Trig. Past Papers Unit 2 Outcome 3

Written Questions

[SQA] 1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

[SQA] 2. Solve the equation $\cos 2x^\circ + 5\cos x^\circ - 2 = 0$, $0 \leq x < 360$. **5**

[SQA] 3. Find the exact solutions of the equation $4\sin^2 x = 1$, $0 \leq x < 2\pi$. **4**

[SQA] 4. Solve the equation $2\cos^2 x = \frac{1}{2}$, for $0 \leq x \leq \pi$. **3**

[SQA] 5. Solve the equation $\cos 2x^\circ + \cos x^\circ = 0$, $0 \leq x < 360$. **5**

[SQA] 6. Solve $2\sin 3x^\circ - 1 = 0$ for $0 \leq x \leq 180$. **4**

[SQA] 7.

(*a*) Show that $2\cos 2x^\circ - \cos^2 x^\circ = 1 - 3\sin^2 x^\circ$. **2**

(*b*) Hence solve the equation $2\cos 2x^\circ - \cos^2 x^\circ = 2\sin x^\circ$ in the interval $0 \leq x < 360$. **4**

[SQA] 8. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$. **5**

[SQA] 9. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3\cos(2x^\circ - 40^\circ) - 1 = 0$. **5**

[SQA] 10.

(*a*) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(*b*) Show that there are no real roots for θ. **1**

[SQA] 11. If $f(a) = 6\sin^2 a - \cos a$, express $f(a)$ in the form $p\cos^2 a + q\cos a + r$.

Hence solve, correct to three decimal places, the equation $6\sin^2 a - \cos a = 5$ for $0 \leq a \leq \pi$. **4**

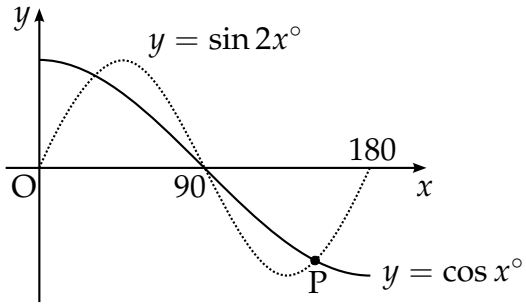
[SQA] 12. Find the values of t, where $0 < t < 2\pi$, for which $4\cos\left(2t - \frac{\pi}{4}\right)$ has its maximum value. **4**

[SQA] 13. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1$, $0 \leq x < 2\pi$. **4**

[SQA] 14. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

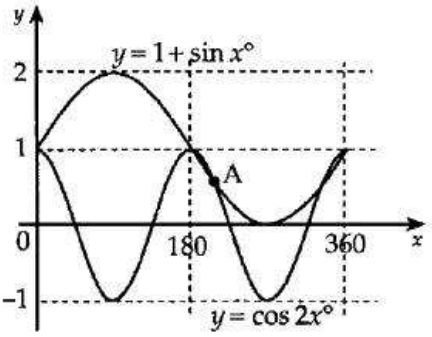
(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



[SQA] 15. The diagram shows two curves with equations $y = \cos 2x^\circ$ and $y = 1 + \sin x^\circ$ where $0 \leq x \leq 360$.

Find the x-coordinate of the point of intersection at A. **4**



[SQA] 16. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

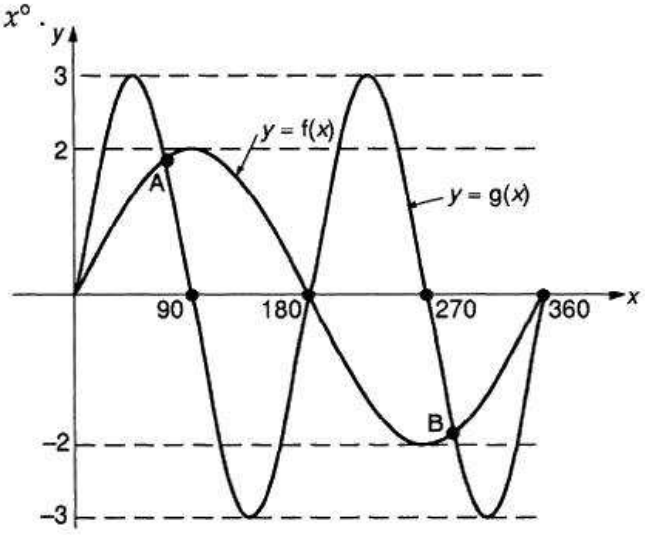
(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

