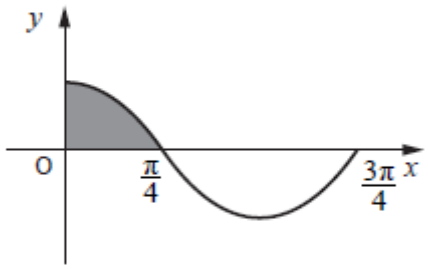
Higher : Further Calculus

2018 P1 Q3	Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$.	3
Ans	$-3\sqrt{3}$	
2017 P1 Q3	Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$.	2
Ans	$48(4x-1)^{11}$	
2017 P2 Q11b	(a) Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. (b) Hence, differentiate $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \frac{\pi}{2}$.	3 3
Ans	*(a) Proof (b)* $3\sin^2 x\cos x$	
2016 P1 Q5	Find $\int 8\cos(4x+1)\,dx$.	2
Ans	$2\sin(4x+1) + c$	
2016 P2 Q10	(a) Given that $y = (x^2+7)^{\frac{1}{2}}$, find $\frac{dy}{dx}$. (b) Hence find $\int \frac{4x}{\sqrt{x^2+7}}\,dx$.	2 1
Ans	*(a)* $x(x^2+7)^{-\frac{1}{2}}$ *(b)* $4(x^2+7)^{\frac{1}{2}} + c$	
2016 P2 Q11b	(a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$. (b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$.	4 2
Ans	*(a) Proof (b)* $f'(x) = 2\sin 2x$ *or* $f'(x) = 4\sin x\cos x$	

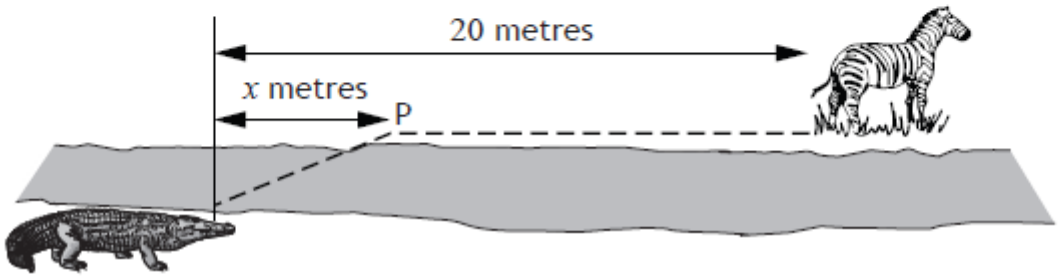
2015 NH P1 Q12	The diagram shows part of the graph of $y = a\cos bx$. The shaded area is $\frac{1}{2}$ unit².  What is the value of $\int_0^{\frac{3\pi}{4}}(a\cos bx)dx$?	2
Ans	$-\frac{1}{2}$	
2015 NH P2 Q7	(a) Find $\int(3\cos 2x+1)dx$. (b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$. (c) Hence, or otherwise, find $\int(\sin^2 x - 2\cos^2 x)dx$.	2 2 2
Ans	*(a)* $\frac{3}{2}\sin 2x + x + c$ *(b) Proof* *(c)* $-\frac{3}{4}\sin 2x - \frac{1}{2}x + c$	

2015 NH P2 Q8

A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land. 1

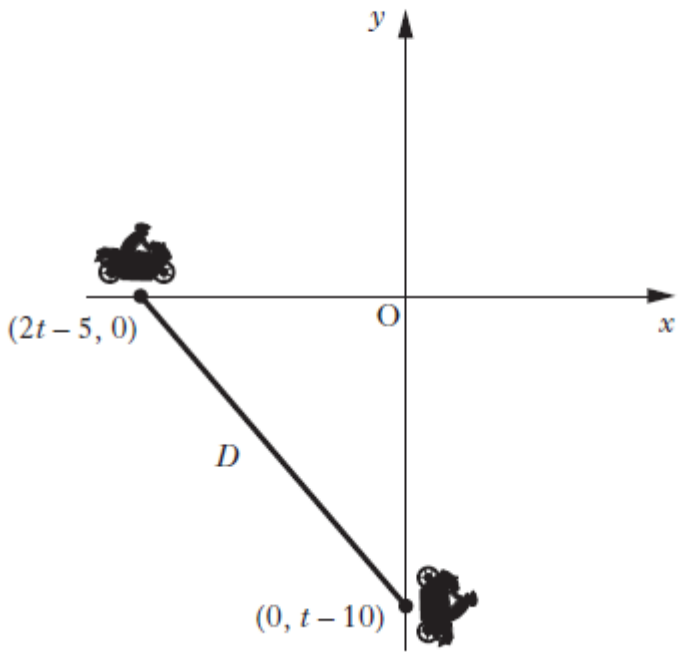
(ii) Calculate the time taken if the crocodile swims the shortest distance possible. 1

