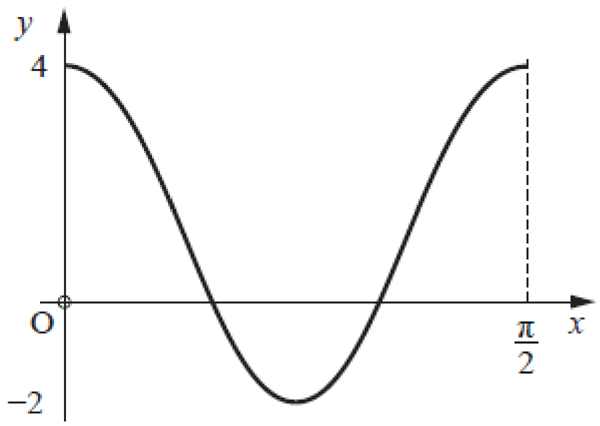
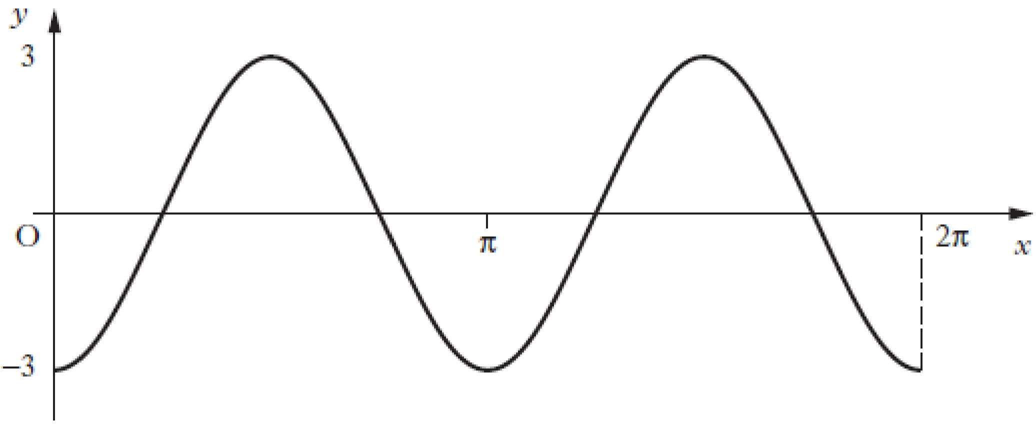
Higher : Trig. Graphs & Equations

2018 P2 Q6(b)	Functions, f and g, are given by $f(x) = 3 + \cos x$ and $g(x) = 2x$, $x \in \mathbb{R}$. (a) Find expressions for (i) $f(g(x))$ and (ii) $g(f(x))$. (b) Determine the value(s) of x for which $f(g(x)) = g(f(x))$ where $0 \le x < 2\pi$.	 2 1 6
Ans	(a)(i) $3 + \cos 2x$ (ii) $2(3 + \cos x)$ (b) $x = \pi$	
2015 NH P1 Q4	The diagram shows part of the graph of the function $y = p\cos qx + r$.  Write down the values of p, q and r.	3
Ans	*p* = 3 , *q* = 4 , *r* = 1	

2015 P1 Q4

The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \leq x \leq 2\pi$.



What is the equation of this graph?

A $y = -3\cos 2x$

B $y = -3\cos 3x$

C $y = 3\cos 2x$

D $y = 3\cos 3x$

2

Ans *A*

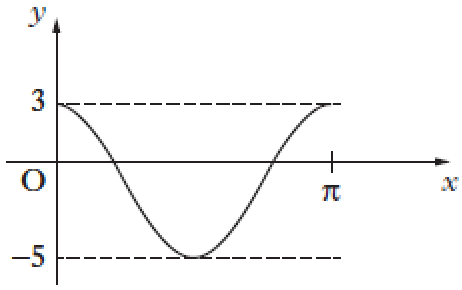
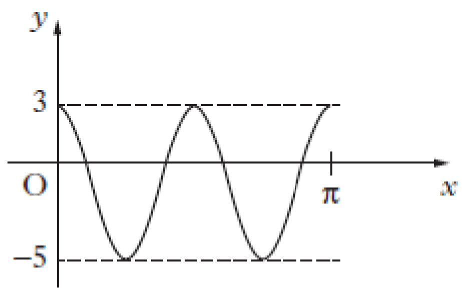
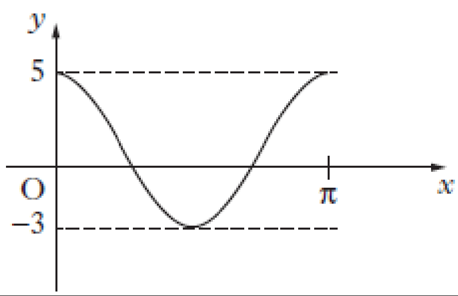
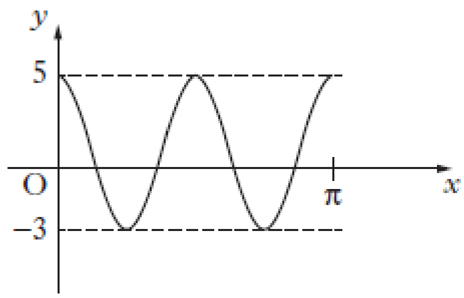
2015 P1 Q9

