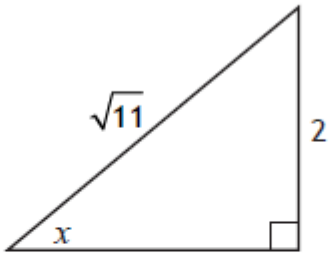
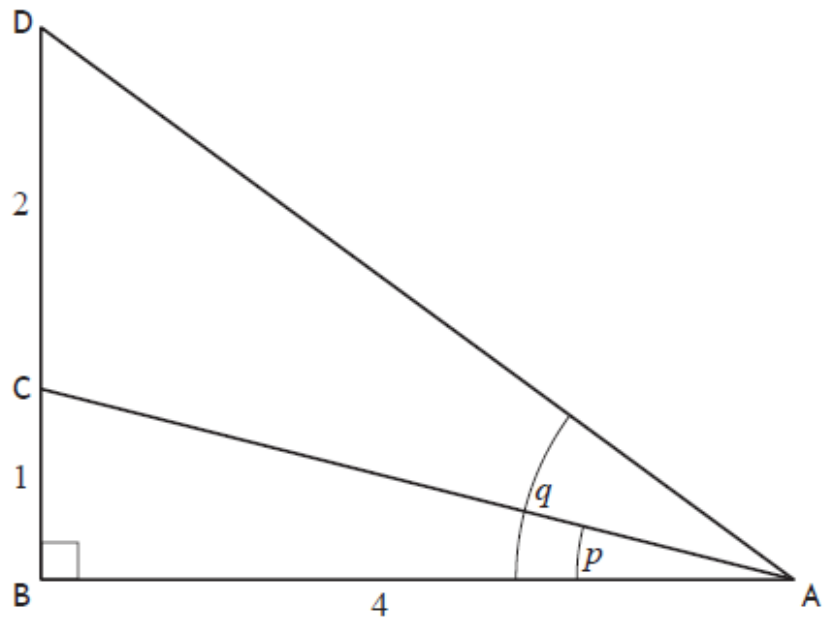
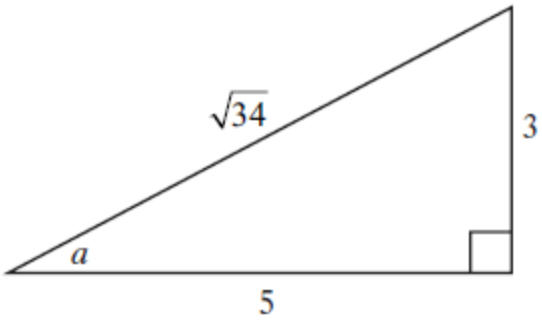
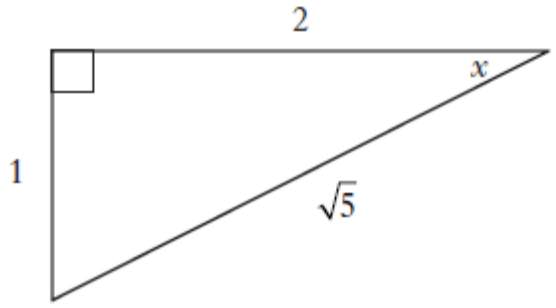
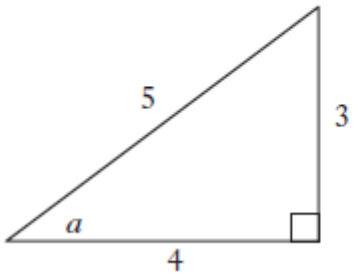
Higher : Addition Formulae

