097\ldots$ and $4{\cdot}712\ldots$	
2015 P2 Q8	The blades of a wind turbine are turning at a steady rate. The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula $h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65.$ Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form $k\sin(1{\cdot}5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$, and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn.	8
Ans	$t = 1{\cdot}006$ and $t = 1{\cdot}615$	

<table>
<tr><td>2014 P1 Q4</td><td>If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?
<table>
<tr><th></th><th>$k\cos a$</th><th>$k\sin a$</th></tr>
<tr><td>A</td><td>-3</td><td>4</td></tr>
<tr><td>B</td><td>3</td><td>-4</td></tr>
<tr><td>C</td><td>4</td><td>-3</td></tr>
<tr><td>D</td><td>-4</td><td>3</td></tr>
</table></td><td>2</td></tr>
<tr><td>Ans</td><td>D</td><td></td></tr>
<tr><td>2014 P1 Q9</td><td>$\sin x + \sqrt{3}\cos x$ can be written as $2\cos\left(x - \frac{\pi}{6}\right)$.
The maximum value of $\sin x + \sqrt{3}\cos x$ is 2.
What is the maximum value of $5\sin 2x + 5\sqrt{3}\cos 2x$?
A 20
B 10
C 5
D 2</td><td>2</td></tr>
<tr><td>Ans</td><td>B</td><td></td></tr>
<tr><td>2013 P1 Q23</td><td>(a) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\ \sin(x - a)°$, where $k > 0$ and $0 \le a < 360$.
Calculate the values of k and a.
(b) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \le x < 360$.</td><td>4
2</td></tr>
<tr><td>Ans</td><td>(a) $k = 2$, $a = 30°$ (b) 14</td><td></td></tr>
<tr><td>2012 P1 Q22a</td><td>(a) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a < 2\pi$.
Calculate the values of k and a.</td><td>4</td></tr>
<tr><td>Ans</td><td>$k = 2$, $a = \pi/3$</td><td></td></tr>
<tr><td>2011 P2 Q6a</td><td>(a) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \le a < 2\pi$.
Calculate the values of R and a.</td><td>4</td></tr>
</table>

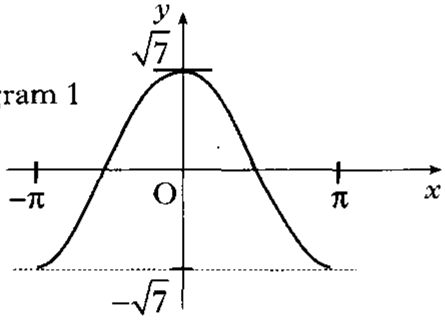
Ans	(*a*) $R = \sqrt{34}$, $a = 5{\cdot}3$	
2010 P2 Q2	(*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \le a < 360$. Calculate the values of k and a. (*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$. (ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur.	4 3
Ans	*(a) k = 13 , a = 226 (b) (i) 13 and -13 (ii) max. at x=337·4º and min. at x=157·4 º*	
2009 P1 Q13	k and a are given by $k\sin a° = 1$ $k\cos a° = \sqrt{3}$ where $k > 0$ and $0 \le a < 90$. What are the values of k and a? | | k | a | A 2 60 B 2 30 C $\sqrt{10}$ 60 D $\sqrt{10}$ 30	2
Ans	B	

Options table for 2009 P1 Q13:

	k	a
A	2	60
B	2	30
C	$\sqrt{10}$	60
D	$\sqrt{10}$	30

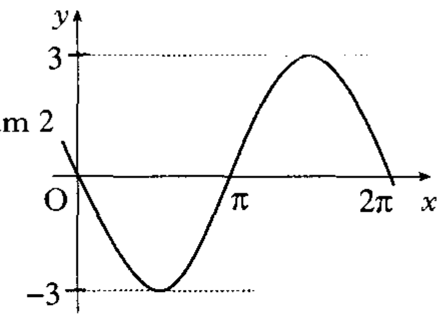
<table>
<tr><td>2008 P2 Q3</td><td>(a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1


(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2


(b) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$.

(c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression.</td><td>2

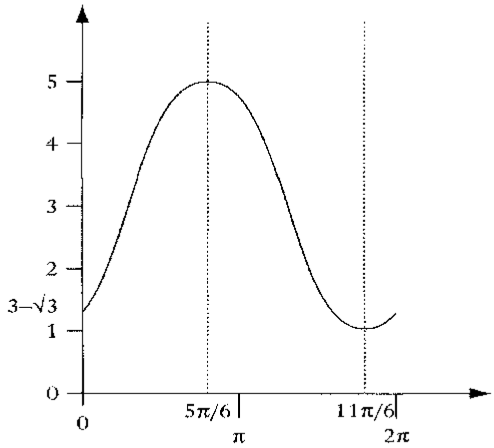
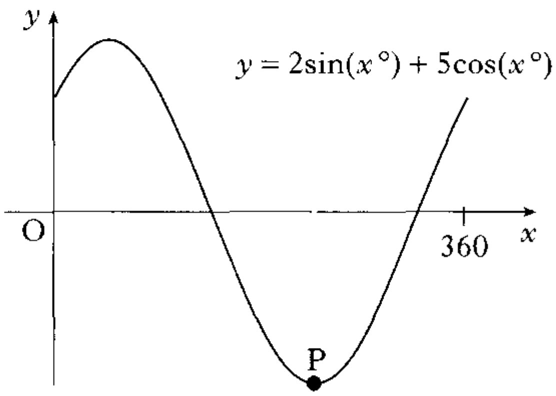
4