Find the minimum value of $3\sin 2x + 5$ and the value of x where this occurs in the interval $0 \leq x < \pi$.

	min value	x
A	2	$\frac{3\pi}{2}$
B	2	$\frac{3\pi}{4}$
C	4	$\frac{3\pi}{2}$
D	4	$\frac{3\pi}{4}$

2

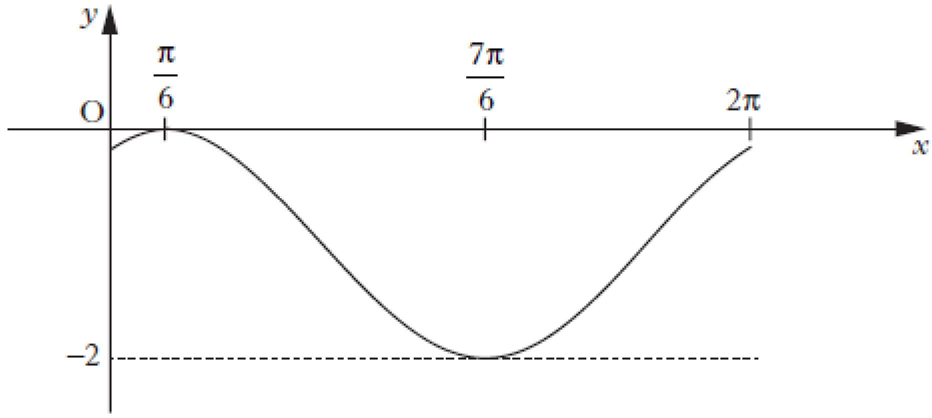
Ans *B*

2015 P1 Q10	Solve $2\cos x + 1 = 0$ for x, where $\pi \le x \le \frac{3\pi}{2}$. A $\frac{5\pi}{6}$ B $\frac{7\pi}{6}$ C $\frac{5\pi}{4}$ D $\frac{4\pi}{3}$	2
Ans	*D*	
2014 P1 Q13	What is the value of $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$? A $\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$ B $\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$ C $\frac{1}{2} - \frac{1}{\sqrt{2}}$ D $\frac{1}{2} + \frac{1}{\sqrt{2}}$	2
Ans	*B*	
2013 P1 Q4	Which of the following shows the graph of $y = 4\cos 2x - 1$, for $0 \le x \le \pi$? A  B  C  D 	2

Ans	*A*	
2013 P1 Q15	Solve $\tan\left(\frac{x}{2}\right) = -1$ for $0 \leq x < 2\pi$. A $\frac{\pi}{2}$ B $\frac{7\pi}{8}$ C $\frac{3\pi}{2}$ D $\frac{15\pi}{8}$	2
Ans	*C*	
2013 P2 Q8	Solve algebraically the equation $\sin 2x = 2\cos^2 x \qquad \text{for } 0 \leq x < 2\pi$	6
Ans	$\frac{p}{4}, \frac{p}{2}, \frac{5p}{4}, \frac{3p}{2}$	

2012 P1 Q9

The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \leq x \leq 2\pi$.



What is the equation of this curve?

A $\quad y = \cos\left(x - \frac{\pi}{6}\right) - 1$

B $\quad y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C $\quad y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D $\quad y = \cos\left(x + \frac{\pi}{6}\right) + 1$

2

Ans *A*

2012 P1 Q22(b)

(*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

Calculate the values of k and a. 4

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \leq x \leq 2\pi$. 3

Ans (a) $k = 2$, $a = \frac{p}{3}$ (b) $\frac{p}{6}, \frac{7p}{6}$

<table>
<tr><td>2012 P2 Q6</td><td>For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$
(a) Why do these sequences have a limit?

