Higher Mathematics Nightly Questions

You should attempt a short question every night during the school week. Your teacher will tell you what question to attempt each night.

The questions will usually take you between 5 and 10 minutes.

Doing a small amount of Higher Maths every night will improve your knowledge and confidence.

Your teacher will go over the answer to the nightly question in class the next day.

Straight Line

1. Given the points A(–2, 1), B(4, 9) and C(–6, –1), use the distance formula to calculate the lengths of AB and AC.

2. The line joining the points (–2, –3) and (6, *k*) has gradient $\frac{2}{3}$.

What is the value of *k*?

3. Use gradients to prove that the points A(–2, –1), B(4, 3) and C(16, 11) are collinear.

4. (a) Find the gradient of the straight line joining the points A(–1, 0) and B(3, 8).

(b) Hence calculate the size of the angle that the line AB makes with the positive direction of the *x*-axis.

5. Line L has equation $2x + y = 8$.

(a) What is the gradient of line L?

(b) Find the equation of the line parallel to L which passes through the point (3, 1).

6. Find the equation of the perpendicular bisector of the line AB, where A is the point (2, –1) and B is (8, 3).

7. (a) Sketch triangle ABC with vertices A(4, 3), B(6, 1) and C(–2, –3).

(b) Find the equation of the median from A.

8. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$.

9. Given that the points A(–1, 0), B(1, 3) and C(5, *k*) are collinear, find the value of *k*.

10. A and B are the points (–3, –1) and (5, 5) respectively.

(a) Find the equation of the line AB.

(b) Find the equation of the perpendicular bisector of AB.

11. The line L passes through the point (–2, –1) and is parallel to the line with equation $5x + 3y - 6 = 0$. Find the equation of line L in the form $ax + by + c = 0$.

12. (a) Find the equation of the straight line joining the points A(−3, 0) and B(5, 4).

(b) The line makes an angle of $a°$ with the positive direction of the x-axis. Find the value of a correct to 1 decimal place.

13. Use gradients to show that the points A(−2, −3), B(1, 1) and C(7, 9) are collinear.

14. P is the point (3, −3) and Q is (−1, 9).
The line l is parallel to PQ and passes through the point R(1, −2).
Find the equation of line l.

15. Triangle ABC has vertices A(4, 6), B(5, −1) and C(10, 4).

(a) Calculate the length of side AB.

(b) Show that triangle ABC is isosceles but not equilateral.

16. A line joins the points P(−4, 3) and Q(2, −7).
Find the equation of the perpendicular bisector of PQ.

17. The line with equation $2y-3x=4$ makes an angle of $a°$ with the positive direction of the x-axis. Calculate the value of a.

18. (a) Find the gradient of the line with equation $3x+4y=2$.

(b) The line L passes through the point (1, 1) and is perpendicular to the line with equation $3x+4y=2$. Find the equation of line L.

19. A straight line makes an angle of $120°$ with the positive direction of the x-axis.
Find the exact value of the gradient of this line.

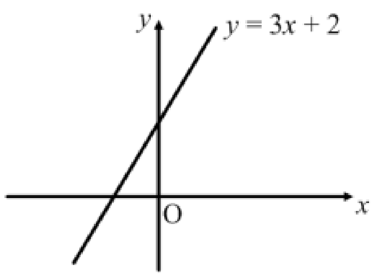
20. (a) Sketch triangle ABC with vertices A(−4, 1), B(12, 3) and C(7, −7).

(b) Find the equation of the median CM, where M lies on AB.

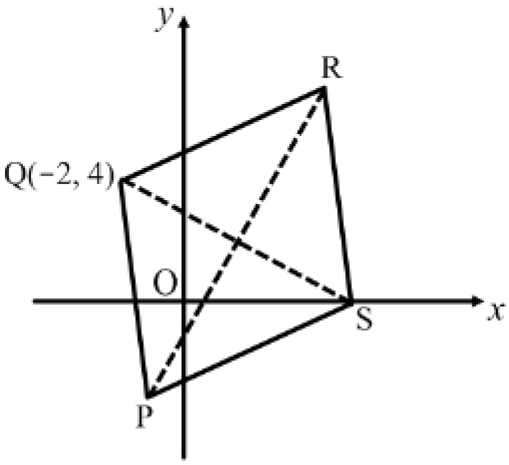
(c) Find the equation of the altitude AD, where D lies on BC.

(d) Find the coordinates of the point of intersection of CM and AD.

21. Calculate the size of the **obtuse** angle between the line $y=3x+2$ and the x-axis.



22. The diagram shows a rhombus PQRS with diagonals PR and QS.
Point Q has coordinates (–2, 4) and diagonal PR has equation $y = 3x - 1$.



Find the equation of diagonal QS.

Functions & Graphs

23. Functions f and g are defined on the set of real numbers by $f(x) = 2x + 4$ and $g(x) = 3x - 2$. Find simplified expressions for $f(g(x))$ and $g(f(x))$.

