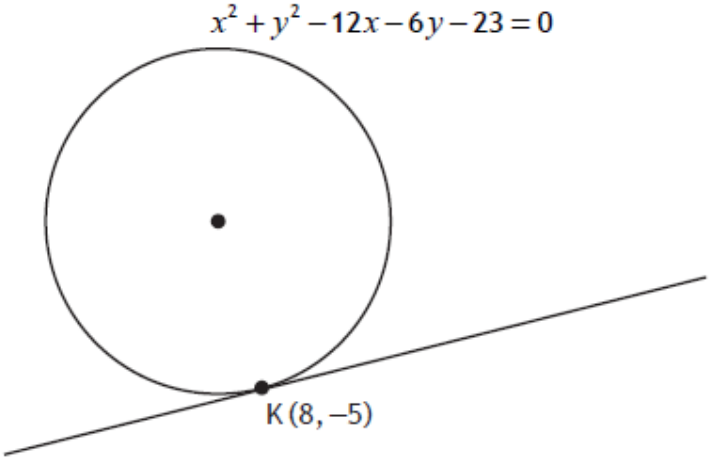
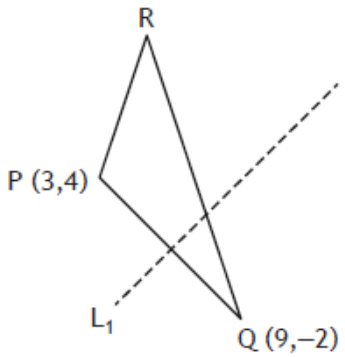
Higher : Circle

2018 P1 Q4	The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.  Find the equation of the tangent to the circle at K.	4
Ans	$y = \frac{1}{4}x - 7$	

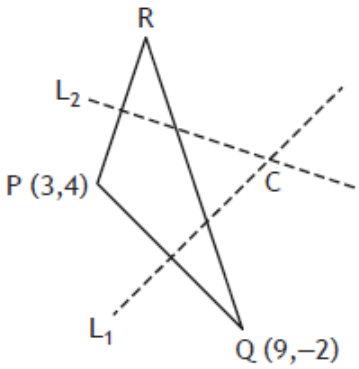
2018 P2 Q5(c)

PQR is a triangle with P(3,4) and Q(9,–2).



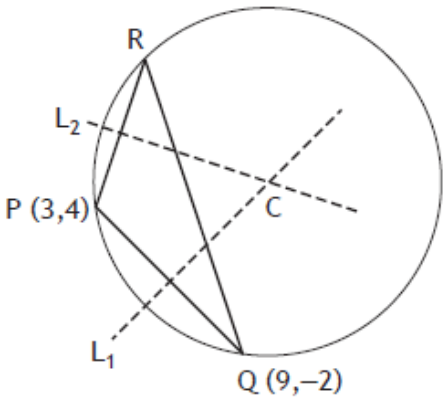
(a) Find the equation of L_1, the perpendicular bisector of PQ. 3

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2. 2

C is the centre of the circle which passes through the vertices of triangle PQR.



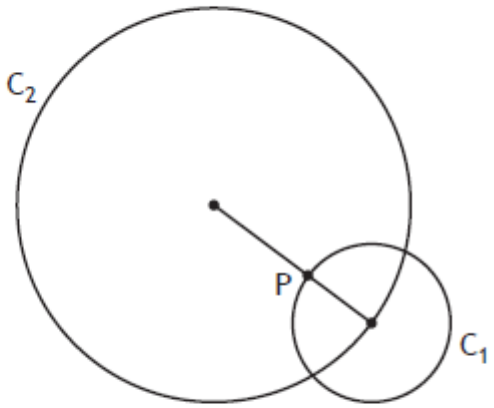
(c) Determine the equation of this circle. 2

Ans (a) $y = x - 5$ (b) C(5,10) (c) $(x - 10)^2 + (y - 5)^2 = 50$

2018 P2 Q12

Circle C_1 has equation $(x-13)^2+(y+4)^2=100$.

Circle C_2 has equation $x^2+y^2+14x-22y+c=0$.