(b) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.
Find the value(s) of x.</td><td>2

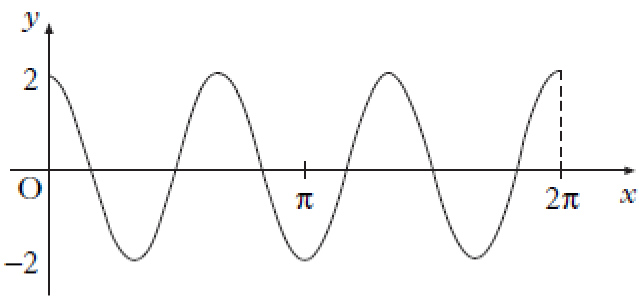
7</td></tr>
<tr><td>Ans</td><td colspan="2">(a) -1 < Sin x < 1 and in the interval, Sin x < 1   (b) x = 0·730</td></tr>
<tr><td>2011 P1 Q10</td><td>Solve $2\cos x = \sqrt{3}$ for x, where $0 \le x < 2\pi$.

A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

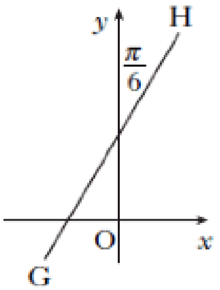
C $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D $\frac{\pi}{6}$ and $\frac{11\pi}{6}$</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">D</td></tr>
<tr><td>2010 P1 Q4</td><td>The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \le x \le 2\pi$.



What is the equation of this graph?

A $y = 2\cos 3x$
B $y = 2\cos 2x$
C $y = 3\cos 2x$
D $y = 4\cos 3x$</td><td>2</td></tr>
<tr><td>Ans</td><td colspan="2">A</td></tr>
</table>

2009 P1 Q11	How many solutions does the equation $$(4\sin x - \sqrt{5})(\sin x + 1) = 0$$ have in the interval $0 \le x < 2\pi$? A 4 B 3 C 2 D 1	2
Ans	*B*	
2009 P1 Q14	If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$? A $-1 \le f(x) \le 11$ B $2 \le f(x) \le 8$ C $3 \le f(x) \le 7$ D $-3 \le f(x) \le 7$	2
Ans	*C*	
2009 P1 Q15	The line GH makes an angle of $\frac{\pi}{6}$ radians with the y-axis, as shown in the diagram. What is the gradient of GH?  A $\sqrt{3}$ B $\frac{1}{2}$ C $\frac{1}{\sqrt{2}}$ D $\frac{\sqrt{3}}{2}$	2
Ans	*A*	

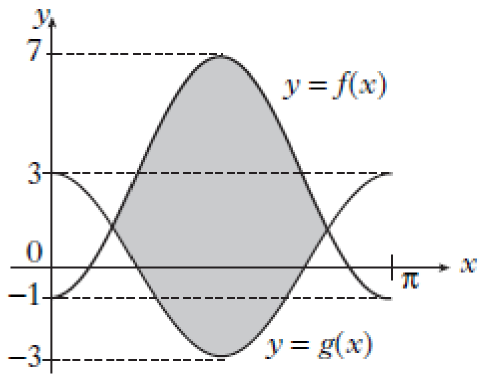
2009 P2 Q5a,b

The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(*a*) Write down the values of m and n. 1