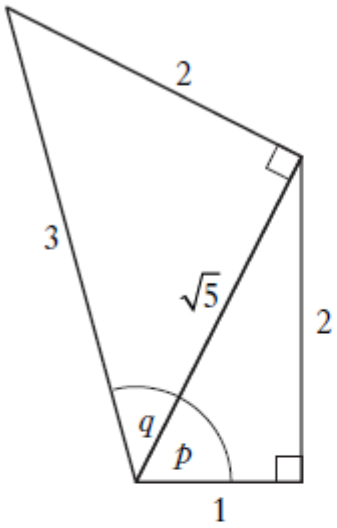
2018 P1 Q13	The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.  (a) Find the exact value of: (i) $\sin 2x$ (ii) $\cos 2x$. (b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$.	 3 1 3
Ans	(a)(i) $\frac{4\sqrt{7}}{11}$ (ii) $\frac{3}{11}$ (b) $\frac{34}{11\sqrt{11}}$	
2017 P2 Q6	Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$.	5
Ans	*x = 0·848 , 2·29*	
2017 P2 Q11a	Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$.	3
Ans	*Proof*	

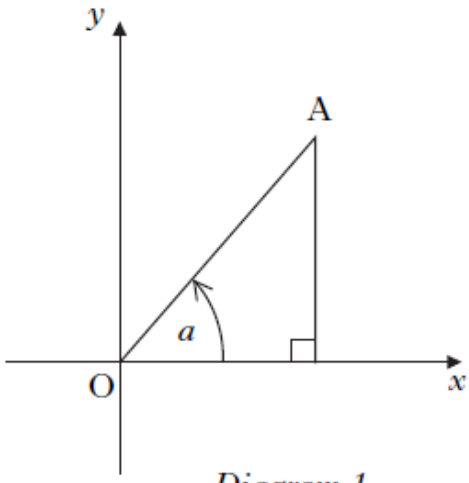
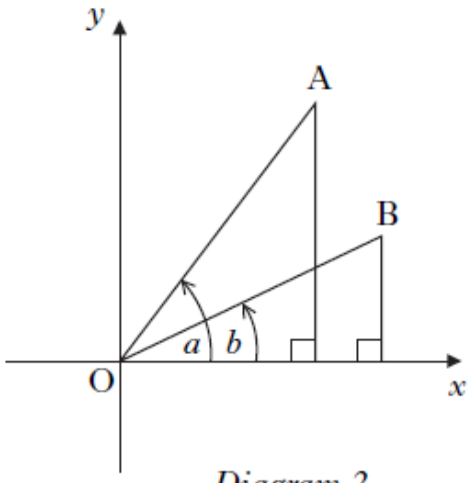
2016 P1 Q13	Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.  Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$.	5
Ans	*Proof*	
2016 P2 Q11a	Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$.	4
Ans	*Proof*	
2015(NH) P1 Q10	Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of (a) $\cos 2x$ (b) $\cos x$.	 1 2
Ans	*(a)* $\frac{4}{5}$ *(b)* $\frac{3}{\sqrt{10}}$	

2015 P1 Q14	If $\cos x = -\frac{2}{5}$, what is the value of $\cos 2x$? A $\frac{33}{25}$ B $\frac{17}{25}$ C $-\frac{4}{5}$ D $-\frac{17}{25}$	2
Ans	*D*	
2014 P1 Q7	A right-angled triangle has sides and angles as shown in the diagram.  What is the value of $\sin 2a$? A $\frac{8}{17}$ B $\frac{3}{\sqrt{34}}$ C $\frac{15}{17}$ D $\frac{6}{\sqrt{34}}$	2
Ans	*C*	

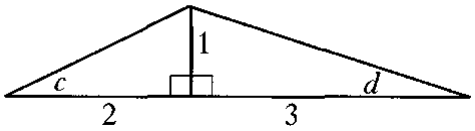
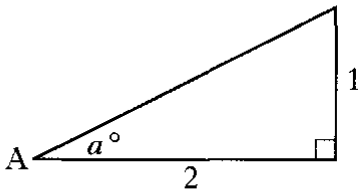
2014 P1 Q18	What is the value of $1 - 2\sin^2 15°$? A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{\sqrt{3}}{2}$ D $\frac{7}{8}$	2
Ans	*C*	
2014 P2 Q6	Solve the equation $\sin x - 2\cos 2x = 1$ for $0 \le x < 2\pi$.	5
Ans	$x = 0{\cdot}848\ ,\ 2{\cdot}29\ ,\ \frac{3\pi}{2}$	
2013 P1 Q10	If $0 < a < 90$, which of the following is equivalent to $\cos(270 - a)°$? A $\cos a°$ B $\sin a°$ C $-\cos a°$ D $-\sin a°$	2
Ans	*D*	