24. Functions f and g are defined on the set of real numbers by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$.

Find simplified expressions for:

(a) $f(g(x))$ (b) $g(f(x))$ (c) $f(f(x))$ (d) $g(g(x))$

25. (a) Express $x^2 + 6x + 14$ in the form $(x + a)^2 + b$.

(a) Hence **write down** the coordinates of the turning point on the parabola with equation $y = x^2 + 6x + 14$.

26. (a) The function f is defined on the set of real numbers by $f(x) = 2x + 3$.
Find an expression for the inverse function $f^{-1}(x)$.

(b) Find $f(f^{-1}(x))$.

27. The point with coordinates $A(3, 2)$ is on the graph with equation $y = f(x)$.

Write down the image of the point A on the graph with equation:

(a) $y = -f(x - 1)$ (b) $y = 2f(x) + 1$

28. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) The function h is defined by $h(x) = f(g(x)) + g(f(x))$.
Find an expression for $h(x)$ in its simplest form.

29. Solve the equation $2\cos x = \sqrt{3}$, where $0 \le x < 2\pi$.

30. (a) Express $2x^2 + 8x + 7$ in the form $a(x+b)^2 + c$.

(b) Hence **write down** the coordinates of the turning point on the parabola with equation $y = 2x^2 + 8x + 7$.

31. The function f is defined by $f(x) = \dfrac{5}{x^2 - 3x + 2}$.

Which two values of x must be excluded from the domain of the function f?

32. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ in the interval $\frac{\pi}{2} \le x \le \pi$?

33. (a) Express $3x^2 - 12x + 10$ in the form $a(x+b)^2 + c$.

(b) Hence **write down** the coordinates of the turning point on the parabola with equation $y = 3x^2 - 12x + 10$.

34. Find the solution of the equation $\sqrt{2}\cos x + 1 = 0$ in the interval $\pi \le x \le \frac{3\pi}{2}$.

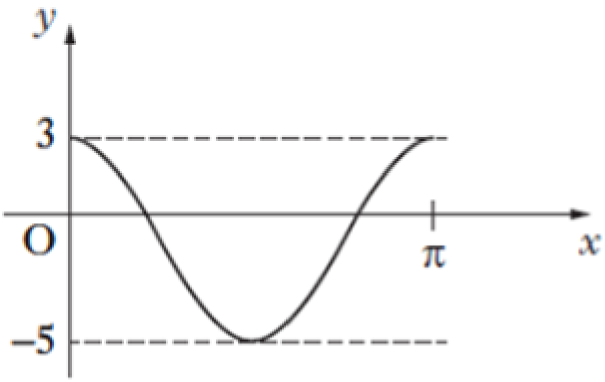
35. Functions f and g are defined on the set of all real numbers by $f(x) = 2x + 1$ and $g(x) = 4x - 2$. Find $f(f(x)) + g(g(x))$ in the form $ax + b$.

36. The function f is defined on the set of real numbers by $f(x) = 3x - 8$.
Find an expression for the inverse function $f^{-1}(x)$ and find the value of $f^{-1}(7)$.

37. Express $f(x) = (2x-1)(2x+5)$ in the form $a(x+b)^2 + c$.

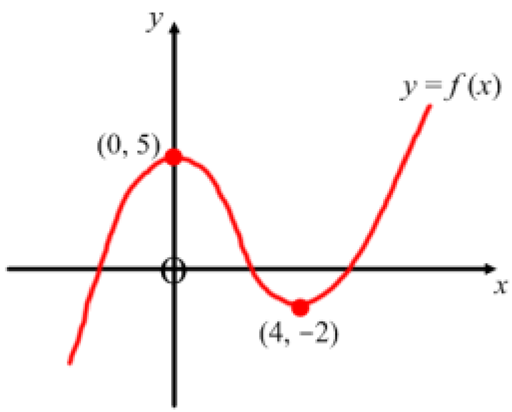
38. Functions f and g are given by $f(x) = 3x^2 - 1$ and $g(x) = x^2 + 2$.
Express $f(g(x))$ in the form $ax^4 + bx^2 + c$.

39. The diagram below shows the graph of $y = a\cos(bx) + c$



Write down the values of *a*, *b* and *c*.

40. The diagram shows the graph of $y = f(x)$.
The graph has a maximum turning point at (0, 5) and a minimum turning point at (4, −2).



Sketch the graph of $y = f(x-1) + 2$.

41. **Do not use a calculator in this question!**

(a) State the value of $\sin\frac{\pi}{4}$.

(b) Work out the value of $\cos\frac{2\pi}{3}$.

(c) Hence evaluate $4\sqrt{2}\sin\frac{\pi}{4}\cos\frac{2\pi}{3}$.

42. Functions f and g are defined on the set of all real numbers by $f(x) = x^2 + 3$ and $g(x) = x + 4$.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Show that the equation $f(g(x))+g(f(x))=0$ has no real roots.