(a) (i) Write down the coordinates of the centre of C_1. 1

(ii) The centre of C_1 lies on the circumference of C_2. 1

Show that $c=-455$.

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. 2

(ii) Hence, or otherwise, determine the coordinates of P. 2

P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. 1

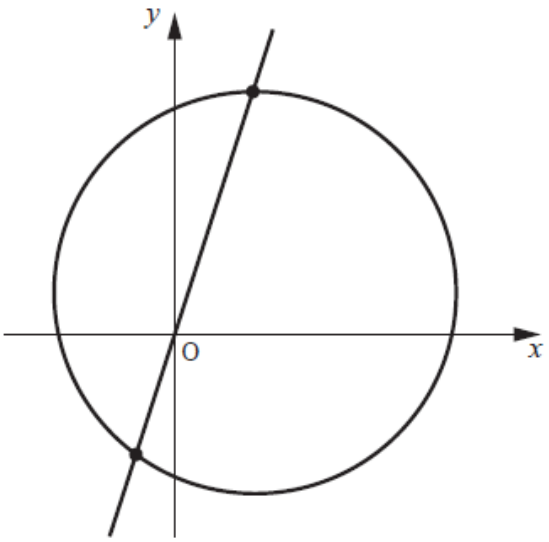
Ans (a)(i) (13,-4) (ii) C = -455 (b)(i) 3:2 or 2:3 (ii) P(5,2) (c) $(x-5)^2+(y-2)^2=1600$

2017 P1 Q2

The point P (–2, 1) lies on the circle $x^2+y^2-8x-6y-15=0$.

Find the equation of the tangent to the circle at P. 4

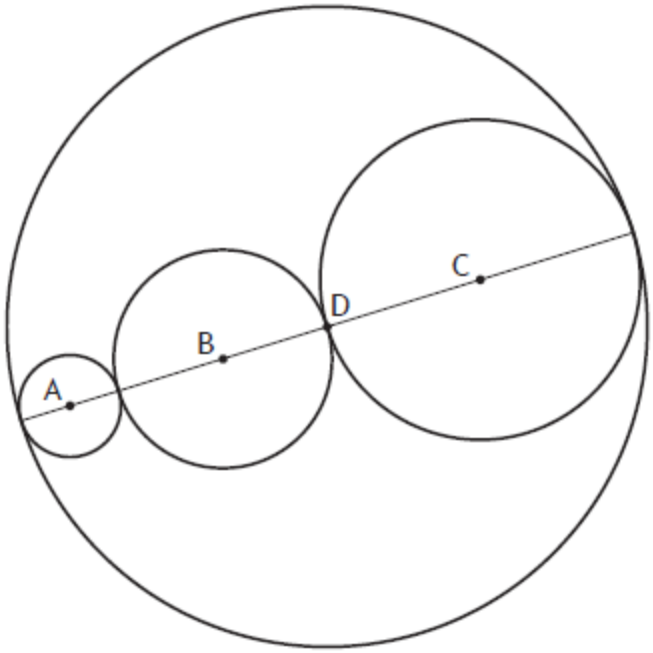
Ans *y = -3x - 5*

2017 P2 Q3	The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.  Find the coordinates of the points of intersection.	5
Ans	*(-1 , -3) and (2 , 6)*	

2017 P2 Q10

(a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear. 3

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.