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. 5



Ans *(a) m = 3 , n = 2 (b) (0·6 , 1·3) , (2·6 , 1·3)*

2008 P1 Q6

What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \le x \le \pi$? 2

A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

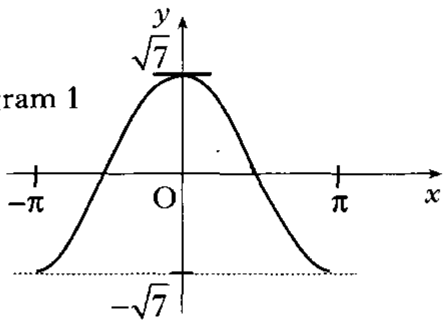
Ans *B*

2008 P2 Q3

(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1

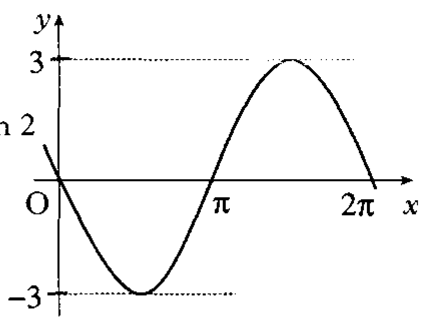


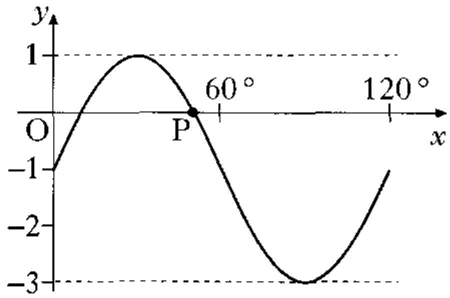
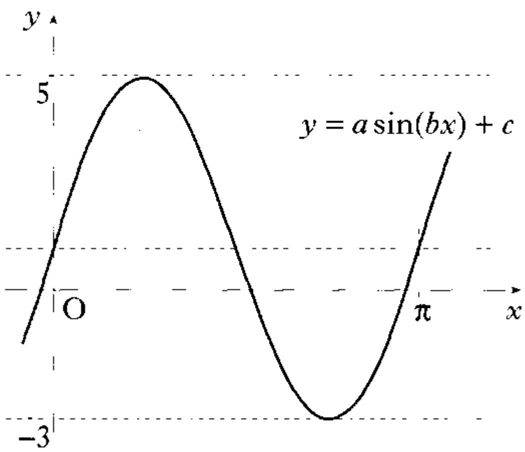
2

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



Ans	$p = \sqrt{7}$, $q = -3$	
2007 P2 Q4	The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$. (*a*) Write down the values of a, b and c. (*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. 	3 3
Ans	(*a*) $a = 2$, $b = 3$, $c = -1$ (*b*) $x_P = 50°$	
2004 P1 Q3	Find all the values of x in the interval $0 \le x \le 2\pi$ for which $\tan^2(x) = 3$.	4
Ans	$x = \frac{\pi}{3}$ and $x = \frac{4\pi}{3}$ $x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$	
2003 P2 Q2	The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$. Determine the values of a, b and c. 	3
Ans	a = 4, b = 2, c = 1	
2002W P1 Q4	(*a*) Write down the exact values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$. (*b*) If $\tan x = 4\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$, find the exact values of x for $0 \le x \le 2\pi$.	1 2
Ans	(*a*) $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$ (*b*) $\frac{\pi}{3}$, $\frac{4\pi}{3}$	

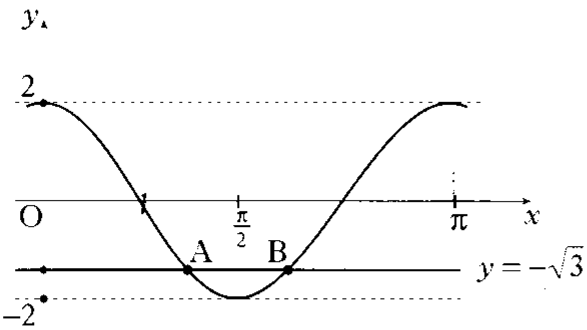
2002 P1 Q8

The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. 1

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. 3



Ans

(*a*) $y = 2\cos(2x)$

(*b*) $\mathrm{B}\left(\frac{7\pi}{12}, -\sqrt{3}\right)$

2002 P1 Q9

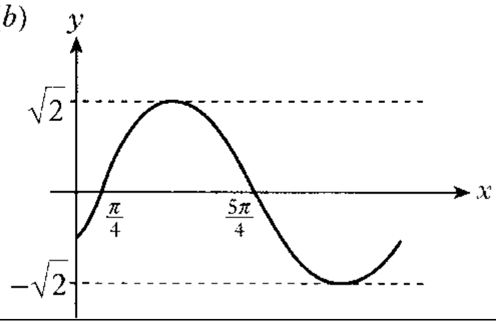
(*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. 4

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. 3

Ans

(*a*) $\sqrt{2}\sin(x - \frac{\pi}{4})$

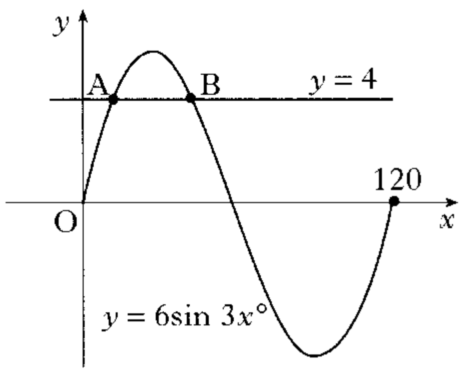
(*b*)



Specimen 2 P2 Q3

The diagram shows part of the graph of $y = 6\sin 3x$ and the line with equation $y = 4$.

Find the x-coordinates of A and B. 3



Ans

$6\sin 3x = 4$

$3x = 41{\cdot}8,\ 138{\cdot}2,\ (401{\cdot}8)$

$x = 13{\cdot}9,\ 46{\cdot}1,\ (133{\cdot}6)$

$x_A = 13{\cdot}9,\ x_B = 46{\cdot}1$

Specimen 1 P1 Q8	Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \le x < 360$.	4
Ans