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. 8

Ans *(a)(i)10·4 or 104 (ii) 11 or 110 (b) 9·8 or 98 *answer acceptable in seconds or tenths of seconds**

2015 P1 Q25

A stunt performed by two members of a motorcycle display team requires them to travel, at speed, at right angles to each other across the arena.



The positions of the motorcyclists, relative to suitable axes, t seconds after the stunt begins, are $(2t - 5, 0)$ and $(0, t - 10)$.

(*a*) Show that, at any given moment, the distance, D, between them is given by

$$D = \sqrt{5t^2 - 40t + 125}.$$

2

(*b*) Determine whether the distance between the motorcyclists is increasing or decreasing 5 seconds after the start of the stunt.

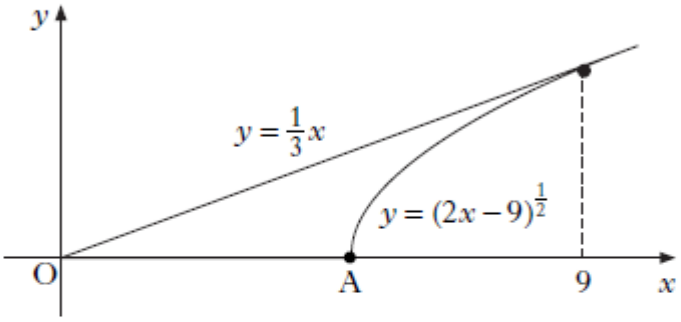
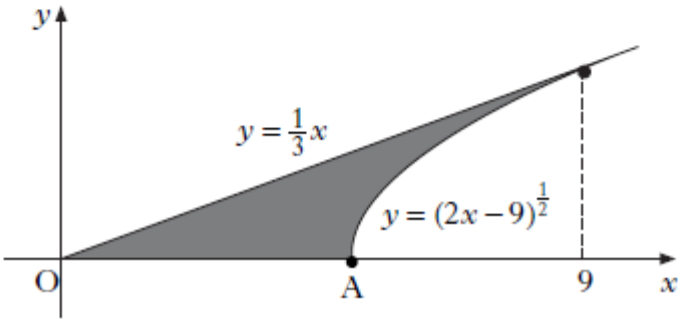
4

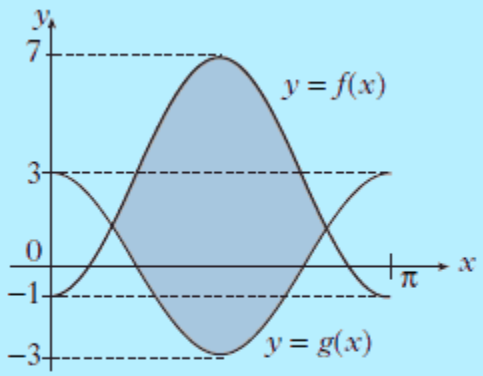
Ans *(a) Clear Proof* *(b)* $D'(5) = \dfrac{10}{2\sqrt{50}} > 0 \Rightarrow$ increasing .

