# Higher : Quadratic Theory

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|---|---|---|
| 2018 P2 Q4 | Express $-3x^2-6x+7$ in the form $a(x+b)^2+c$. | 3 |
| *Ans* | $-3(x+1)^2+10$ | |
| 2018 P2 Q10 | The equation $x^2+(m-3)x+m=0$ has two real and distinct roots.<br><br>Determine the range of values for $m$. | 4 |
| *Ans* | $m<1$ , $m>9$ | |
| 2017 P1 Q4 | Find the value of $k$ for which the equation $x^2+4x+(k-5)=0$ has equal roots. | 3 |
| *Ans* | $k=9$ | |
| 2017 P2 Q4(a) | Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. | 3 |
| *Ans* | $3(x+4)^2+2$ | |
| 2016 P1 Q12b | The functions $f$ and $g$ are defined on $\mathbb{R}$, the set of real numbers by<br><br>$f(x)=2x^2-4x+5$ and $g(x)=3-x$.<br><br>(a) Given $h(x)=f(g(x))$, show that $h(x)=2x^2-8x+11$. | 2 |
| | (b) Express $h(x)$ in the form $p(x+q)^2+r$. | 3 |
| *Ans* | (a) Proof    (b) $2(x-2)^2+3$ | |
| 2016 P2 Q2 | Find the range of values for $p$ such that $x^2-2x+3-p=0$ has no real roots. | 3 |
| *Ans* | $p<2$ | |

ABCD is a rectangle with sides of lengths $x$ centimetres and $(x-2)$ centimetres, as shown.



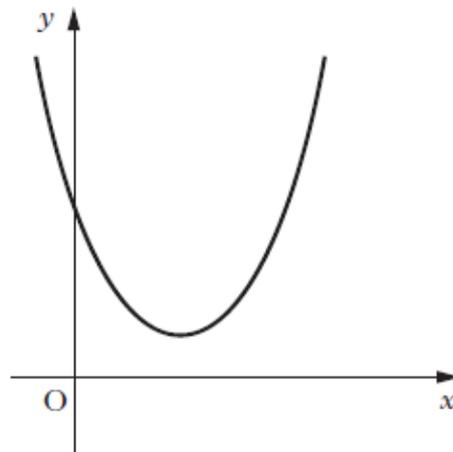A ————— $x$ cm ————— B

$(x-2)$ cm

D ————————————— C

4
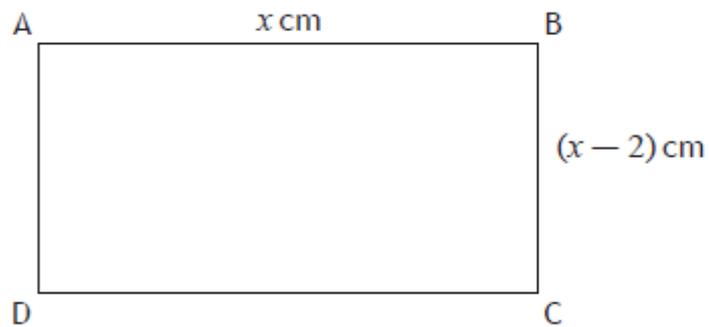
If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of $x$.

| Ans | $2 < x < 5$ |
|---|---|

The diagram shows a curve with equation of the form $y = ax^2 + bx + c$.



2

Here are two statements about $a$, $b$ and $c$:

(1)    $a > 0$

(2)    $b^2 - 4ac > 0$

Which of the following is true?

A    Neither statement is correct.

B    Only statement (1) is correct.

C    Only statement (2) is correct.

