Higher : Recurrence Relations

2018 P2 Q7(b)(c)	(a) (i) Show that $(x-2)$ is a factor of $2x^3-3x^2-3x+2$. (ii) Hence, factorise $2x^3-3x^2-3x+2$ fully. The fifth term, u_5, of a sequence is $u_5 = 2a-3$. The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$. (b) Show that $u_7 = 2a^3-3a^2-a-1$. For this sequence, it is known that • $u_7 = u_5$ • a limit exists. (c) (i) Determine the value of a. (ii) Calculate the limit.	2 2 1 3 1
Ans	(a)(i) Proof (ii) $(x-2)(2x-1)(x+1)$ (b) $2a^3-3a^2-a-1$ (c)(i) $a=\frac{1}{2}$ (ii) -2	
2017 P1 Q9	A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant. (a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. (b) (i) Explain why this sequence approaches a limit as $n \to \infty$. (ii) Calculate this limit.	2 1 2
Ans	*(a) m = 0·25* *(b)(i) a limit exists since the recurrence relation is linear and -1 < 0·25 < 1* *(ii) 8*	
2017 P2 Q8	Sequences may be generated by recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$. (a) Show that $u_2 = 5k^2 - 20k - 20$. (b) Determine the range of values of k for which $u_2 < u_0$.	2 4
Ans	*(a) Proof (b) -1 < k < 5 with justificationof sketch or table of signs.*	

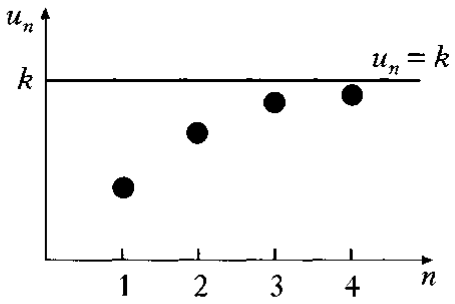
2016 P1 Q3	A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$. (a) Find the value of u_4. (b) Explain why this sequence approaches a limit as $n \to \infty$. (c) Calculate this limit.	 1 1 2
Ans	*(a)* $u_4 = 12$ *(b)* $1 < \frac{1}{3} < 1$ *(c) 15*	
2015 NH P2 Q3	A version of the following problem first appeared in print in the 16th Century. A frog and a toad fall to the bottom of a well that is 50 feet deep. Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well. The toad climbs 13 feet each day before resting. Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well. Their progress can be modelled by the recurrence relations: • $f_{n+1} = \frac{1}{3}f_n + 32, \qquad f_1 = 32$ • $t_{n+1} = \frac{3}{4}t_n + 13, \qquad t_1 = 13$ where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in. (a) Calculate t_2, the height of the toad at the end of the second day. (b) Determine whether or not either of them will eventually escape from the well.	 1 5
Ans	*(a) 22.75 (b) The toad will escape since 52 > 50.*	

2015 P1 Q5	A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}2u_n + 9$, $u_5 = 11$. What is the value of u_3? A 11·24 B 9·4 C 5 D 4	2
Ans	*C*	
2014 P1 Q1	A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$. What is the value of u_4? A $2\frac{1}{9}$ B $2\frac{1}{3}$ C 3 D 30	2
Ans	*C*	
2014 P1 Q10	A sequence is defined by the recurrence relation $u_{n+1} = (k-2)u_n + 5$ with $u_0 = 3$. For what values of k does this sequence have a limit as $n \to \infty$? A $-3 < k < -1$ B $-1 < k < 1$ C $1 < k < 3$ D $k < 3$	2
Ans	*C*	

2013 P1 Q8	A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$. Here are two statements about this sequence: (1) $u_0 = 9{\cdot}1$; (2) The sequence has a limit as $n \longrightarrow \infty$. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	2
Ans	*C*	
2013 P2 Q1	The first three terms of a sequence are 4, 7 and 16. The sequence is generated by the recurrence relation $u_{n+1} = mu_n + c$, with $u_1 = 4$. Find the values of m and c.	4
Ans	*m = 3 , c = -5*	
2012 P1 Q1	A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$. Find the value of u_2. A 7 B 10 C 25 D 35	2
Ans	*C*	
2012 P2 Q6	For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation $u_{n+1} = (\sin x)u_n + \cos 2x$, with $u_0 = 1$. (*a*) Why do these sequences have a limit? (*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x.	2 7
Ans	(*a*) *-1 < Sin x < 1 and in the interval, Sin x < 1* (*b*) *x = 0·730*	