43. The functions *f* and *g* are defined on suitable domains by

$$f(x)=4x-3 \quad \text{and} \quad g(x)=\sqrt{x+1}.$$

(a) A third function, *h*, is defined by $h(x)=g(f(x))$.
Find an expression for $h(x)$.

(b) State the largest possible domain for *h*.

44. The function *f* is defined on the set of all real numbers by $f(x)=3x-10$.
Find a formula for the inverse function $f^{-1}(x)$

45. Functions *f* and *g* are defined by $f(x)=3x+5$ and $g(x)=2-x$.
Find an expression for $f(g(x))$ and find the value of *x* for which $f(g(x))=32$.

46. Write each quadratic expression in the form $p(x+q)^2+r$:

(a) $2x^2+4x+7$ (b) $15+2x-x^2$

47. What is the value of $\sin\frac{5\pi}{6}-\tan\frac{3\pi}{4}$?
Do not use a calculator!

<u>Differentiation</u>

48. Find $f'(x)$ when:

(a) $f(x)=2x^3+4x^2-3x+1$ (b) $f(x)=\frac{1}{2}x^4+\frac{1}{3}x^3+x$

49. Given $f(x)=3x^2(2x-1)$, find the value of $f'(-1)$.

50. Find the gradient of the tangent to the curve with equation $y=x^3-4x+1$ at the point where $x=-2$.

51. Find $f'(x)$ when: (a) $f(x)=\frac{2}{x^5}$ (b) $f(x)=8\sqrt{x}$

52. Find $f'(x)$ when: (a) $f(x)=2x^{\frac{3}{2}}+\frac{3}{x^2}$ (b) $f(x)=\frac{1}{4x^3}$

53. Find the equation of the tangent to the curve $y=x^2-3x+6$ at the point (4, 10).

54. Recall that $f'(x)$ measures the rate of change of a function $f(x)$

(a) If $s(t) = t^2 - 5t + 8$, find the rate of change of s with respect to t when $t = 3$.

(b) The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

Find the rate of change of V with respect to r when $r = 2$.

55. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature.

56. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find the value of $f'(4)$.

57. Find the equation of the tangent to the curve $y = x^3 - 9x$ at the point where $x = -2$.

58. Find the stationary points on the curve with equation $y = x^3 - 9x^2 + 24x - 2$ and determine their nature.

59. If $p = \frac{4}{x^3}$, find the rate of change of p with respect to x when $x = 2$.

60. Given $f(x) = 2\sqrt{x} + \frac{3}{x^2}$, find the exact value of $f'(4)$.

61. Find $f'(x)$ when: (a) $f(x) = x(x-1)^2$ (b) $f(x) = \frac{2x^6 - 5}{x^2}$

62. A parabola has equation $y = \frac{1}{2}x^2 - 8x + 34$.

The gradient of the tangent to the parabola at the point P is 4.
Find the coordinates of P.

63. Find the coordinates of the stationary points of the curve $y = x^3 - 3x^2 + 4$ and justify their nature.

64. The amount of a drug, in milligrams, remaining in a person's bloodstream t minutes after being administered is given by $A(t) = 300\sqrt{t} - 15t$.

Find the rate of change of A with respect to t when $t = 36$.

65. Given that $f(x) = 4x^{\frac{3}{2}} - 2\sqrt{x}$, find $f'(x)$.

66. Find the equation of the tangent to the curve with equation $y = \frac{8}{x}$ at the point P where $x = 4$.

67. Show that the line $y = 8x - 11$ is a tangent to the parabola $y = 3x^2 - 4x + 1$ and find the coordinates of the point of contact.

68. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 12.

(b) Hence find the equation of the tangent at P.

69. A ball is thrown vertically upwards.
The height, h metres, of the ball after t seconds is given by the formula $h = 30t - 5t^2$.

(a) The velocity, v metres per second, of the ball after t seconds is given by $v = \frac{dh}{dt}$.
Find a formula for v in terms of t and hence find the velocity of the ball after 3 seconds.

(b) Explain your answer to part (a) in terms of the ball's movement.

70. A curve has equation $y = x^4 - 2x^3 + 5$.
Find the equation of the tangent to the curve at the point where $x = 2$.

71. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds.

A mathematical model suggests that P and x are related by the formula $P = 12x^3 - x^4$ for $0 < x < 12$.

Find the value of x in the interval $0 < x < 12$ which gives the maximum profit, justifying your answer.

72. Find the stationary points on the curve with equation $y = x^3 - 3x^2 - 24x - 28$ and justify their nature.

73. The area, A cm^2, of tin required to make a box with a square base is given by the formula

$$A = x^2 + \frac{250}{x}, \qquad x > 0,$$

where x cm is the side length of the base.

Find the value of x for which this area is a minimum, justifying your answer.

74. The equation of a curve is $y = 2x^3 - 3x^2$.

(a) Find the coordinates of the two points where the curve crosses the x-axis.

(b) Find the coordinates of the stationary points on the curve and determine their nature.