D    Both statements are correct.

| Ans | B |
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| *2015 P1 Q24* | Find the range of values for $k$ such that $kx^2 + 3x + 9k = 0$ has real roots. | 4 |
| *Ans* | $-\dfrac{1}{2} \leq k \leq \dfrac{1}{2}$ | |
| *2014 P1 Q17* | $3x^2 + 12x + 17$ is expressed in the form $3(x + p)^2 + q$.<br><br>What is the value of $q$?<br><br>A     1<br><br>B     5<br><br>C    17<br><br>D   &minus;19 | 2 |
| *Ans* | B | |
| *2014 P2 Q3* | Functions $f$ and $g$ are defined on suitable domains by<br><br>$$f(x) = x(x - 1) + q \text{ and } g(x) = x + 3.$$<br><br>(a)  Find an expression for $f(g(x))$.<br><br>(b)  Hence, find the value of $q$ such that the equation $f(g(x)) = 0$ has equal roots. | 2<br><br><br>4 |
| *Ans* | (a) $(x + 3)(x + 2) + q$ *or equivalent*    (b) $q = \dfrac{1}{4}$ | |
| *2013 P1 Q3* | Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.<br><br>A  &minus;32<br><br>B  &minus;24<br><br>C    48<br><br>D    56 | 2 |
| *Ans* | B | |

Solve $1 - 2x - 3x^2 > 0$, where $x$ is a real number.

A    $x < -1$ or $x > \dfrac{1}{3}$

B    $-1 < x < \dfrac{1}{3}$

C    $x < -\dfrac{1}{3}$ or $x > 1$

D    $-\dfrac{1}{3} < x < 1$

2

Ans   B

Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$.

3

Ans   $2(x + 3)^2 - 17$

If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of $q$?

A   $-22$

B   $5$

C   $14$

D   $50$

2

Ans   B

A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.
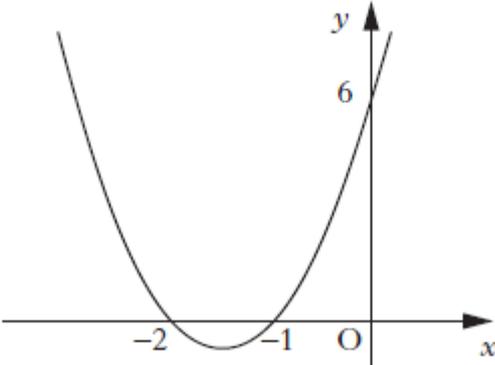


What is the equation of the parabola?

A   $y = 6(x - 1)(x - 2)$

B   $y = 6(x + 1)(x + 2)$

C   $y = 3(x - 1)(x - 2)$

D   $y = 3(x + 1)(x + 2)$

2

| Ans | D |

Solve $6 - x - x^2 < 0$.

A   $-3 < x < 2$

B   $x < -3, x > 2$

C   $-2 < x < 3$

D   $x < -2, x > 3$

2

| Ans | B |

A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.

(a)   (i)   Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii)   Show that $\cos \hat{ABC} = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$.

7

(b)   If angle $ABC = 30°$, find the possible values of $k$.

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| Ans | (a)(i) $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$    (ii) *Proof*    (b) $k = -2$ , $k = 4$ |

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| *2011 P1 Q5* | If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of $q$?<br><br>A  −9<br><br>B  −1<br><br>C  7<br><br>D  23 | 2 |
| *Ans* | A | |
| *2011 P1 Q9* | The discriminant of a quadratic equation is 23.<br><br>Here are two statements about this quadratic equation:<br><br>(1) the roots are real;<br><br>(2) the roots are rational.<br><br>Which of the following is true?<br><br>A  Neither statement is correct.<br><br>B  Only statement (1) is correct.<br><br>C  Only statement (2) is correct.<br><br>D  Both statements are correct. | 2 |
| *Ans* | B | |
| *2011 P1 Q18* | If $f(x) = (x - 3)(x + 5)$, for what values of $x$ is the graph of $y = f(x)$ above the $x$-axis?<br><br>A  $-5 < x < 3$<br><br>B  $-3 < x < 5$<br><br>C  $x < -5, x > 3$<br><br>D  $x < -3, x > 5$ | 2 |
| *Ans* | C | |
| *2010 P1 Q5* | When $x^2 + 8x + 3$ is written in the form $(x + p)^2 + q$, what is the value of $q$?<br><br>A  −19<br><br>B  −13<br><br>C  −5<br><br>D  19 | 2 |
| *Ans* | B | |

The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of $k$?