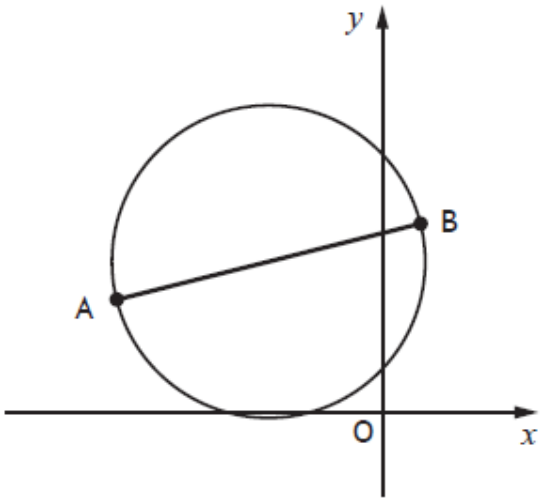


The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

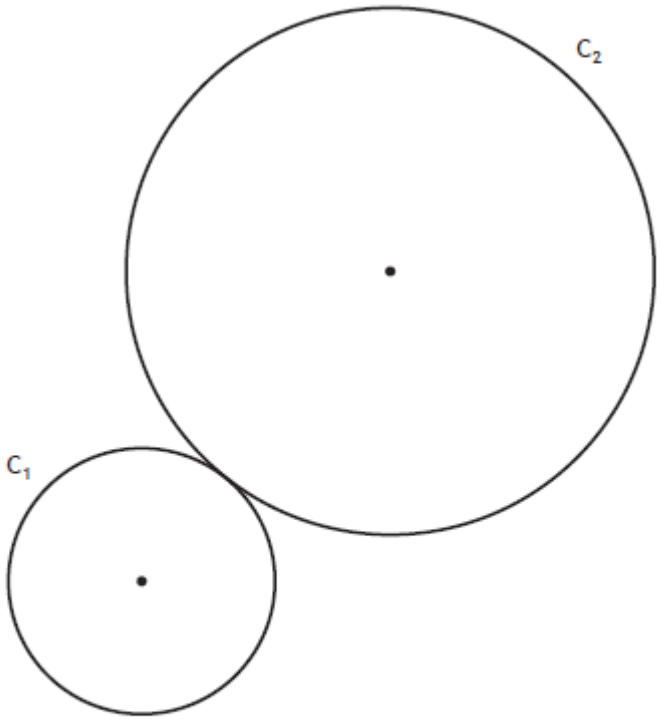
- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

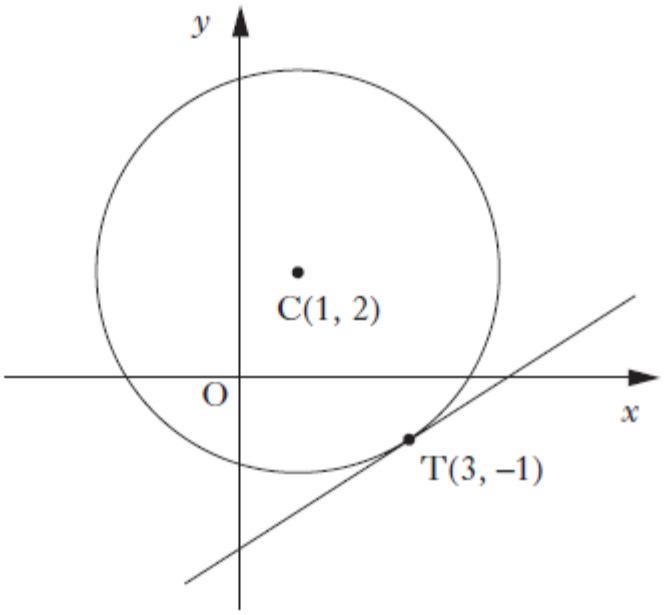
(b) Determine the equation of the circle with centre D. 4

Ans $(x-8)^2 + (y-3)^2 = 360$

2016 P1 Q4	A and B are the points $(-7, 3)$ and $(1, 5)$. AB is a diameter of a circle.  Find the equation of this circle.	3
Ans	$(x + 3)^2 + (y - 4)^2 = 17$	
2016 P1 Q8	Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact.	5
Ans	*Proof and (2, 1)*	
2016 P2 Q4	Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively. (a) Write down the centres and radii of C_1 and C_2. (b) Show that C_1 and C_2 do not intersect.	4 3
Ans	*(a) C_1 (-5, 6) and r = 3 , C_2 (3, 0) and r = 5* *(b) Proof using 'sum of radii' < 'distance between centres' , 8 < 10*	

2015 NH P1 Q11	T(−2, −5) lies on the circumference of the circle with equation $(x+8)^2+(y+2)^2=45$. (a) Find the equation of the tangent to the circle passing through T. (b) This tangent is also a tangent to a parabola with equation $y=-2x^2+px+1-p$, where $p>3$. Determine the value of p.	4 6
Ans	*(a) y = 2x – 1 (b) p = 10*	
2015 NH P1 Q14	The circle with equation $x^2+y^2-12x-10y+k=0$ meets the coordinate axes at exactly three points. What is the value of k?	2
Ans	*k = 25*	