Recurrence Relations

75. (a) A sequence is defined by the recurrence relation $u_{n+1} = 2u_n - 5$ with $u_0 = 6$. Find the value of u_3.

(b) A second sequence us defined by the recurrence relation $v_{n+1} = \frac{1}{3}v_n + 4$.

If $v_2 = 10$, find the value of v_1.

76. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 8u_n + 12$ with $u_0 = 4$.

(a) Explain why this sequence has a limit.

(b) Find the value of the limit.

77. (a) Calculate the limit as $n \to \infty$ of the sequence defined by $u_{n+1} = 0 \cdot 9u_n + 10$ with $u_0 = 1$.

(b) Determine the least value of *n* for which u_n is greater than half of this limit.

78. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

(a) Write down the condition on *k* for the sequence to have a limit.

(b) The sequence tends to a limit of 5 as $n \to \infty$.
Find the value of *k*.

79. A slow puncture causes a tyre to lose 11% of its pressure each day.
To compensate for this, every morning the tyre is inflated by adding 3 units of pressure.

(a) If u_n is the pressure of the tyre after *n* days, write down a recurrence relation for u_{n+1} in terms of u_n.

(b) If this pattern continues indefinitely, explain why the pressure in the tyre will approach a limit and find the value of this limit correct to 1 decimal place.

80. A sequence u_1, u_2, u_3, … is generated by the recurrence relation $u_{n+1} = au_n + b$.

(a) The first two terms are $u_1 = 4$ and $u_2 = 5$.
Write down an equation in *a* and *b*.

(b) The third term is $u_3 = 7$.
Write down a second equation in *a* and *b*.

(c) Find the values of *a* and *b*.

81. At the same time every day a doctor gives a patient a 250 mg dose of an antibiotic. It is known that over a 24 hour period, the amount of antibiotic in the bloodstream is reduced by 80%.

(a) Write down a recurrence relation for the amount of antibiotic in the patient's bloodstream immediately after a dose.

(b) It is known that more than 350 mg of the antibiotic in the bloodstream will result in unpleasant side effects.

Is it safe to administer this antibiotic over an extended period of time?
Explain your answer.

82. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 6$ with $u_2 = 14$.
Find the value of u_0.

83. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 4u_n + 90$.

(a) Explain why the sequence will approach a limit as $n \to \infty$.

(b) Find the limit of this sequence.

84. A toad falls to the bottom of a well.
Each day, the toad climbs 13 feet and then rests overnight. During the night, it slides down $\frac{1}{4}$ of its height above the floor of the well.

(a) Write down a recurrence relation for t_{n+1} in terms of t_n, where t_n is the height reached by the toad at the end of *n* days.

(a) Given that the well is 50 feet deep, determine whether or not the toad will eventually escape from the well.

Polynomials & Quadratics

85. Calculate the discriminant and state the nature of the roots of each quadratic equation.

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 + 2x + 7 = 0$ (c) $4x^2 - 12x + 9 = 0$

86. The equation $2x^2 + kx + k = 0$ has equal roots, where $k \neq 0$.
Find the value of *k*.

87. A function *f* is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

Find the remainder when $f(x)$ is divided by:

(a) $(x-1)$ (b) $(x+2)$

88. Show that $(x-3)$ is a factor of $f(x) = x^3 + 4x^2 - 11x - 30$ and hence factorise $f(x)$ fully.

89. Given that $(x-3)$ is a factor of $2x^3-9x^2+kx-3$, find the value of k.

90. A function f is defined by $f(x)=2x^3-7x^2+9$ where x is a real number.

(a) Show that $(x-3)$ is a factor of $f(x)$ and hence factorise $f(x)$ fully.

(b) Find the coordinates of all the points where the curve with equation $y=f(x)$ crosses the coordinate axes.

91. Find the two values of k for which the quadratic equation $x^2+(k-3)x+k=0$ has equal roots.

92. Find the quotient and remainder when $2x^4-3x^3-3x+1$ is divided by $(x-2)$.

93. $f(x)=6x^3-5x^2-17x+6$

(a) Show that $(x-2)$ is a factor of $f(x)$.

(b) Hence express $f(x)$ in its fully factorised form.

94. Factorise fully $f(x)=x^3-4x^2-7x+10$.

95. Given that $(x+1)$ is a factor of $2x^3+3x^2+kx-6$, find the value of k.

96. Show that $(x-2)$ is a factor of $f(x)=x^3-6x^2+3x+10$ and hence factorise $f(x)$ fully.

97. A parabola crosses the x-axis at the points (2, 0) and (5, 0).
Given that the parabola also passes through the point (4, 4), find the equation of the parabola.

[You may find it helpful to draw a sketch of the parabola.]

98. (a) Show that $x=1$ is a root of $x^3+8x^2+11x-20=0$.

(b) Hence factorise $x^3+8x^2+11x-20$ fully and solve the equation $x^3+8x^2+11x-20=0$.