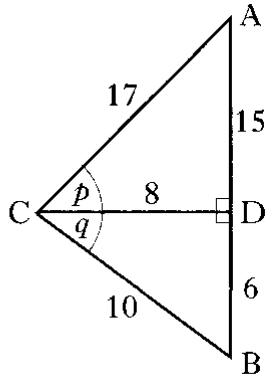
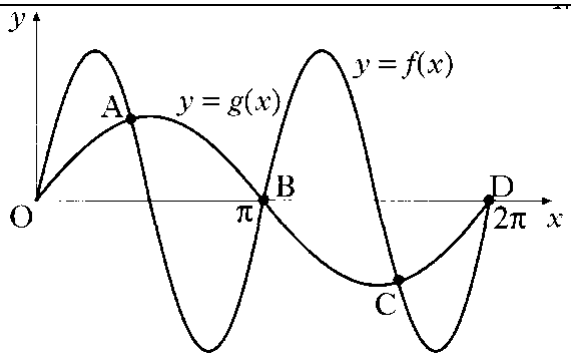
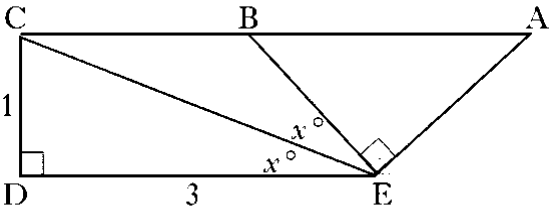
2013 P1 Q9	The diagram shows a right-angled triangle with sides and angles as marked.  Find the value of $\sin 2x$. A $\frac{4}{5}$ B $\frac{2}{5}$ C $\frac{2}{\sqrt{5}}$ D $\frac{1}{\sqrt{5}}$	2
Ans	*A*	
2012 P1 Q5	The diagram shows a right-angled triangle with sides and angles as marked.  What is the value of $\cos 2a$? A $\frac{7}{25}$ B $\frac{3}{5}$ C $\frac{24}{25}$ D $\frac{6}{5}$	2
Ans	*A*	

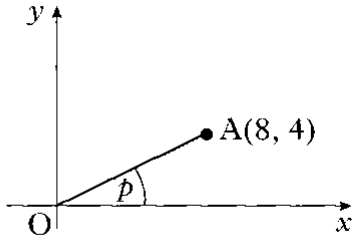
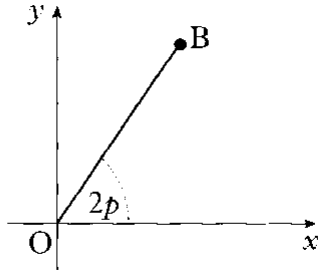
2011 P1 Q12	The diagram shows two right-angled triangles with sides and angles as given.  What is the value of $\sin(p+q)$? A $\frac{2}{\sqrt{5}}+\frac{2}{3}$ B $\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{3}$ C $\frac{2}{3}+\frac{2}{3\sqrt{5}}$ D $\frac{4}{3\sqrt{5}}+\frac{1}{3}$	2
Ans	*C*	

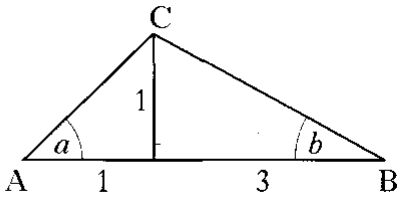
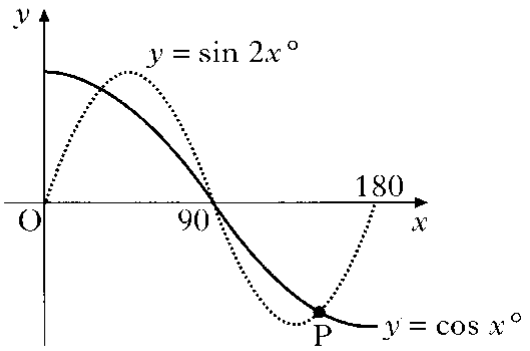
2010 P1 Q23	(*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$. (i) Show that $\tan a = \frac{3}{2}$. (ii) Find the value of $\sin a$.  *Diagram 1*	4
	(*b*) A second right angled triangle is added as shown in Diagram 2. The line OB has equation $3x - 4y = 0$. Find the values of $\sin b$ and $\cos b$.  *Diagram 2*	4
	(*c*) (i) Find the value of $\sin(a - b)$. (ii) State the value of $\sin(b - a)$.	4
Ans	*(a) 3/√13 (b)3/5 and 4/5 (c) (i) 6/(5√13) (ii) -6/(5√13)*	
2010 P2 Q4	Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$.	5
Ans	*2·419, 3·864*	