A    $-\dfrac{9}{8}$

B    $-\dfrac{8}{9}$

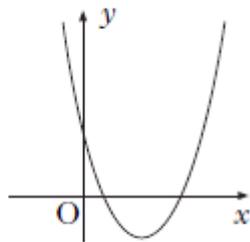C    $\dfrac{8}{9}$

D    $\dfrac{9}{8}$

2

Ans    D

Which of the following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where
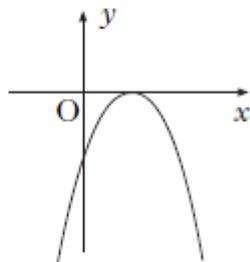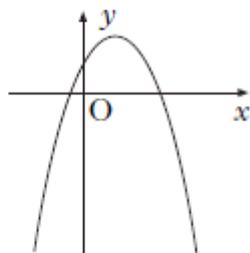
- $a > 0$
- $b^2 - 4ac > 0$?

A



B



C



D



2
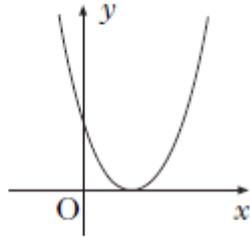
| Ans | B |
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What is the solution of $x^2 + 4x > 0$, where $x$ is a real number?

A   $-4 < x < 0$

B   $x < -4, x > 0$

C   $0 < x < 4$

D   $x < 0, x > 4$

2

| Ans | B |
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| *2009 P1 Q12* | A function $f$ is given by $f(x) = 2x^2 - x - 9$.<br><br>Which of the following describes the nature of the roots of $f(x) = 0$?<br><br>A   No real roots<br><br>B   Equal roots<br><br>C   Real distinct roots<br><br>D   Rational distinct roots | 2 |
| *Ans* | C | |
| *2009 P1 Q19* | For what values of $x$ is $6 + x - x^2 < 0$?<br><br>A   $x > 3$ only<br><br>B   $x < -2$ only<br><br>C   $x < -2, x > 3$<br><br>D   $-3 < x < 2$ | 2 |
| *Ans* | C | |
| *2008 P1 Q10* | Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:<br><br>(1)   the roots are equal;<br><br>(2)   the roots are real.<br><br>Which of the following is true?<br><br>A   Neither statement is correct.<br><br>B   Only statement (1) is correct.<br><br>C   Only statement (2) is correct.<br><br>D   Both statements are correct. | 2 |
| *Ans* | A | |

The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.
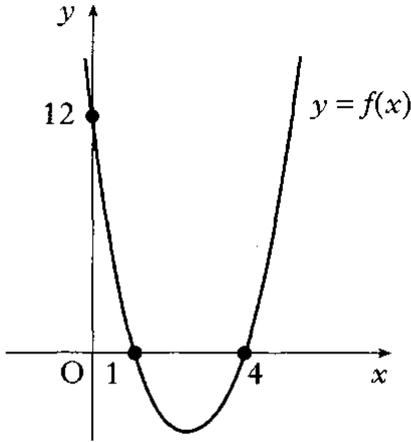


What is the equation of the graph?

A $\quad y = 3(x - 1)(x - 4)$

B $\quad y = 3(x + 1)(x + 4)$

C $\quad y = 12(x - 1)(x - 4)$

D $\quad y = 12(x + 1)(x + 4)$

2

| Ans | A |
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$2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of $q$?

A $\quad$ 5

B $\quad$ 7

C $\quad$ 9

D $\quad$ 11

2

| Ans | A |
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Find the range of values of $k$ such that the equation $kx^2 - x - 1 = 0$ has no real roots.

4

| Ans | $k < -\frac{1}{4}$ |
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(a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$.

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(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.