99. Find the quotient and remainder when $3x^4-4x^2+2x-3$ is divided by $(x-2)$.

100. $f(x)=2x^3+px^2+qx+4$

(a) Given that $(x-2)$ is a factor of $f(x)$, write down an equation in p and q.

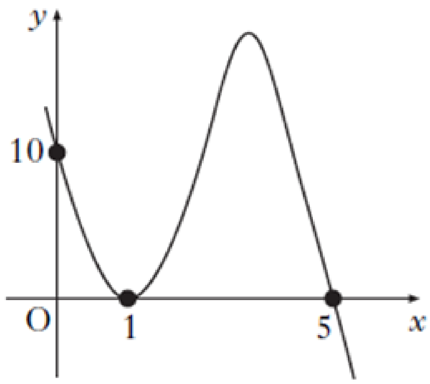
(b) The remainder when $f(x)$ is divided by $(x+1)$ is 9.
Write down a second equation in p and q.

(b) Find the values of p and q.

101. For what value of *k* does the quadratic equation $kx^2 + (2k+1)x + k = 0$ have equal roots?

[*Hint*: remember that $b^2 - 4ac = 0$ for equal roots]

102. The graph of a cubic function is shown below.



Find the equation of the graph.

103. Solve each quadratic inequality by first sketching the graph of the quadratic.

(a) $x^2 - 2x - 15 < 0$ (b) $6 - x - x^2 < 0$

104. For the cubic polynomial $f(x) = 6x^3 + 7x^2 + ax + b$,

- $(x+1)$ is a factor of $f(x)$
- the remainder when $f(x)$ is divided by $(x-2)$ is 72.

Find the values of *a* and *b*.

<u>Integration</u>

105. Find: (a) $\int (6x^2 + 2x + 3)dx$ (b) $\int \frac{8}{x^3}dx$ (c) $\int 6\sqrt{x}dx$

106. At any point (*x*, *y*) on a curve, $\frac{dy}{dx} = 3x^2 + 4x$.

Given that the curve passes through the point (−1, 5), express *y* in terms of *x*.

107. Evaluate $\int_1^2 (x^3 - 2x)dx$.

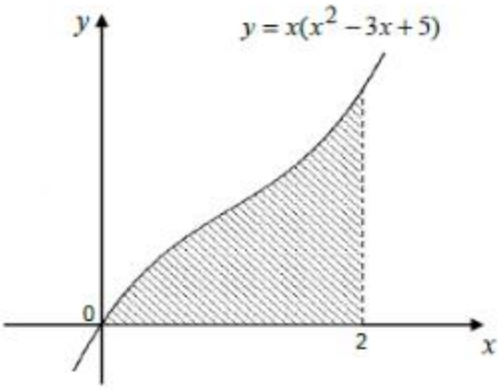
108. For a curve $y = f(x)$, it is known that $\frac{dy}{dx} = 4x^3 - 3x^2 - 1$ and the curve passes through the point (2, 0). Find the equation of the curve.

109. Evaluate $\int_4^9 \sqrt{x}\,dx$.

110. (a) Given $f(x) = \dfrac{6x^5 - 1}{x^2}$, find $f'(x)$.

(b) Find $\int (2x+3)(2x-5)\,dx$.

111. The curve with equation $y = x(x^2 - 3x + 5)$ is shown below.

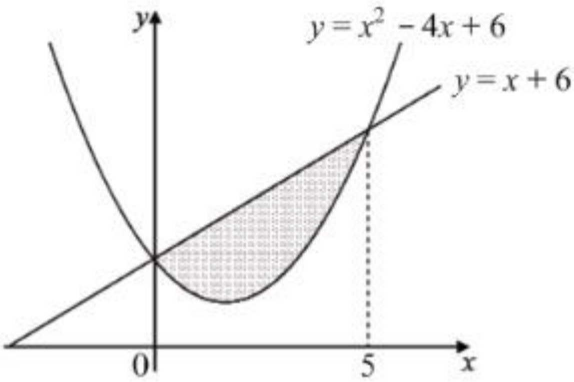


Calculate the shaded area.

112. Given that $\int_1^a (2x+5)\,dx = 18$, where $a > 1$, find the value of a.

113. Find: (a) $\int \left(6\sqrt{x} + \dfrac{1}{x^3}\right) dx$ (b) $\int \dfrac{4x^3 - 1}{x^2}\,dx$

114. The line with equation $y = x + 6$ and the curve with equation $y = x^2 - 4x + 6$ intersect where $x = 0$ and $x = 5$, as shown in the diagram below.



Calculate the shaded area.

115. A curve with equation $y = f(x)$ is such that $\dfrac{dy}{dx} = 3x^2 - x$.

If the curve passes through the point (2, 11), express *y* in terms of *x*.

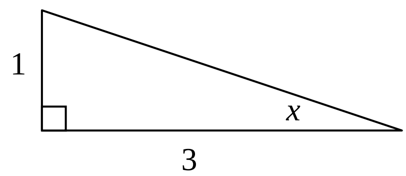
116. (a) Given that $\int_0^p (6x^2 + 6x - 5)dx = 6$, where $p > 0$, show that *p* satisfies the equation $2p^3 + 3p^2 - 5p - 6 = 0$.