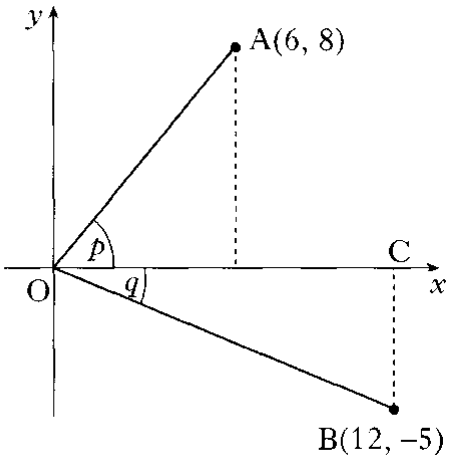
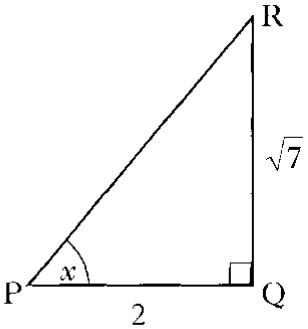
2009 P1 Q7	If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$. A $-\frac{3}{5}$ B $-\frac{2}{\sqrt{5}}$ C $\frac{2}{\sqrt{5}}$ D $\frac{3}{5}$	2
Ans	*A*	
2009 P1 Q24	(*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. (*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$. (*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$. (ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$.	3 2 4
Ans	*(a)* $\frac{\sqrt{3}+1}{2\sqrt{2}}$ *(b) Proof* *(c)(i)* $\frac{\pi}{3} - \frac{\pi}{4}$ *(ii)* $\frac{\sqrt{6}}{2}$	
2008 P1 Q9	**Given that $0 \le a \le \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.** A $\sin x + \frac{3}{5}$ B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$ C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$ D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$	2
Ans	*B*	
2008 P2 Q5	Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$.	5
Ans	*90 , 199·5 , 340·5*	

2007 P1 Q6	Solve the equation $\sin 2x° = 6\cos x°$ for $0 \le x \le 360$.	4
Ans	$x = 90, 270$	
2007 P2 Q2	The diagram shows two right-angled triangles with angles c and d marked as shown.  (a) Find the exact value of $\sin(c + d)$. (b) (i) Find the exact value of $\sin 2c$. (ii) Show that $\cos 2d$ has the same exact value.	4 4
Ans	(a) $\sqrt{\frac{1}{2}}$ (b) (i) $\frac{4}{5}$	
2006 P1 Q7	Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$.	4
Ans	$x = 0, 180, 360, \quad 60, 300$	
2006 P2 Q8	The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.  (a) Find the exact values of: (i) $\sin a°$; (ii) $\sin 2a°$. (b) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$.	4 4
Ans	(a) (i) $\sin a° = \frac{1}{\sqrt{5}}$ (ii) $\sin 2a° = \frac{4}{5}$ (b) $\sin 3a° = \frac{11}{5\sqrt{5}}$	
2005 P1 Q9	If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.	4
Ans	$\cos(x) = \frac{4}{5}$ $\sin(x) = \frac{3}{5}$	