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| *Ans* | (a) $2(x+1)^2 - 5$ <br><br> (b) $(-1, -5)$ | |
| *2006 P2 Q2* | Find the value of $k$ such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. | 4 |
| *Ans* | $k = 24$ | |
| *2006 P2 Q3* | The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5). <br><br> (a) Find the equation of this tangent. <br><br><br><br><br><br><br><br> (b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. | 4 <br><br><br><br><br><br><br><br><br><br><br><br><br><br> 5 |
| *Ans* | (a) $y - 5 = 2(x - 8)$ <br><br> (b) Q = (4, –3) | |
| *2004 P1 Q8* | (a) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. <br><br> (b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. | 2 <br><br> 4 |
| *Ans* | (a) $(x - 5)^2 + 2$ <br><br> (b) $g'(x) = x^2 - 10x + 27$ <br> $\quad\quad\quad = (x - 5)^2 + 2$ <br> $g'(x) > 0$ for all $x$ and so $g(x)$ is always increasing. | |

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| *2004 P1 Q11* | The diagram shows a parabola passing through the points $(0, 0)$, $(1, -6)$ and $(2, 0)$.<br><br>(*a*) The equation of the parabola is of the form $y = ax(x - b)$.<br>Find the values of $a$ and $b$.  | 3 |
| *Ans* | (*a*) $a = 6$, $b = 2$<br><br>(*b*) $f(x) = 2x^3 - 6x^2 + 8$ | |
| *2004 P2 Q3* | Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of $p$. | 4 |
| *Ans* | $b^2 - 4ac = p^2 - 4 \times 2 \times (-3)$<br>$\qquad\qquad = p^2 + 24$<br>$p^2$ is positive, so $b^2 - 4ac$ is positive too<br>and roots are real. | |
| *2003 P1 Q2* | (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$.<br>(*b*) Hence or otherwise sketch the graph of $y = f(x)$. | 2<br>2 |
| *Ans* | (*a*) $(x + 3)^2 + 2$<br><br>(*b*)<br> | |
| *2003 P1 Q7* | Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. | 5 |
| *Ans* | $x^2 + 3x + 4 = 2x + 1$<br>$x^2 + x + 3 = 0$<br>$b^2 - 4ac = -11$<br>$b^2 - 4ac < 0$ therefore no intersection | |

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| *2002W P2 Q6* | The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown.<br><br>Find the value of $a$ correct to one decimal place. |  | 3 |
| *Ans* | Evaluate $f(0\cdot1)$ and $f(0\cdot5)$, for example, to start with<br><br>$a = 0\cdot2$ | |
| *2002 P1 Q7* | (a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. | 2 |
| *Ans* | (a) $f(x) = (x-2) + 1$ | |
| *2002 P2 Q9* | Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of $k$. | 5 |
| *Ans* | discriminant $= (-5k)^2 - 4(1 - 2k)(-2k)$<br>$\qquad\qquad = 9k^2 + 8k$<br><br>for real roots, discriminant $\geq 0$<br><br>ie $9k^2 + 8k \geq 0$<br>$\qquad k(9k + 8) \geq 0$<br>$\qquad k \geq 0$ or $k \leq \frac{-8}{9}$<br><br>no integers between 0 and $\frac{-8}{9}$<br><br>hence no integral values of $k$<br><br>give non-real roots | |
| *2001 P1 Q2* | For what value of $k$ does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? | 3 |
| *Ans* | $k = \dfrac{1}{4}$ | |
| *2001 P1 Q4* | Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. | 2 |
| *Ans* | $(x + 1)^2 - 9$ | |