(b) Solve the equation to find the value of *p*.

<u>Trigonometry</u>

117. Expand and simplify: (a) $\cos\left(x + \dfrac{\pi}{6}\right)$ (b) $\sin\left(x + \dfrac{\pi}{2}\right)$

118. The diagram shows a right-angled triangle.
Find the exact value of $\cos 2x$.

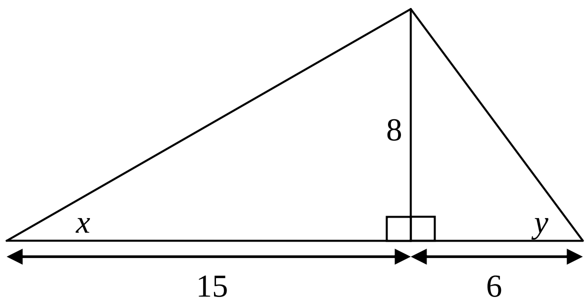


119. The acute angle *A* is such that $\cos A = \dfrac{1}{\sqrt{5}}$.

Find the exact value of : (a) $\sin 2A$ (b) $\cos 2A$

120. Solve the equation $\sin 2x° - \sin x° = 0$ in the interval $0 \le x \le 360$.
[*Hint*: start by replacing $\sin 2x°$ with a double angle formula]

121. The diagram below shows two right-angled triangles.



Find the exact value of $\cos(x + y)$.

122. Expand and simplify $2\sin\left(x + \dfrac{\pi}{6}\right) - 2\cos x$.

[*Hint*: use the formula $\sin(A+B)=\sin A\cos B+\cos A\sin B$]

123. The acute angles A and B are such that $\sin A=\frac{4}{5}$ and $\sin B=\frac{2}{\sqrt{5}}$.

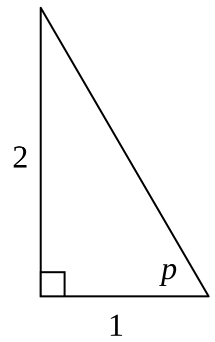
Find the exact value of $\sin(A+B)$.

[*Hint*: use the formula $\sin(A+B)=\sin A\cos B+\cos A\sin B$.]

124. Solve the equation $\sin 2x-\sqrt{3}\sin x=0$ for $0\le x\le 2\pi$.

[*Hint*: remember that $\sin 2x=2\sin x\cos x$]

125. The diagram shows a right-angled triangle.
Find the exact values of $\sin 2p$ and $\cos 2p$.



126. Solve each equation for $0\le x\le 360$:

(a) $\cos 2x°+3\cos x°+2=0$ (b) $\cos 2x°+7\sin x°=4$

[*Hint*: replace $\cos 2x°$ with an appropriate formula in each equation]

127. If $\cos 2x=\frac{7}{25}$ and $0<x<\frac{\pi}{2}$, find the values of $\cos x$ and $\sin x$.

[*Hint*: use a formula for $\cos 2x$]

Wave Function

128. Express $3\cos x°+2\sin x°$ in the form $k\cos(x-a)°$ where $k>0$ and $0\le a<360$.

129. The expression $\sqrt{3}\cos x+\sin x$ can be expressed in the form $k\cos(x-a)$, where $k>0$ and $0<a<\frac{\pi}{2}$. Find the values of k and a.

130. (a) Write $2\sin x°+\sqrt{5}\cos x°$ in the form $k\sin(x+a)°$, where $k>0$ and $0<x<90$.

(b) Hence write down the maximum and minimum values of the function $f(x)=2\sin x°+\sqrt{5}\cos x°+1$, where x is a real number.

131. Express $2\sqrt{2}\sin x + 2\sqrt{2}\cos x$ in the form $k\sin(x+a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$.

Further Calculus

132. Use the chain rule to find $f'(x)$ when:

(a) $f(x) = (5x+2)^4$ (b) $f(x) = (x^2+1)^6$

133. Given that $y = 3\sin x + \cos 2x$, find $\frac{dy}{dx}$.

134. Given that $f(x) = \sqrt{3x^2+2}$, use the chain rule to find $f'(x)$.

135. Given $f(x) = 4\sin 3x$, find the value of $f'(0)$.

136. If $y = \frac{1}{x^3} - \cos 2x$, find $\frac{dy}{dx}$.

137. (a) Given that $f(x) = \sin^3 x$, find $f'(x)$.
[*Hint*: write $f(x) = (\sin x)^3$ and use the chain rule]

(a) If $y = (1+\cos 2x)^4$, find $\frac{dy}{dx}$.

