Higher : Polynomials
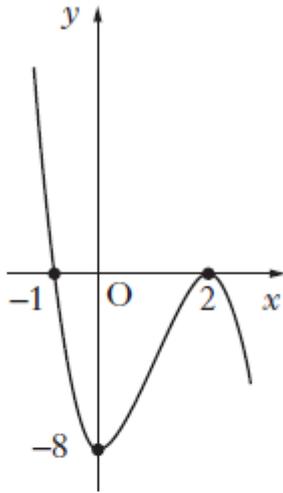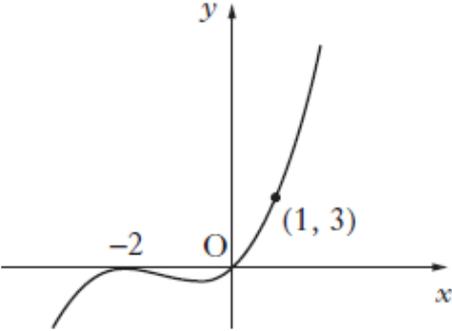
A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$

- $x = 2$ is a repeated root of $f(x)$

- $f'(-2) = 0$

- $f'(x) > 0$ where the graph with equation $y = f(x)$ crosses the y-axis

Sketch a possible graph of $y = f(x)$ on the diagram in your answer booklet.

4

Ans

(i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$.

2

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully.

2

Ans *(i) Proof   (ii) $(x-2)(2x-1)(x+1)$*

The diagram below shows the graph with equation $y = f(x)$, where

$$f(x) = k(x-a)(x-b)^2.$$

(a) Find the values of a, b and k.

3

(b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.

1

Ans (a) a = 4 , b = -5 , k = -1/12   (b) d > 9

2

(i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$.

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$.

3

Ans (i) Proof   (ii) x = -1, 2, 3·5

Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully.

4

Ans (x + 3)(x − 4)(x − 2)

2015 P1 Q3	If $2x^3 + x^2 - 4x + 1$ is divided by $(x - 2)$, what is the remainder? A $\quad$ −11 B $\quad$ 0 C $\quad$ 1 D $\quad$ 13	2
Ans	D	

The graph of a cubic function, $y = f(x)$, is shown below.

It passes through the points $(-2, 0)$, $(-1, 0)$, $(3, 0)$ and $(0, -3)$.
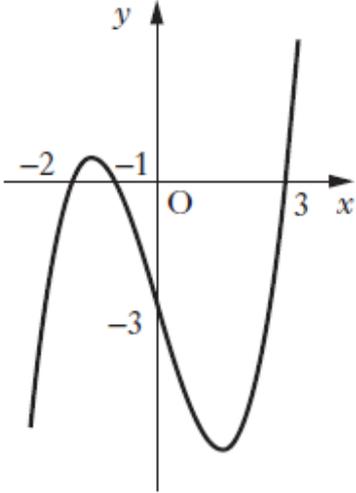
What is the equation of this curve?

A $\quad y = \frac{1}{2}(x - 3)(x + 1)(x + 2)$

B $\quad y = 2(x - 3)(x + 1)(x + 2)$

C $\quad y = -\frac{1}{2}(x + 3)(x - 1)(x - 2)$

D $\quad y = -2(x + 3)(x - 1)(x - 2)$

2

Ans	A	
2015 P1 Q21a	Show that $(x - 1)$ is a factor of $x^3 - 6x^2 + 9x - 4$ and hence factorise $x^3 - 6x^2 + 9x - 4$ fully.	4
Ans	$(x - 1)^2(x - 4)$	

The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).

What is the equation of the curve?

A $\quad y = -2(x+1)^2(x+2)$

B $\quad y = -2(x+1)(x-2)^2$

C $\quad y = 4(x+1)(x-2)$

D $\quad y = -8(x+1)(x-2)^2$

2

Ans	B

For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(a) Determine the values of a and b.

4

(b) Hence factorise the polynomial completely.

3

Ans	*(a) a = -1 , b = -2 (b) (x + 1)(3x + 2)(2x – 1)*

What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

A　0

B　3

C　4

D　8

2

Ans　C

The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.