2015 NH P2 Q5	Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$. The centre of circle C_2 is (9, 11). Circles C_1 and C_2 touch externally.  (a) Determine the radius of C_2.	4
	A third circle, C_3, is drawn such that: • both C_1 and C_2 touch C_3 internally • the centres of C_1, C_2 and C_3 are collinear. (b) Determine the equation of C_3.	4
Ans	*(a) 15 (b) $(x-6)^2 + (y-7)^2 = 400$*	
2015 P1 Q18	The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points. What is the value of k? A 5 B 6 C 25 D 36	2
Ans	*C*	

2014 P1 Q2	The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).  What is the gradient of this tangent? A $\frac{1}{4}$ B $\frac{2}{3}$ C $\frac{3}{2}$ D 4	2
Ans	*B*	
2014 P1 Q23	(*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. (*b*) T is the centre of C_1. Show that PT and QT are perpendicular. (*c*) A second circle C_2 passes through P, Q and T. Find the equation of C_2.	4 3 3
Ans	*(a) (1 ,-2) , (3 ,4) (b) Proof (eg.using gradients : 2×-0·5 = -1) (c) $(x - 2)^2 + (y - 1)^2 = 10$*	

2014 P2 Q8	Given that the equation $x^2 + y^2 - 2px - 4py + 3p + 2 = 0$ represents a circle, determine the range of values of p.	5
Ans	$p < -\frac{2}{5},\ p > 1$	
2013 P1 Q22	A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$. (*a*) Write down the centre and calculate the radius of C_1.	2
	(*b*) The point P(3, 2) lies on the circle C_1. Find the equation of the tangent at P.	3
	(*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1. Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$.	3
	(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2.	4
Ans	*(a) (-1, -2) , √32 (b) y = -x +5 (c) Proof (d) Proof*	

2012 P2 Q2

(*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

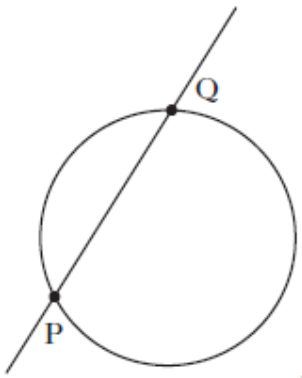


Diagram 1

Find the coordinates of P and Q. 6

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.

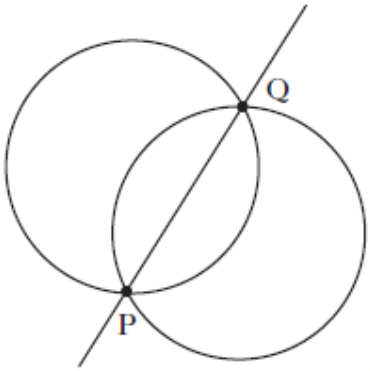


Diagram 2

Determine the equation of this second circle. 6

Ans *(a) P(-3,-1) , Q(1,7) (b) $(x + 5)^2 + (y - 5)^2 = 40$*

2011 P1 Q6

The point P(2, –3) lies on the circle with centre C as shown.

The gradient of CP is –2.

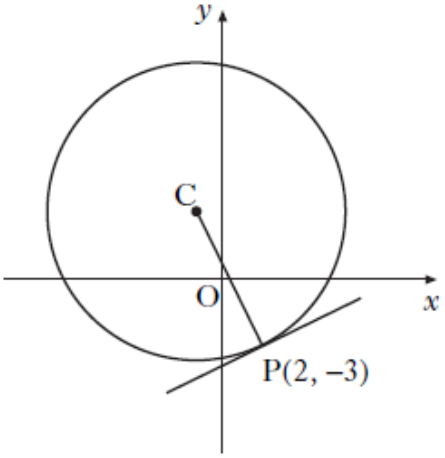
What is the equation of the tangent at P?