2005 P2 Q2	Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram. (*a*) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$. (*b*) Calculate the exact values of: (i) $\cos(p+q)$; (ii) $\tan(p+q)$.  	4 3
Ans	(*a*) $\cos(p) = \frac{8}{17}$, $\sin(p) = \frac{15}{17}$ $\cos(q) = \frac{8}{10}$, $\sin(q) = \frac{6}{10}$ From $\sin(A+B) =$ $\sin(A)\cos(B) + \cos(A)\sin(B)$ $\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \frac{84}{85}$ (*b*) (i) $-\frac{13}{85}$ (ii) $-\frac{84}{13}$	
2005 P2 Q8	Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$. The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D. Show that, at A and C, $\cos x = \frac{1}{2k}$.  	5
Ans	$k\sin(2x) = \sin(x)$ $k \times 2\sin(x)\cos(x)$ $\sin(x) \times (2k\cos(x) - 1) = 0$ $\sin(x) = 0$ **and** $\cos(x) = \frac{1}{2k}$ $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$ at (O), B and D **and** $\cos(x) = \frac{1}{2k}$ at A and C	
2004 P1 Q10	In the diagram angle DEC = angle CEB = $x°$ and angle CDE = angle BEA = 90°. CD = 1 unit; DE = 3 units. By writing angle DEA in terms of $x°$, find the exact value of $\cos(D\hat{E}A)$.  	7

Ans	$\cos D\hat{E}A = -\frac{6}{10}$	
2003 P1 Q10	A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis. (*a*) Find the exact values of: (i) $\sin(2p)$; (ii) $\cos(2p)$.  The line OB is inclined at an angle $2p$ radians to the x-axis. (*b*) Write down the exact value of the gradient of OB.  	5 1
Ans	(*a*) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (*b*) $\frac{4}{3}$	
2003 P2 Q10	Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \le x \le \pi$, correct to 2 decimal places.	5
Ans	1·23 radians only	
2002W P2 Q5	Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \le x < 2\pi$.	4
Ans	$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	
2002 P1 Q3	Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \le x \le 360$.	 2 5

Ans	(*a*) (i) $\sin(2x°)$ (ii) $2\sin(x°)$ (*b*) $4\sin(x°)\cos(x°) - 2\sin(x°) = 0$ $2\sin(x°)(2\cos(x°) - 1) = 0$ $\sin(x°) = 0,\ \cos(x°) =$ $0°, 60°, 180°, 300°, 360°$	
2002 P1 Q5	In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$.  	4
Ans	• $AC = \sqrt{2}$ **and** $BC = \sqrt{10}$ • $\sin(a+b) = \sin a \cos b + \cos a \sin b$ • $= \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ • $= \frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4} \times \sqrt{5}} = \frac{2}{\sqrt{5}}$	
2001 P1 Q5	(*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. (*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$. Use your solutions in (*a*) to write down the coordinates of the point P.  	4 1
Ans	(*a*) 30, 90, 150 (*b*) $\left(150, -\frac{\sqrt{3}}{2}\right)$	

2001 P1 Q7	Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers. (*a*) Find expressions for: (i) $f(h(x))$; (ii) $g(h(x))$. (*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$. (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$.	2 5
Ans	(*a*) (i) $\sin\left(x + \frac{\pi}{4}\right)$; (ii) $\cos\left(x + \frac{\pi}{4}\right)$ (*b*) (i) $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	
2000 P1 Q1	On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q. Find the exact value of $\sin(p + q)$.  	4
Ans		
Specimen 2 P1 Q7	Using triangle PQR, as shown, find the exact value of $\cos 2x$.  	3
Ans	$\cos x = \frac{2}{\sqrt{11}}$ $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1 = -\frac{3}{11}$	

Specimen 1 P1 Q4	If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$.	3
Ans	$\sin x° \cos 30° + \cos x° \sin 30°$ $= \frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2}$ $= \frac{4\sqrt{3}+3}{10}$	
Specimen 1 P2 Q7	(*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. (*b*) **Hence** (i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$ (ii) solve this equation in the interval $0 \le x < 90$.	2 3
Ans	(*a*) $2(1 - 2\sin^2 x°) - \cos^2 x°$ $= 2 - 4\sin^2 x° - \cos^2 x°$ $= 2 - 4\sin^2 x° - (1 - \sin^2 x°)$ $= 1 - 3\sin^2 x°$ (*b*) (i) $3\sin^2 x° + 2\sin x° - 1 = 0$ (ii) 19·5	