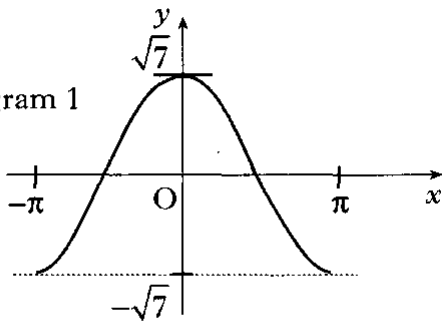
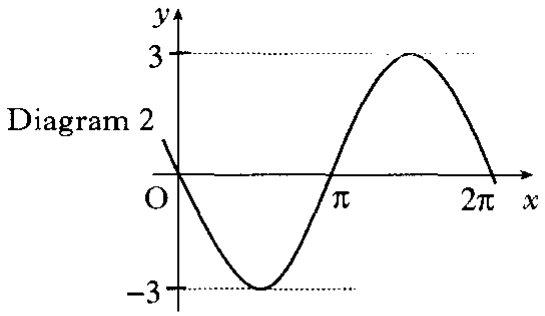
2014 P1 Q5	Find $\int (2x+9)^5\,dx$. A $10\,(2x+9)^4 + c$ B $\frac{1}{4}(2x+9)^4 + c$ C $10\,(2x+9)^6 + c$ D $\frac{1}{12}(2x+9)^6 + c$	2
Ans	*D*	
2014 P1 Q8	What is the derivative of $\left(4-9x^4\right)^{\frac{1}{2}}$? A $-\frac{9}{2}\left(4-9x^4\right)^{-\frac{1}{2}}$ B $\frac{1}{2}\left(4-9x^{-4}\right)^{-\frac{1}{2}}$ C $2\left(4-9x^4\right)^{-\frac{1}{2}}$ D $-18x^3\left(4-9x^4\right)^{-\frac{1}{2}}$	2
Ans	*D*	
2014 P2 Q5	Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t.	5
Ans	*t = 15*	

2014 P2 Q9	Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. (*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. (*c*) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.	3 2 3
Ans	*(a)* $a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$ *(b) Increasing since a(10) > 0* *(c)* $c = 8$ *so* $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	
2013 P1 Q16	Find $\int (1-6x)^{-\frac{1}{2}} dx$ where $x < \frac{1}{6}$. A $\frac{1}{9}(1-6x)^{-\frac{3}{2}} + c$ B $3(1-6x)^{-\frac{3}{2}} + c$ C $-\frac{1}{3}(1-6x)^{\frac{1}{2}} + c$ D $-3(1-6x)^{\frac{1}{2}} + c$	2
Ans	*C*	
2013 P1 Q18	Given that $y = \sin(x^2 - 3)$, find $\frac{dy}{dx}$. A $\sin 2x$ B $\cos 2x$ C $2x \sin(x^2 - 3)$ D $2x \cos(x^2 - 3)$	2
Ans	*C*	

2013 P2 Q6	Given that $\int_0^a 5\sin 3x\ dx = \frac{10}{3}$, $0 \le a < \pi$, calculate the value of a.	5
Ans	*C*	
2012 P1 Q14	Find $\int (2x-1)^{\frac{1}{2}}\ dx$ where $x > \frac{1}{2}$. A $\frac{1}{3}(2x-1)^{\frac{3}{2}}+c$ B $\frac{1}{2}(2x-1)^{-\frac{1}{2}}+c$ C $\frac{1}{2}(2x-1)^{\frac{3}{2}}+c$ D $\frac{1}{3}(2x-1)^{-\frac{1}{2}}+c$	2
Ans	*A*	
2012 P1 Q16	If $y = 3\cos^4 x$, find $\frac{dy}{dx}$. A $12\cos^3 x \sin x$ B $12\cos^3 x$ C $-12\cos^3 x \sin x$ D $-12\sin^3 x$	2
Ans	*C*	
2011 P2 Q6(b)	(*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \le a < 2\pi$. Calculate the values of R and a. (*b*) Hence find the value of t, where $0 \le t \le 2$, for which $\int_0^t (3\cos x + 5\sin x)\ dx = 3.$	4 7
Ans	(*a*) $R = \sqrt{34}$, $a = 5{\cdot}3$ (b) $t = 0{\cdot}7$	

2010 P1 Q9	Find $\int (2x^{-4} + \cos 5x)\,dx$. A $-\frac{2}{5}x^{-5} - 5\sin 5x + c$ B $-\frac{2}{5}x^{-5} + \frac{1}{5}\sin 5x + c$ C $-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$ D $-\frac{2}{3}x^{-3} - 5\sin 5x + c$	2
Ans	*C*	
2010 P2 Q6	(*a*) A curve has equation $y = (2x-9)^{\frac{1}{2}}$. Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$.	5
	(*b*) Diagram 1 shows part of the curve and the tangent. The curve cuts the x-axis at the point A.  *Diagram 1* Find the coordinates of point A.	1
	(*c*) Calculate the shaded area shown in diagram 2.  *Diagram 2*	7
Ans	*(a) y = x/3 (b) A(4·5 , 0) (c) 4·5*	