What are the values of a and k?

	a	k
A	−2	$\frac{1}{3}$
B	−2	3
C	2	$\frac{1}{3}$
D	2	3

2

Ans　C

(a)　Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.

4

Ans　*(x − 1) (x² + 4x + 5) with valid reason for quadratic factor.*

(a) (i) Show that $(x-4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

6

Ans | (a)(ii) $(x-4)(x-2)(x+1)$ (iii) 4, 2, -1

A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x-1)$?

A   0

B   2

C   3

D   4

2

Ans | D

The diagram shows the graph of a cubic.
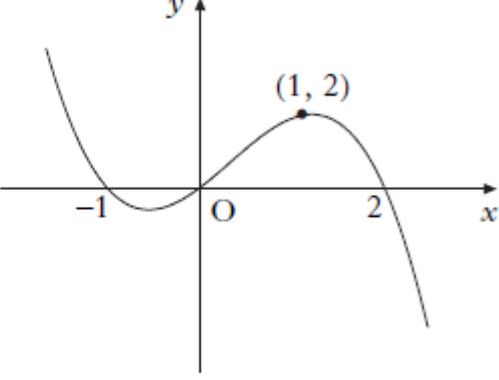
What is the equation of this cubic?

A $y = -x(x+1)(x-2)$

B $y = -x(x-1)(x+2)$

C $y = x(x+1)(x-2)$

D $y = x(x-1)(x+2)$

2

Ans | A

Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(a) Find $g(f(x))$.

2

1

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.

(c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

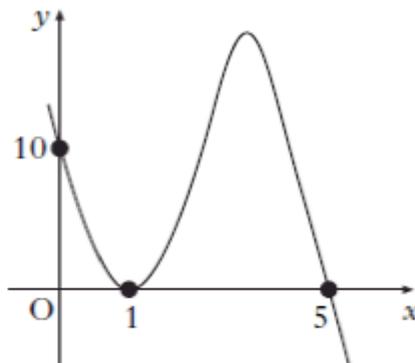(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.

5

(d) Hence solve $g(f(x)) + xh(x) = 0$.

1

Ans (a) $3x^3 - 2$ (b) $3x^3 + 4x^2 - 5x - 2$ (c)(i) Proof (ii) $(x - 1)(3x + 1)(x + 2)$ (d) $x = 1$, $-\frac{1}{3}$, -2

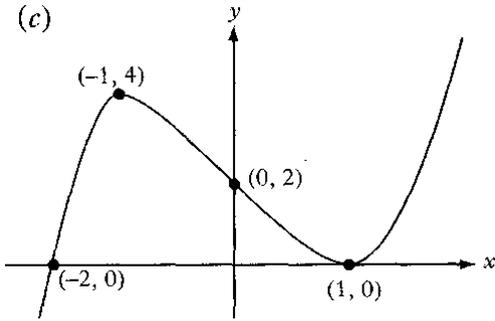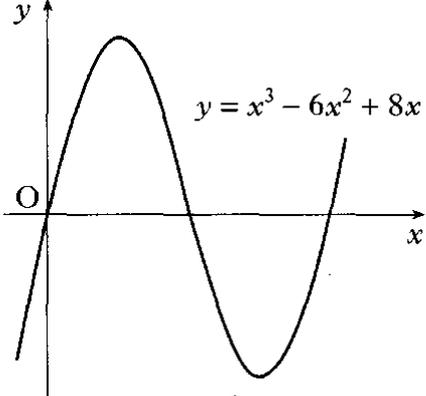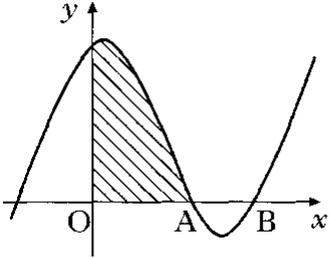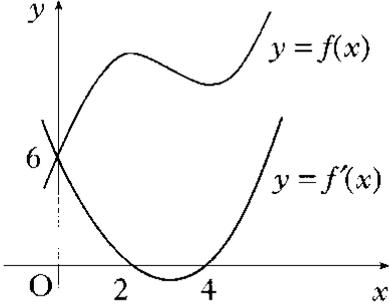The diagram shows the graph with equation $y = k(x - 1)^2(x + t)$.

