Higher : Sets & Functions

2018 P1 Q2	A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by $$g(x)=\frac{1}{5}x-4.$$ Find the inverse function, $g^{-1}(x)$.	3
Ans	$g^{-1}(x) = 5(x+4)$	
2018 P2 Q6(a)	Functions, f and g, are given by $f(x)=3+\cos x$ and $g(x)=2x$, $x\in\mathbb{R}$. (a) Find expressions for (i) $f(g(x))$ and (ii) $g(f(x))$.	2 2
Ans	(a)(i) $3 + \cos 2x$ *(ii)* $2(3 + \cos x)$	
2017 P1 Q1	Functions f and g are defined on suitable domains by $f(x)=5x$ and $g(x)=2\cos x$. (a) Evaluate $f(g(0))$. (b) Find an expression for $g(f(x))$.	1 2
Ans	*(a) 10 (b) 2cos5x*	
2017 P1 Q6	A function, h, is defined by $h(x)=x^3+7$, where $x\in\mathbb{R}$. Determine an expression for $h^{-1}(x)$.	3
Ans	$h^{-1}(x)=\sqrt[3]{x-7}$	
2016 P1 Q6	Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x)=3x+5$, find $f^{-1}(x)$. (b) If $g(2)=7$, write down the value of $g^{-1}(7)$.	3 1
Ans	*(a)* $f^{-1}(x)=\frac{x-5}{3}$ *(b) 2*	

2016 P1 Q12a	The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x)=2x^2-4x+5$ and $g(x)=3-x$. (a) Given $h(x)=f(g(x))$, show that $h(x)=2x^2-8x+11$.	2
Ans	*Proof*	
2015 NH P1 Q5	A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$. (a) Determine an expression for $g^{-1}(x)$. (b) Write down an expression for $g(g^{-1}(x))$.	2 1
Ans	*(a)* $g^{-1}(x)=\frac{6-x}{2}$ *(b) x*	
2015 NH P2 Q2	Functions f and g are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$. (a) Find an expression for $f(g(x))$. (b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. (c) Another function h is given by $h(x)=\frac{1}{f(g(x))}$. What values of x cannot be in the domain of h?	2 3 2
Ans	*(a) 10 + (1 + x)(3 – x) + 2 (b) -1(x – 1)² + 16 (c) 5 and -3*	
2014 P1 Q12	A function f, defined on a suitable domain, is given by $f(x)=\frac{6x}{x^2+6x-16}$. What restrictions are there on the domain of f? A $x \neq -8$ or $x \neq 2$ B $x \neq -4$ or $x \neq 4$ C $x \neq 0$ D $x \neq 10$ or $x \neq 16$	2
Ans	*A*	

2014 P2 Q3(a)	Functions f and g are defined on suitable domains by $f(x) = x(x-1) + q$ and $g(x) = x + 3$. (*a*) Find an expression for $f(g(x))$.	2
Ans	*(x + 3)(x + 2) + q (or equivalent)*	
2013 P1 Q1	The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers. Find $g(f(x))$. A $3x^2 - 1$ B $9x^2 - 15$ C $9x^2 + 17$ D $3x^3 - 4x^2 + 3x - 4$	2
Ans	*A*	
2013 P1 Q13	A function f is defined on a suitable domain by $f(x) = \frac{x+2}{x^2-7x+12}$. What value(s) of x cannot be in this domain? A 3 and 4 B -3 and -4 C -2 D 0	2
Ans	*A*	
2012 P2 Q1	Functions f and g are defined on the set of real numbers by • $f(x) = x^2 + 3$ • $g(x) = x + 4$. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.	3 3
Ans	*(a)(i) $(x + 4)^2 + 3$ (ii) $x^2 + 7$ (b) discriminant = $\sqrt{-144}$, does not exist so no real roots*	

2010 P1 Q11	Functions f and g are defined on suitable domains by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$. What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$? A $\frac{1}{2} + \frac{\pi}{6}$ B $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$ C $\frac{\sqrt{3}}{2}$ D $\frac{1}{2}$	2
Ans	*D*	
2009 P2 Q2a	Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$. (*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$. (ii) Find $q(x)$ where $q(x) = g(f(x))$.	3
Ans	*(a)(i)* $3(x^2 - 2) + 1$ *(ii)* $(3x + 1)^2 - 2$	
2008 P1 Q17	A function f is given by $f(x) = \sqrt{9 - x^2}$. What is a suitable domain of f? A $x \geq 3$ B $x \leq 3$ C $-3 \leq x \leq 3$ D $-9 \leq x \leq 9$	2
Ans	*C*	
2008 P1 Q23	Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (*a*) Find expressions for $h(f(x))$ and $h(g(x))$.	3
Ans	(*a*) $h(f(x)) = \log_2(x^2 - x + 10)$ $h(g(x)) = \log_2(5 - x)$	
2007 P1 Q3	Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$. Find: (*a*) $g(f(x))$; (*b*) $g(g(x))$.	2 2

Ans	(*a*) $g(f(x)) = -2x^2 - 1$ (*b*) $(g(g(x)) = 4x - 1$	
2006 P1 Q3	Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.	3 2
Ans	(*a*) $f(g(x)) = 2(2x - 3) + 3$ $g(f(x)) = 2(2x + 3) - 3$ (*b*) $16x^2 - 9$ minimum value $= -9$	
2005 P1 Q4	Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers. (*a*) Find $h(x)$ where $h(x) = g(f(x))$. (*b*) (i) Write down the coordinates of the minimum turning point of $y = h(x)$. (ii) Hence state the range of the function h.	2 2
Ans	(*a*) $(3x - 1)^2 + 7$ (*b*) (i) $\left(\frac{1}{3}, 7\right)$ (ii) $y \geq 7$	
2003 P1 Q9	Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains. (*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. (*b*) Write down any restriction on the domain of h.	2 1
Ans	(*a*) $\frac{1}{2x-1}$ (*b*) $x \neq \frac{1}{2}$	
2002W P1 Q9	The function f, defined on a suitable domain, is given by $f(x) = \frac{3}{x+1}$. (*a*) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form. (*b*) Describe any restriction on the domain of h.	3 1
Ans	(*a*) $\frac{3(x+1)}{x+4}$ (*b*) $x \neq -4$	

2002 P1 Q3	Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$.	2
Ans	(*a*) (i) $\sin(2x°)$ (ii) $2\sin(x°)$	
2001 P1 Q7	Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers. (*a*) Find expressions for: (i) $f(h(x))$; (ii) $g(h(x))$.	2
Ans	(*a*) (i) $\sin\left(x + \frac{\pi}{4}\right)$; (ii) $\cos\left(x + \frac{\pi}{4}\right)$	
2000 P2 Q3	$f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$. (*a*) Find $p(x)$ where $p(x) = f(g(x))$. (*b*) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.	2 3
Ans	(*a*) $3 - \frac{3}{x}$ (*b*) x	
Specimen 2 P1 Q8	Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ $g(x) = x^2$. (*a*) Find formulae for (i) $f(g(x))$ (ii) $g(f(x))$.	3
	(*a*) $f(g(x)) = x^2 - 1$, $g(f(x)) = (x-1)^2$	