2011 P2 Q3	(*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$. Write down the values of u_1 and u_2. (*b*) A second sequence is given by 4, 5, 7, 11, ... It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$. Find the values of p and q. (*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit. (i) Calculate this limit. (ii) Why does the other sequence not have a limit?	1 3 3
Ans	*(a) $u_1 = 8$, $u_2 = -4$ (b) $p = 2$, $q = -3$ (c) (i) limit = 0 (ii) 2 does not lie between -1 and 1*	
2010 P1 Q2	A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$. What is the value of u_2? A 7 B 10 C 13 D 16	2
Ans	*C*	
2010 P1 Q7	A sequence is generated by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 7$, with $u_0 = -2$. What is the limit of this sequence as $n \to \infty$? A $\frac{1}{28}$ B $\frac{28}{5}$ C $\frac{28}{3}$ D 28	2
Ans	*C*	
2009 P1 Q1	A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$. What is the value of u_3? A 34 B 21 C 18 D 13	2

Ans	*A*	
2009 P1 Q6	A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}7u_n + 10$. What is the limit of this sequence as $n \to \infty$? A $\frac{100}{3}$ B $\frac{100}{7}$ C $\frac{17}{100}$ D $\frac{3}{10}$	2
Ans	*A*	
2008 P1 Q4	A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$. What is the limit of this sequence as $n \to \infty$? A -800 B -400 C 200 D 400	2
Ans	*B*	
2008 P1 Q1	A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}3u_n + 6$ with $u_{10} = 10$. What is the value of u_{12}? A $6{\cdot}6$ B $7{\cdot}8$ C $8{\cdot}7$ D $9{\cdot}6$	2
Ans	*C*	

2007 P1 Q7	A sequence is defined by the recurrence relation $$u_{n+1} = \frac{1}{4}u_n + 16,\ u_0 = 0.$$ (*a*) Calculate the values of u_1, u_2 and u_3. Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph. As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.  (*b*) (i) Give a reason why this sequence has a limit. (ii) Find the exact value of k.	3 3
Ans	(*a*) $u_1 = 16$ $u_2 = 20$ $u_3 = 21$ (*b*) (i) $-1 < \frac{1}{4} < 1$ (ii) $\frac{64}{3}$	
2006 P1 Q4	A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$. (*a*) State why this sequence has a limit. (*b*) Find this limit.	 1 2
Ans	(*a*) sequence has limit since $-1 < 0{\cdot}8 < 1$ (*b*) limit = 60	
2005 P1 Q6	(*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. (*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$. (i) Express u_1 and u_2 in terms of m. (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit.	2 5
Ans	(*a*) $k = -\frac{1}{4}$ (*b*) (i) $u_1 = 3m + 5$, $u_2 = m(3m + 5) + 5$ (ii) $m = -2$	
2004 P2 Q4	A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$. (*a*) Write down the condition on k for this sequence to have a limit. (*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k.	 1 3

Ans	(*a*) $-1 < k < 1$ (*b*) $k = \frac{2}{5}$	
2003 P1 Q4	A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$. (*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. (*b*) Find the limit of this recurrence relation as $n \to \infty$.	 2 2
Ans	(*a*) $p = \frac{1}{3}$, $q = 11$ (*b*) 16½	
2002W P2 Q3	(*a*) Calculate the limit as $n \to \infty$ of the sequence defined by $u_{n+1} = 0{\cdot}9u_n + 10$, $u_0 = 1$. (*b*) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.	3 2
Ans	(*a*) 100 (*b*) $n = 7$, $u_7 = 52{\cdot}65$	
2002 P2 Q4	A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year. (*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? (*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition?	 3 3
Ans	(*a*) $-1 < 0{\cdot}8 < 1$ and limit = 2·5 metres (*b*) trim 25%	

2001 P2 Q3	On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan. (*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made. Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. (*b*) Find the date and the amount of the final payment.	2 4
Ans	(*a*) $u_{n+1} = 1{\cdot}015u_n - 300,\ u_0 = 2500$ (*b*) Dec 1st, £290·68	
2000 P1		4 1
Ans	$a = \frac{3}{5},\ L = 25$	
Specimen 2 P2 Q4	Two sequences are defined by the recurrence relations $u_{n+1} = 0{\cdot}2u_n + p,\quad u_0 = 1$ and $v_{n+1} = 0{\cdot}6v_n + q,\quad v_0 = 1.$ (*a*) Explain why each of these sequences has a limit. (*b*) If both sequences have the same limit, express p in terms of q.	3
Ans	(*a*) $-1 < 0{\cdot}2 < 1$ and $-1 < 0{\cdot}6 < 1$ (*b*) Limit $= \dfrac{p}{0{\cdot}8}$ and Limit $= \dfrac{q}{0{\cdot}4} \Rightarrow p = 2q$	
Specimen 1 P1 Q2	A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}3u_n + 5$ with first term u_1. (*a*) Explain why this sequence has a limit as n tends to infinity. (*b*) Find the **exact** value of this limit.	1 2

Ans	(*a*) $-1 < 0{\cdot}3 < 1$ (*b*) $\frac{50}{7}$	
Specimen 1 P2 Q2	Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees. A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees. Which pesticide will be more effective in the long term?	5
Ans	Pestkill	