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y **Quest**

# Wave Fctn Past Papers Unit 3 Outcome 4

Written Questions

[SQA] 1. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x° + a°)$ where $k > 0$ and $0 < a < 360$. **4**

[SQA] 2. Express $2\sin x° - 5\cos x°$ in the form $k\sin(x - \alpha)°$, $0 \le \alpha < 360$ and $k > 0$. **4**

[SQA] 3. (a) Express $\sin x° - 3\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a. **4**

(b) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **2**

[SQA] 4. Solve the simultaneous equations $k\sin x° = 5$

$k\cos x° = 2$ where $k \ge 0$ and $0 \le x \le 360$ **4**

[SQA] 5. Solve the equation $2\sin x° - 3\cos x° = 2\cdot 5$ in the interval $0 \le x < 360$. **8**

[SQA] 6. $f(x) = 2\cos x° + 3\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. **(4)**

(b) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**

[SQA] 7. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$. **6**

[SQA] 8.

(a) Show that $2\cos(x° + 30°) - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **3**

(b) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x° + \alpha°)$ where $k > 0$ and $0 \le \alpha \le 360$ and find the values of k and α. **4**

(c) Hence, or otherwise, solve the equation $2\cos(x° + 30°) = \sin x° + 1$, $0 \le x \le 360$. **3**

[SQA] 9. The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight.

(a) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)**

(b) (i) Use your result from part (a) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$.

(ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)**

(c) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)**

(d) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

[SQA] 10. The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$.

(a) Express d in the form $k\cos(20t° - \alpha°)$, where $k > 0$ and $0 \le \alpha \le 360$. **4**

(b) Sketch the graph of d for $0 \le t \le 18$. **4**

(c) Find, correct to one decimal place, the values of t, $0 \le t \le 18$, for which the displacement is 1·5 units. **3**

[SQA] 11. (a) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \le a \le 2\pi$ **4**

(b) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

[SQA] 12. The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(a) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x + \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)**

(b) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \le x < 360$. **(4)**

(c) Write down the minimum value of $(f(x))^2$. **(1)**

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[SQA] 13. A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.
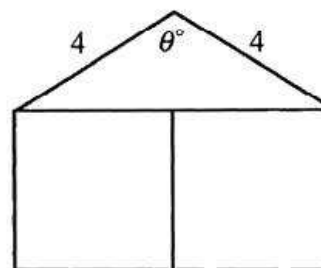
(a) If $\theta°$ is the angle shown in the diagram and A is the area (in square metres) of the gable end, show that $A = 8\left(2 + \sin\theta° - 2\cos\theta°\right)$. **(5)**

(b) Express $8\sin\theta° - 16\cos\theta°$ in the form $k\sin(\theta - \alpha)°$. **(4)**

(c) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**

[SQA] 14. (a) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b\sin ax°$, where a and b are constants.
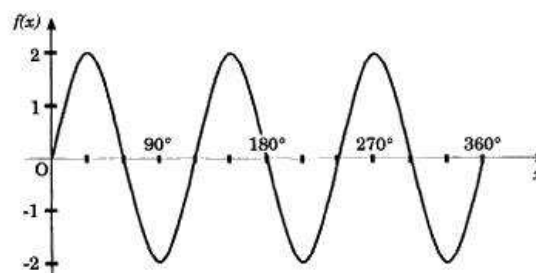Write down the values of a and b.

Diagram 1

(ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d\cos cx°$, where c and d are constants.
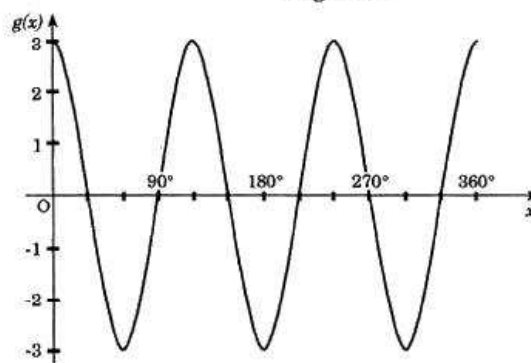Write down the values of c and d. **(4)**

Diagram 2

(b) The function h is defined by $h(x) = f(x) + g(x)$.
Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q\sin(px + r)°$ and find the values of p, q and r. **(5)**

[SQA] **15.** An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.
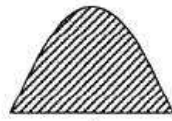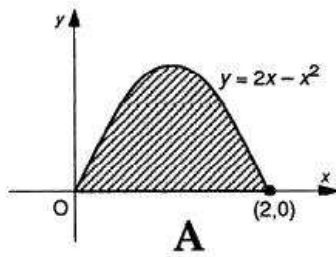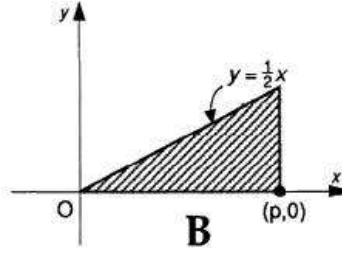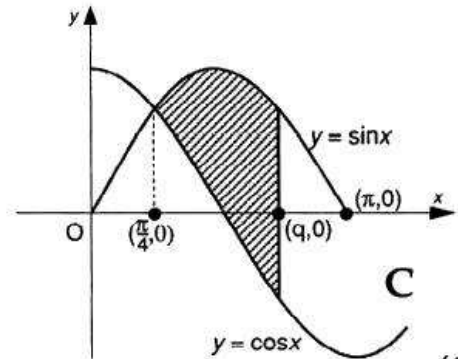

Relative to x,y–axes, the shapes are positioned as shown below.

(a) Find the area shaded under $y = 2x - x^2$. **(4)**

(b) Use the area found in part (a) to find the value of p. **(2)**

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. **(10)**

[END OF WRITTEN QUESTIONS]

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