What are the values of k and t?

	k	t
A	-2	-5
B	-2	5
C	2	-5
D	2	5

2

Ans A

2010 P1 Q22	(*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$. (ii) Hence factorise $f(x)$ fully. (*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. (*c*) The line with equation $y = 2x-3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G. Find the coordinates of G. (*d*) This tangent meets the curve again at the point H. Write down the coordinates of H.	5 1 5 1
Ans	(*a*) (ii) $(x-1)(x-1)(2x+5)$ (*b*) $x = 1$, $x = -2.5$	
2009 P2 Q3a	(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$. (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.	4
Ans	$(x-1)(x+4)(x+5)$	
2008 P1 Q21	A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (*b*) (i) Show that $(x-1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4

Ans	(*a*) (−1, 4) maximum
	     (1, 0) minimum
	
	(*b*) (i) $x = 1$, $f(x) = 0$
	        so $(x-1)$ is a factor
	     (ii) $(x-1)(x-1)(x+2)$
		(*c*)

2008 P1 Q22	The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$. (a) Find the coordinates of the points on the curve where the gradient of the tangent is -1. (b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A.	
		5
		2
Ans	(a) (1,3), (3,-3) (b) (1,3)	
2007 P1 Q8	The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. (a) Show that the graph cuts the x-axis at $(3, 0)$. (b) Hence or otherwise find the coordinates of A.	
		1
		3
Ans	(a) To cut the x-axis, $y = 0$. So $\quad 0 = x^3 - 4x^2 + x + 6$ $\quad\quad = (x - 3)(x^2 - x - 2)$ $\quad\quad = (x - 3)(x - 2)(x + 1)$ So graph cuts x-axis at $x = -1, 3, 2$. (b) (2,0)	
2007 P2 Q10	The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point $(0, 6)$. The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$. (a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$: (i) write down the values of a and b; (ii) find the value of k.	
		3
Ans	(a) (i) $a = 2$, $b = 4$ (ii) $k = \frac{3}{4}$	

2005 P1 Q8	A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number. (*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. (*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. (*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.	5 2 5
Ans	(*a*)$(x - 3)(2x - 3)(x + 1)$ (*b*)$(-1,0)$, $(-\frac{3}{2}, 0)$, $(3, 0)$ (*c*)greatest value $= 9$ least value $= -35$	
2005 P2 Q11	(*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. (*b*) Hence find the range of values of p for which all the roots of the cubic equation are real.	1 7
Ans	(*a*)$f(-1) = -1 + p - p + 1 = 0$ (*b*)$p \leq -1$, $p \geq 3$	
2004 P1 Q2	$f(x) = x^3 - x^2 - 5x - 3$. (*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$. (ii) Hence or otherwise factorise $f(x)$ fully. (*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.	5 1
Ans	$(x + 1)(x + 1)(x - 3)$ $(-1, 0)$	
2003 P2 Q1	$f(x) = 6x^3 - 5x^2 - 17x + 6$. (*a*) Show that $(x - 2)$ is a factor of $f(x)$. (*b*) Express $f(x)$ in its fully factorised form.	4
Ans	(*b*) $(x - 2)\,(2x + 3)\,(3x - 1)$	
2002W P1 Q5	Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.	5
Ans	$c = -19$, $d = 6$	
2001 P2 Q1	(*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. (*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.	3 2

Ans	(a) $k = -5$ (b) $x = -2, \frac{1}{2}, 1$	
2000 P2 Q1	The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$. (a) Find the equation of the tangent to this curve at the point where $x = 1$. (b) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. $y = x^3 - 3x^2 + 2x$	5 5
Ans	(a) $x + y = 1$ (b) $(-1, -6)$	
Specimen 2 P1 Q1	Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4
Ans	$\begin{array}{r\|rrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$ remainder $= 0 \Rightarrow x = 2$ is a root $2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{1}{2}, -3$	
Specimen 1 P1 Q3	(a) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (b) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.	3 1
Ans	(a) $f(1) = 0$, $(x - 4)$, $(x - 1)$ (b) $(1,0)$, $(4,0)$, $(0, -4)$	

X