A $y + 3 = -2(x - 2)$

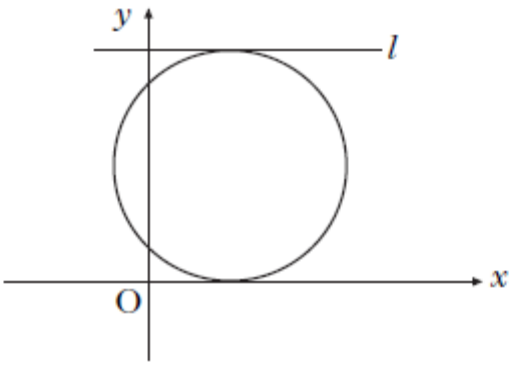
B $y - 3 = -2(x + 2)$

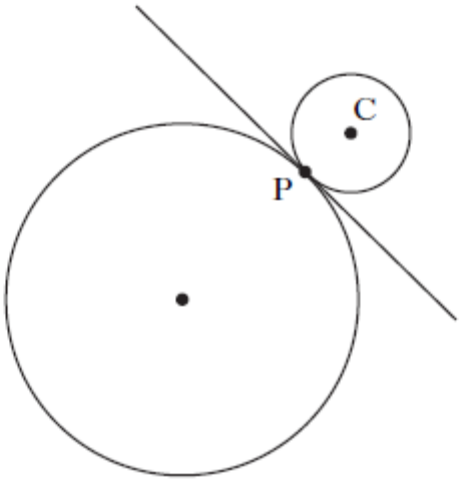
C $y + 3 = \frac{1}{2}(x - 2)$

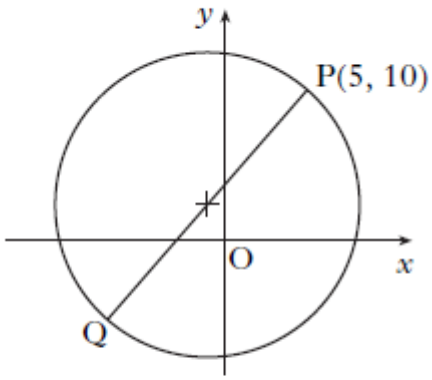
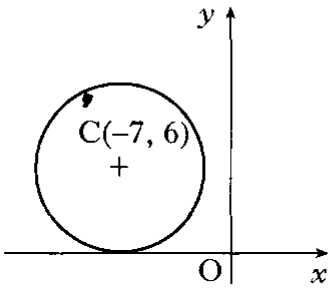
D $y - 3 = \frac{1}{2}(x + 2)$



2

Ans	*C*	
2011 P2 Q7	Circle C_1 has equation $(x+1)^2+(y-1)^2=121$. A circle C_2 with equation $x^2+y^2-4x+6y+p=0$ is drawn inside C_1. The circles have no points of contact. What is the range of values of p?	9
Ans	*-23 < p < 13*	
2010 P1 Q8	The equation of the circle shown in the diagram is $x^2+y^2-6x-10y+9=0$. The x-axis and the line l are parallel tangents to the circle.  What is the equation of line l? A $y=5$ B $y=10$ C $y=18$ D $y=20$	2
Ans	*B*	

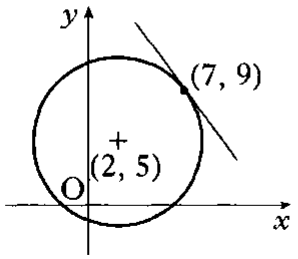
2010 P2 Q3	(*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$. (ii) Find the coordinates of the point of contact, P.	5
	(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.  The line $y = 3 - x$ is a common tangent at the point P. The radius of the larger circle is three times the radius of the smaller circle. Find the equation of the smaller circle.	6
Ans	*(a) P(-1 , 4) (b) $(x - 1)^2 + (y - 6)^2 = 8$*	
2009 P1 Q2	A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$. What is the radius of this circle? A 5 B 10 C $\sqrt{75}$ D $\sqrt{175}$	2
Ans	*B*	
2009 P1 Q9	The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K. What are the x-coordinates of J and K? A $x_J = 1, x_K = -1$ B $x_J = 2, x_K = -2$ C $x_J = 1, x_K = -2$ D $x_J = -1, x_K = 2$	2