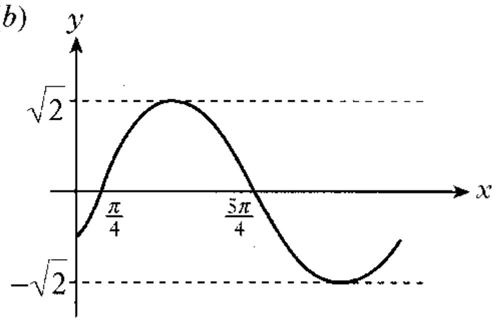
2</td></tr>
<tr><td>Ans</td><td colspan="2">(a) p = √7 , q=-3 (b) 4Cos(x + 0·848) (c) -4Sin(x + 0·848)</td></tr>
<tr><td>2007 P2 Q11</td><td>(a) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$.

(b) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$.</td><td>4

4</td></tr>
<tr><td>Ans</td><td colspan="2">(a) $a = \frac{1}{2}$
(b) $b = \frac{3}{2}$
(c) $y = 3 \times 4^x$
$\log_{10}y = \log_{10}3 + \log_{10}(4^x)$
$= \log_{10}3 + x\log_{10}(4)$
So gradient of line = $\log_{10}(4)$</td></tr>
<tr><td>2006 P2 Q10</td><td>A curve has equation $y = 7\sin x - 24\cos x$.
(a) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$.
(b) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1.</td><td>4

3</td></tr>
<tr><td>Ans</td><td colspan="2">(a) $k = 25$
$a = 1{\cdot}29$
(b) $x = 2{\cdot}82$</td></tr>
</table>

2005 P1 Q10	(*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x-a)$ where $k > 0$ and $0 \le a \le 2\pi$. (*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$.	4 5
Ans	.(*a*) $2\sin(x - \frac{\pi}{3})$ (*b*)  	
2004 P2 Q6	(*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \le a \le 90$. (*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \le x \le 90$.	4 3
Ans	(*a*) $\sqrt{34}\cos(x - 59)°$ (*b*) $x = 12{\cdot}3$	
2003 P2 Q7	Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram. (*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \le a < 360$. (*b*) Find the coordinates of the minimum turning point P.  	4 3
Ans	(*a*) $\sqrt{29}\sin(x + 68{\cdot}2)°$ (*b*) $(201{\cdot}8°, -\sqrt{29})$	
2002W P2 Q4	(*a*) Write $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x+a)°$ where $k > 0$ and $0 \le a < 360$. (*b*) Hence find the maximum value of $5 + \sqrt{3}\sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \le x \le 360$.	4 2

Ans	(*a*) $2\sin(x+30)°$ (*b*) max = 7 when $x = 60$	
2002 P1 Q9	(*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. (*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.	4 3
Ans	(*a*) $\sqrt{2}\sin(x - \frac{\pi}{4})$ (*b*) 	
2001 P2 Q5	Express $8\cos x° - 6\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 < a < 360$.	4
Ans	$10\cos(x + 36{\cdot}9)°$	
2000 P1 Q10	Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$.	6
Ans	max value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$	
Specimen 2 P1 Q6	$f(x) = \sqrt{3}\sin x° - \cos x°$ (*a*) Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a < 360$. (*b*) Hence solve the equation $f(x) = \sqrt{2}$ in the interval $0 \leq a < 360$.	4 3
Ans	(*a*) compare $\sqrt{3}\sin x° - \cos x°$ with $k\sin x°\cos a° - k\cos x°\sin a°$ $k\cos a° = \sqrt{3},\ k\sin a° = 1$ $k = 2,\ \tan a° = \frac{1}{\sqrt{3}} \Rightarrow a = 30$ (*b*) $2\sin(x - 30)° = \sqrt{2}$ $x - 30 = 45,\ 135$ $x = 75,\ 165$	

Specimen 1 P2 Q6	$f(x) = 2\cos x° + 3\sin x°$. (*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$. (*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \leq x < 360$. (*c*) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \leq x < 360$.	4 3 2
Ans	(*a*) $\sqrt{13}\cos(x - 56\cdot3)°$ (*b*) 138·8, 334·3 (*c*) 146·3°	