2009 P1 Q18	Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$. A $-3x(4 - 3x^2)^{-\frac{1}{2}}$ B $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$ C $2(4 - 3x^3)^{\frac{1}{2}}$ D $3x(4 - 3x^2)^{-\frac{3}{2}}$	2
Ans	*D*	
2009 P2 Q5c	The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram. $f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$. (*a*) Write down the values of m and n. (*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. (*c*) Calculate the shaded area. 	1 5 6
Ans	*(a) m = 3 , n = 2 (b) y = 1·3 , 1·3 (c) 12·4*	
2008 P1 Q15	What is the derivative of $(x^3 + 4)^2$? A $(3x^2 + 4)^2$ B $\frac{1}{3}(x^3 + 4)^3$ C $6x^2(x^3 + 4)$ D $2(3x^2 + 4)^{-1}$	2
Ans	*C*	

2008 P2 Q3	(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$. Write down the value of p. Diagram 1  (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$. Write down the value of q. Diagram 2  (*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. (*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression.	2 4 2
Ans	*(a) $p = \sqrt{7}$, $q = -3$ (b) $4\cos(x + 0{\cdot}848)$ (c) $-4\sin(x + 0{\cdot}848)$*	
2007 P1 Q10	Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$.	3
Ans	$\frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \times 6x \ (= 3x(3x^2 + 2)^{-\frac{1}{2}})$	
2007 P2 Q7	Find the value of $\int_0^2 \sin(4x + 1)\,dx$.	4
Ans	0·36	
2006 P1 Q5	A function f is defined by $f(x) = (2x - 1)^5$. Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature.	7
Ans	point of inflexion at $(\frac{1}{2}, 0)$	
2006 P2 Q9	If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$.	4

Ans	$-3x^{-4} + 2\sin 2x$	
2005 P1 Q5	Differentiate $(1 + 2\sin x)^4$ with respect to x.	2
Ans	$4(1 + 2\sin(x))^3$ $\ldots\times 2\cos(x)$	
2004 P1 Q6	Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$.	3
Ans	$3\cos(x) - 2\sin(2x)$	
2004 P1 Q7	Find $\int_0^2 \sqrt{4x+1}\ dx$.	5
Ans	$\frac{13}{3}$	
2003 P1 Q8	Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.	4
Ans	$\frac{2}{3}$	
2003 P2 Q6	If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.	4
Ans	$6 - \sqrt{3}$	
2002W P1 Q10	A function f is defined by $f(x) = 2x + 3 + \frac{18}{x-4}$, $x \neq 4$. Find the values of x for which the function is increasing.	5
Ans	$f'(x) = 2 - \frac{18}{(x-4)^2}$ $f'(x) = 0 \Rightarrow x = 1, 7$ $f'(x) > 0 \Rightarrow x < 1, x > 7$	
2002W P2 Q8	Find $\int_0^1 \left(\cos(3x) - \sin\left(\frac{1}{3}x + 1\right)\right) dx$ correct to 3 decimal places.	3
Ans	$a = -0{\cdot}868$	

2002 P1 Q10	(*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. (*b*) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}\,dx$.	2 1
Ans	(*a*) $f'(x) = -\frac{3}{2}\,x^2(8-x^3)^{-\frac{1}{2}}$ (*b*) $-\frac{2}{3}(8-x^3)^{\frac{1}{2}} + c$	
2002 P2 Q6	Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.	4
Ans	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	
2001 P2 Q10	A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi,\ \sqrt{3}\right)$. Find y in terms of x.	4
Ans	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	
2000 P1 Q8	The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9},\ 1\right)$. If $f'(x) = \sin(3x)$ express y in terms of x.	4
Ans	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	
2000 P2 Q8	Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.	3
Ans		
2000 P2 Q10	Find $\int \frac{1}{(7-3x)^2}\,dx$.	2
Ans	$\frac{1}{3(7-3x)} + c$	
Specimen 1 P1 Q9	Find $\frac{dy}{dx}$ given that $y = \sqrt{1+\cos x}$.	3
Ans	$-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$	