Ans	*A*	
2009 P2 Q4	(*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x+1)^2 + (y-2)^2 = 100$.	1
	(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. 	5
	(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q. The radius of each of these circles is twice the radius of circle C_1. Find the equations of circles C_2 and C_3.	4
Ans	*(a) Proof (b) $4y = -3x - 45$ (c) $(x-5)^2 + (y-10)^2 = 400$ and $(x+19)^2 + (y+22)^2 = 400$*	
2008 P1 Q2	The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.  What is the equation of the circle? A $(x+7)^2 + (y-6)^2 = 1$ B $(x+7)^2 + (y-6)^2 = 49$ C $(x-7)^2 + (y+6)^2 = 36$ D $(x+7)^2 + (y-6)^2 = 36$	2
Ans	D	

2008 P1 Q5

The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

2

Ans A

2008 P2 Q4

(*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. 4

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. 5

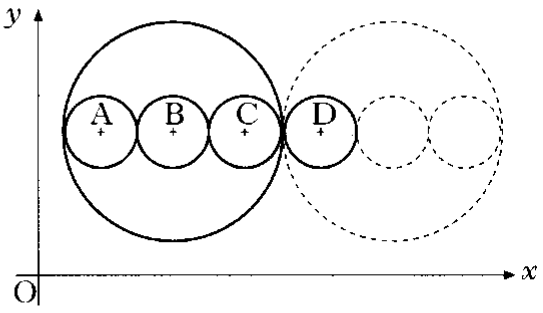
Ans (a) (-4,-2), $\sqrt{58}$ (b) d = $\sqrt{128}$ (c) (3,1), (-1,5)

2007 P1 Q5

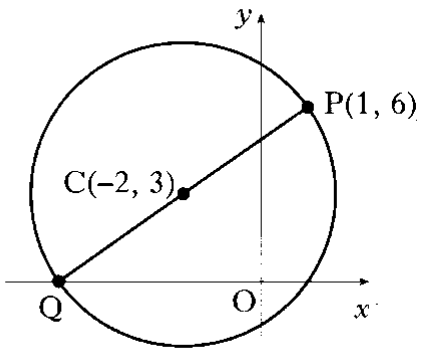
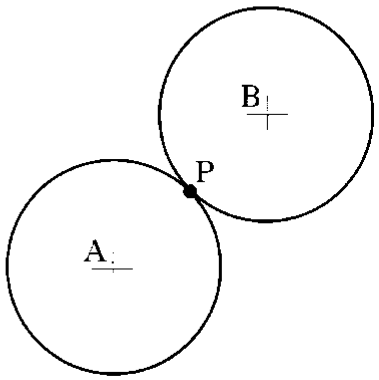
The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

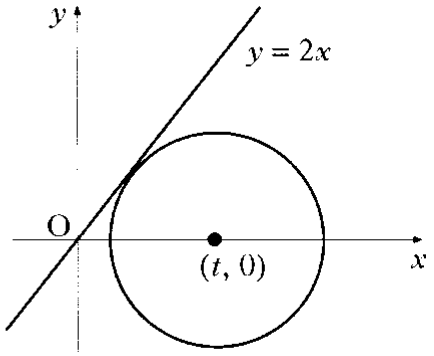
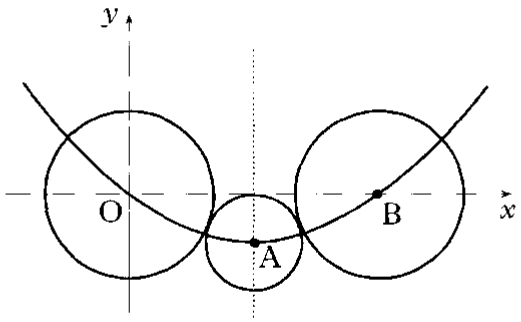
Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.