138. Use the chain rule to find $f'(x)$ when:

(a) $f(x) = (5x+2)^4$ (b) $f(x) = \sqrt{8x+1}$

139. Find: (a) $\int 6\cos 2x dx$ (b) $\int 2\sin 4x dx$

140. A curve has equation $y = \sqrt{x^2+5}$.

(a) Use the chain rule to find $\frac{dy}{dx}$.

(b) Hence find the equation of the tangent to the curve at the point where $x = 2$.

141. (a) A function *f* is defined by $f(x) = (1-x^3)^{\frac{1}{3}}$.
Use the chain rule to find $f'(x)$.

(b) Find $\int 6\cos 2x dx$.

142. (a) Use the chain rule to differentiate $f(x) = (1+\sin x)^4$.

(b) Find $\int\left(\frac{2}{x^4}+\cos 5x\right)dx$.

143. (a) For what value of k does the equation $kx^2-6x+1=0$ have equal roots?

(b) Find $\int\sqrt{6x+1}\,dx$. [*Hint*: use $\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$]

144. An oil production platform is to be connected by a pipeline to a refinery on shore. The length of the underwater part of the pipeline is x kilometres.

The total cost of building a pipeline is $C(x)$ million pounds, where

$$C(x)=2x+100-\sqrt{x^2-243}.$$

Show that $x=18$ gives the minimum cost and find this minimum cost.

145. (a) Given that $f(x)=\sqrt{3x^2+2}$, use the chain rule to find $f'(x)$.

(b) Evaluate $\int_0^{\frac{\pi}{6}}\cos 2x\,dx$.

146. (a) Find $\int\frac{1}{\sqrt{3x+4}}dx$. [*Hint*: use $\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$]

(b) Given that $\int_4^a\frac{1}{\sqrt{3x+4}}dx=2$, find the value of *a*.

Circle

147. A circle with centre C(–2, 1) passes through the point P(5, –2).

(a) Use the distance formula (or otherwise) to find the radius of the circle.

(b) Hence write down the equation of the circle.

148. A circle has equation $x^2+y^2+8x+6y-75=0$.
Find the centre and radius of this circle.

149. A circle has equation $(x-3)^2+(y+2)^2=25$.

(b) Write down the coordinates of C, the centre of the circle.

(b) Find the equation of the tangent at the point P(6, 2) on the circle.

150. Find the coordinates of the two points of intersection of the line with equation $y=2x+5$ and the circle with equation $x^2+y^2-6x-2y-30=0$.

151. Circle C_1 has equation $x^2+y^2+8x+4y-38=0$.

Circle C_2 has equation $(x-4)^2 + (y-6)^2 = 26$.
Find the distance between the centres of these two circles.

152. At what two points does the circle with equation $x^2 + y^2 + 5x - y - 6 = 0$ intersect the *y*-axis? [*Hint*: remember that a graph intersects the *y*-axis when $x = 0$.

153. Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact.

154. The point A(2, 3) lies on the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$.
Find the equation of the tangent to the circle at A.

155. A circle, centre C, has equation $x^2 + y^2 - 4x - 2y - 20 = 0$.

(a) Write down the centre, C, and calculate the radius of this circle.

(b) The point P(5, –3) lies on the circumference of the circle.
Find the equation of the tangent to the circle at P.

156. A circle, centre C, has equation $x^2 + y^2 - 4x - 2y - 20 = 0$.

(a) Find the centre, C, and radius of this circle.

(b) The point P(5, –3) lies on the circumference of the circle.
Find the equation of the tangent to the circle at P.

157. Find the coordinates of the two points of intersection of the line with equation $y = 2x + 1$ and the circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$.

158. A circle with centre C(–1, 5) passes through the point P(2, 1).
Find the equation of the circle.

159. Circle C_1 has equation $x^2 + y^2 - 2x - 4y - 4 = 0$
Circle C_2 has equation $(x-4)^2 + (y-6)^2 = 4$.

(a) Find the centre and radius of each circle.

(b) Hence show that circles C_1 and C_2 touch externally.

160. Show that the line with equation $y = 2x - 12$ is a tangent to the circle with equation $x^2 + y^2 - 6x - 8y + 5 = 0$ and find the coordinates of the point of contact.

<u>Logs & Exponentials</u>

161. (a) Find $\int (3x-1)^5 \, dx$. [*Hint*: use $\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + C$]

(b) A curve has equation $y = \log_b (x+5)$, where $b > 0$.
Given that the curve passes through the point (4, 2), find the value of *b*.

[*Hint*: substitute the coordinates of (4, 2) into the equation of the curve]

162. (a) Find the value of x if $\log_8 x = \frac{2}{3}$.

(b) Use the laws of logarithms to evaluate $\log_3 9 + \log_3 6 - \log_3 18$.

163. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.

164. (a) If $\log_2 9 + \log_2 x = 3$, what is the value of x?

(b) Find the value of x if $\log_3 x - \log_3 4 = 2$.