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| *2001 P2 Q11* | The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.<br><br>(a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$.<br><br>(b) For what value of $p$ will the line $y = x + p$ be a tangent to this curve? |  |
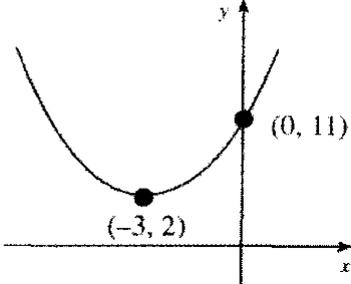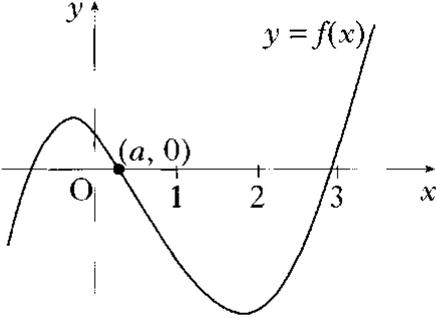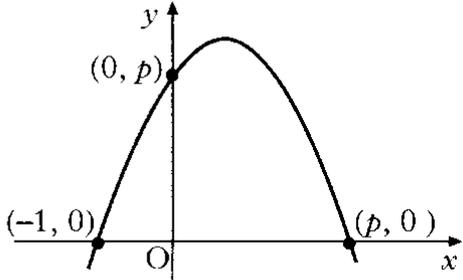
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| *Ans* | (a) $y = k(x + 1)(x - p)$<br>$k = -1$ with justification<br>    ie substitute $(0, p)$<br>$y = -1(x + 1)(x - p)$ and complete<br><br>(b) 2 |

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| *2000 P2 Q4* | The parabola shown crosses the $x$-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.<br><br>The shaded area is bounded by the parabola, the $x$-axis and the lines $x = 2$ and $x = k$.<br><br>(a) Find the equation of the parabola. |  |
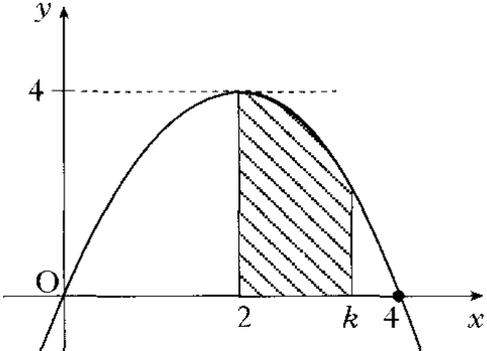
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| *Ans* | (a) $y = 4x - x^2$ |

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| *Specimen 2 P2 Q1* | The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the $x$-axis at the origin and P.<br><br>The line PQ has equation $2y + x = 4$.<br><br>Find the coordinates of P and Q. |  |
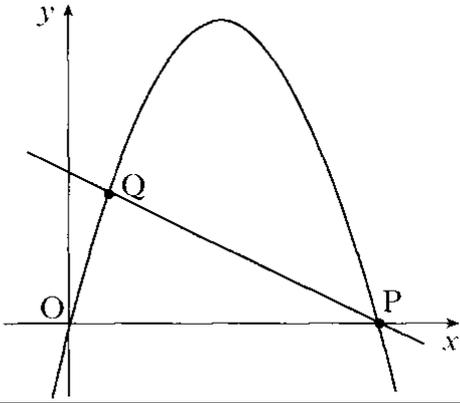
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| *Ans* | $4x - x^2 = 2 - \frac{1}{2}x \Rightarrow 2x^2 - 9x + 4 = 0$<br>$x = \frac{1}{2}, x = 4 \Rightarrow P = (4, 0), \ Q = \left(\frac{1}{2}, \frac{7}{4}\right)$ |

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| *Specimen 1 P2 Q3* | (a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where $a$, $b$ and $c$ are constants.<br><br>(b) Hence, or otherwise, find the coordinates of the turning point of the function $f$. | 3<br><br>1 |

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|---|---|---|
| *Ans* | (a)  $2(x + 2)^2 - 11$ <br><br> (b)  $(-2, -11)$ | |
| *Specimen 1 P2 Q8* | The roots of the equation $(x - 1)(x + k) = -4$ are equal. <br> Find the values of $k$. | 5 |
| *Ans* | $-5, 3$ | |