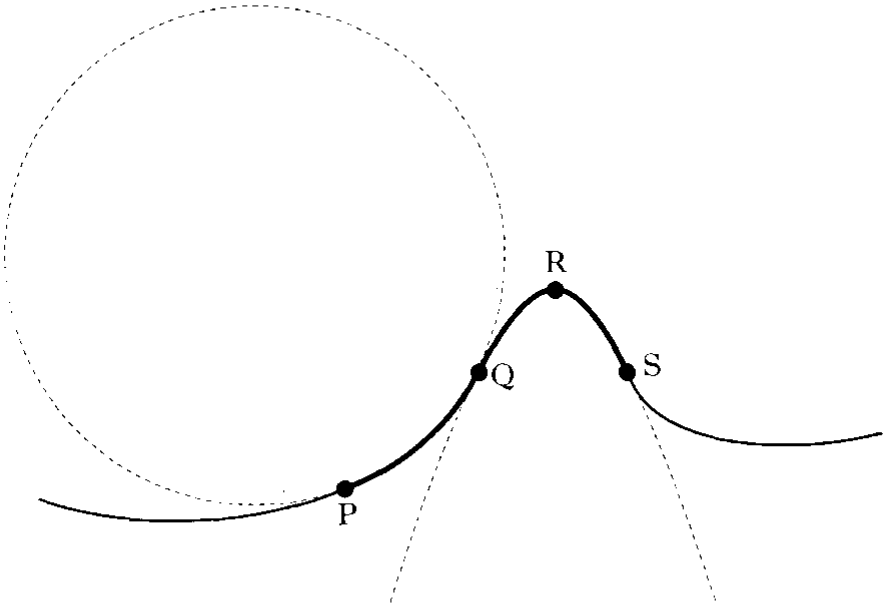
Find the equation of the circle with centre D. 5

Ans	$(x-15)^2+(y-8)^2=2^2$	
2007 P2 Q3	Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle.	6
Ans	$x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$ $x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$ $5x^2 - 10x + 5 = 0$ $5(x^2 - 1) = 0$ Only one root so line is tangential to circle. Point of contact is (1,4).	
2006 P1 Q2	A circle has centre C(–2, 3) and passes through P(1, 6). (*a*) Find the equation of the circle. (*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. 	2 4
Ans	(*a*) $r^2 = 18$ (*b*) $y - 0 = -(x - (-5))$	
2006 P2 Q4	The circles with equations $(x-3)^2 + (y-4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre. Determine the radius of the larger circle.	5
Ans	$\sqrt{37}$, 5 and "2nd" circle	
2005 P1 Q2	Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are $x^2 + y^2 + 6x + 4y - 12 = 0$ and $x^2 + y^2 - 6x - 12y + 20 = 0$. (*a*) Find the coordinates of P. (*b*) Find the length of AB. 	3 2
Ans	(*a*) P = (0,2) (*b*) AB = 10	

2005 P1 Q11	(*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units. Write down the equation of the circle. (*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. 	1 5
Ans	.(*a*) $(x - t)^2 + (y - 0)^2 = 2^2$ (*b*) $t = \sqrt{5}$	
2003 P1 Q11	• O, A and B are the centres of the three circles shown in the diagram below. • The two outer circles are congruent and each touches the smallest circle. • Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$. • The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.  (*a*) (i) State the coordinates of A and find the length of the line OA. (ii) Hence find the equation of the circle with centre B. (*b*) The equation of the parabola can be written in the form $y = px(x + q)$. Find the values of p and q.	 2 3 2
Ans	(*a*) (i) A(12, –5), OA = 13 (ii) $(x - 24)^2 + y^2 = 64$ (*b*) $p = \frac{5}{144}$, $q = -24$	

2002W P1 Q6

The side view of part of a roller coaster ride is shown by the path PQRS.
The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$.
The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$.
The point Q has coordinates (2, 3).



(*a*) Find the equation of the tangent to the circle at Q. 4

(*b*) Show that this tangent to the circle at Q is also the tangent to the parabola at Q. 2

Ans

(*a*) $y - 2x = -1$

(*b*) proof

For parabola

- $\frac{dy}{dx} = -2x + 6$
- $m = 2$

(same as gradient of tangent to circle)

2002W P2 Q10

The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.