[*Hint*: use the laws of logarithms to help]

165. Solve the equation $\log_9 (x+2) = \frac{1}{2} + \log_9 (x-5)$, where $x > 5$.

[*Hint*: write the equation as $\log_9 (x+2) - \log_9 (x-5) = \frac{1}{2}$ and use the laws of logarithms]

166. The number of bacteria, b, in a culture after t hours is given by the formula $b = b_0 e^{kt}$, where b_0 is the initial number of bacteria present and k is a constant.

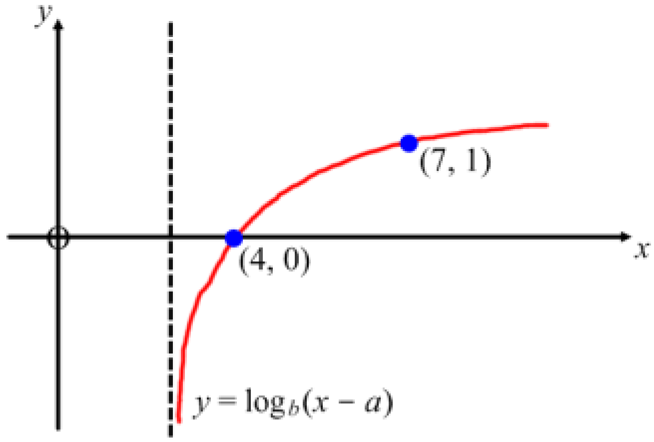
The number of bacteria in the culture increased from 800 to 2400 in 2 hours.
Make up an equation and find the value of k correct to 3 significant figures.

167. (a) The point (q, 2) lies on the graph with equation $y = \log_3 (x-4)$.
Find the value of q.

[*Hint*: substitute the coordinates of (q, 2) into the equation of the curve]

(b) If $\log_{10} y = \log_{10} 5 + 3\log_{10} x$, express y in terms of x.

168. The diagram shows the graph of $y = \log_b (x-a)$.



Find the values of a and b.

169. The graph of $\log_2 y$ against $\log_2 x$ is a straight line passing through the points (0, 5) and (4, 7).

Find the gradient of the straight line and hence show that $y = kx^n$ for some constants k and n.

170. (a) Solve the equation $\log_x(x+2) + \log_x(2x-3) = 2$, where $x > \frac{3}{2}$.

[*Hint*: use the laws of logarithms to help]

(b) A radioactive substance decays according to the law $m = m_0 e^{kt}$, where m is the mass after t years, m_0 is the initial mass and k is a constant.

The time taken for 100 grams of the substance to decay to 50 grams is 5 years.
Make up an equation and find the value of k correct to 3 significant figures.

Vectors

171. (a) Given the vectors $\boldsymbol{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, find the vector $2\boldsymbol{p} - \boldsymbol{q} - \frac{1}{2}\boldsymbol{r}$.

(b) K and L are the points (2, 0, −1) and (−1, 3, 1) respectively.
Find the magnitude of the vector $\overrightarrow{KL}$.

172. (a) Use vectors to prove that the points P(1, –2, 4), Q(3, 4, 0) and R(9, 22, –12) are collinear.

(b) State the ratio $PQ : QR$.

173. A is the point (2, 1, 3) and B is (6, 5, 11).
The point C divides AB in the ratio 3 : 1.
Use the section formula (or other method) to find the coordinates of C.

174. (a) Express $3x^2+6x-4$ in the form $a(x+b)^2+c$.

(b) Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 4\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 3\boldsymbol{i} - 6\boldsymbol{j} + 5\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular.

[*Hint*: remember that if vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular, then $\boldsymbol{u}.\boldsymbol{v} = 0$]

175. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by $\boldsymbol{a} = \begin{pmatrix} 0 \\ 8 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$.

Calculate θ, the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

176. (a) If $\boldsymbol{u} = \begin{pmatrix} 3k \\ k \\ 0 \end{pmatrix}$, where $k>0$, and $\boldsymbol{u}$ is a unit vector, find the value of k.

[*Hint*: remember that a unit vector has magnitude 1, so $|\boldsymbol{u}|=1$]

(b) The vectors $\boldsymbol{a} = \begin{pmatrix} t \\ 3 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ t \\ 3 \end{pmatrix}$ are perpendicular.

Find the value of t.

[*Hint*: remember that if vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, then $\boldsymbol{a}.\boldsymbol{b} = 0$]

177. (a) Find the coordinates of the point where the graph of $y=\log_5(2x+9)$ crosses the x-axis. [*Hint*: remember that a graph crosses the x-axis when $y=0$]

(b) The point P divides the line AB in the ratio 3 : 2 where A is (–3, 2, 6) and B is (7, –3, 1). Use the section formula (or other method) to find the coordinates of P.

178. (a) Find a unit vector, $\boldsymbol{u}$, which is parallel to the vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}$.

[*Hint*: start by finding $|\boldsymbol{a}|$]

(b) Prove that $(1+2\sin x)(1-2\sin x) = 4\cos^2 x - 3$.

[*Hint*: multiply out the brackets and remember that $\sin^2 x + \cos^2 x = 1$]