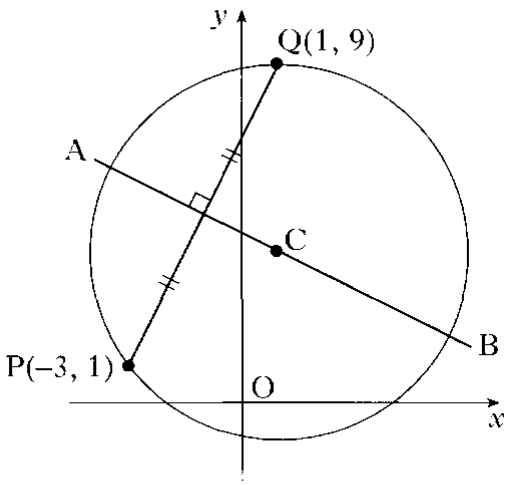
(*a*) Find the value of k. 7

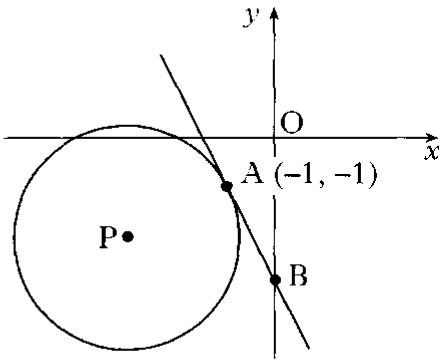
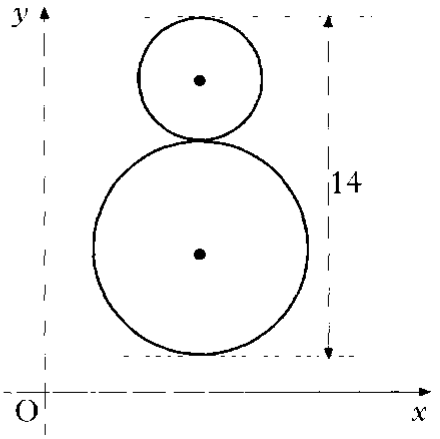
(*b*) Deduce the coordinates of the point of contact. 2

Ans

(*a*) $x^2 + (k - 2x)^2 - 2x - 4 = 0$ and apply discriminant
k=7

(*b*) (3, 1)

2002 P1 Q1	The point P(2, 3) lies on the circle $(x+1)^2 + (y-1)^2 = 13$. Find the equation of the tangent at P.	4
Ans	$2y + 3x = 12$	
2001 P1 Q11	Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$. (*a*) (i) Show that the radius of circle P is $4\sqrt{2}$. (ii) Hence show that circles P and Q touch. (*b*) Find the equation of the tangent to circle Q at the point $(-4, 1)$. (*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.	4 3 3
Ans	(*a*) $r_P = 4\sqrt{2}$ $r_P + r_Q = 6\sqrt{2}$ $C_P = (4,5)$ $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ and "so touch" (*b*) $y = x + 5$ (*c*) $x = 2 \pm 2\sqrt{3}$	
2000 P1 Q6	For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?	5
Ans	for all k	
2000 P2 Q2	(*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(−3, 1) and Q(1, 9). (*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. (*c*) The tangents at P and Q intersect at T. Write down (i) the equation of the tangent at Q (ii) the coordinates of T.  	4 3 2

Ans	(*a*) $x + 2y = 9$ (*b*) $(x-1)^2 + (y-4)^2 = 25$ (*c*) (i) $y = 9$ (ii) T = (–9, 9)	
Specimen 2 P1 Q5	(*a*) The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. Find the equation of the tangent at the point A (–1, –1) on the circle. (*b*) The tangent crosses the *y*-axis at B. Find the equation of the circle with AB as diameter. 	4 3
Ans	(*a*) P = (–3,–2) $m_{PA} = \frac{1}{2} \Rightarrow m_{tgt} = -2$ $y - 1 = -2(x - (-1))$ (*b*) B = (0,3) centre C = $\text{mid}_{AB} = (-\frac{1}{2}, -2)$ $\text{radius}^2 = \text{CB}^2 = \frac{5}{4}$ $(x + \frac{1}{2})^2 + (y + 2)^2 = \frac{5}{4}$	
Specimen 1 P1 Q6	A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the *y*-axis. Find the equation of the "head". 	5
Ans	$(x-5)^2 + (y-13)^2 = 9$	