[SQA] 17. (*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions f and g. State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of x in the interval $0 \le x \le 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**

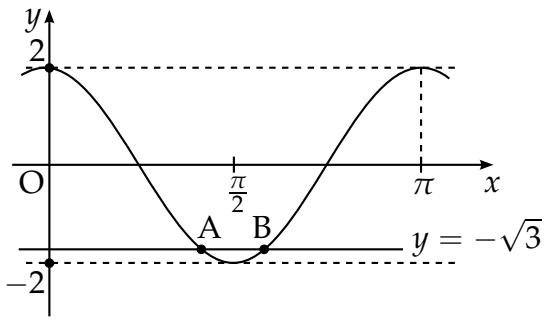


[SQA] 18. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

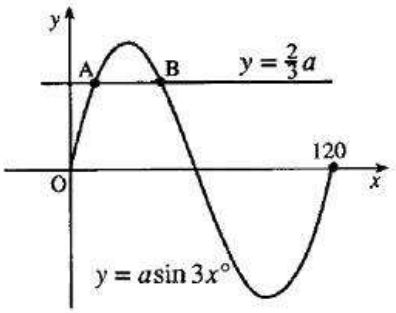
(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**



[SQA] 19. The diagram shows part of the graph of $y = a\sin 3x°$ and the line with equation $y = \frac{2}{3}a$.

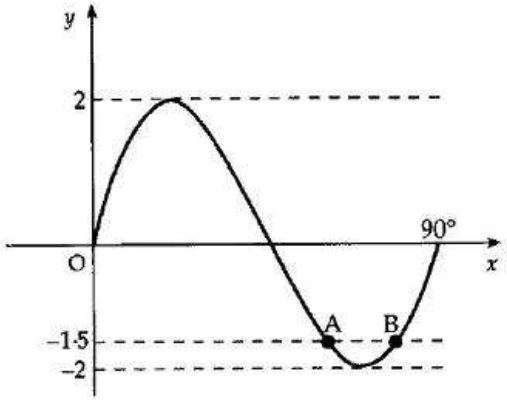
Find the x-coordinates of A and B. **4**



[SQA] 20. The diagram shows the graph of a sine function from 0° to 90°.

(a) State the equation of the graph. 2

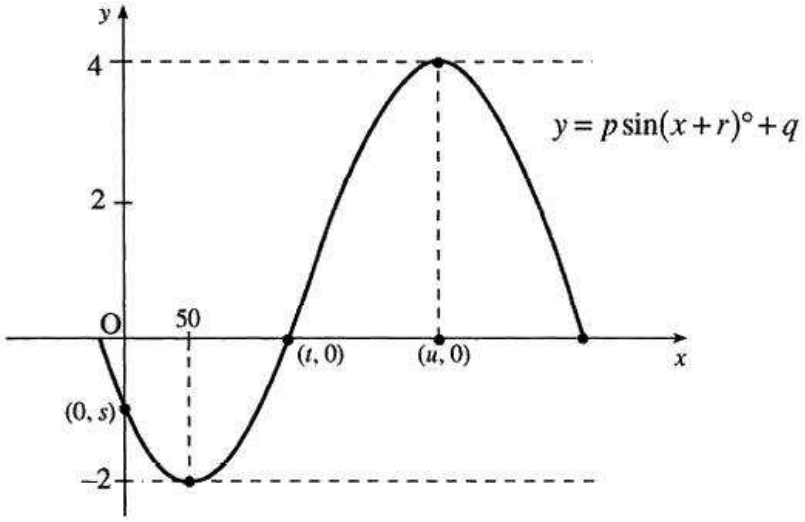
(b) The line with equation y = –1·5 intersects the curve at A and B.
Find the coordinates of A and B. 3



[SQA] 21. The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x+r)° + q$. It crosses the axes at $(0, s)$ and $(t,0)$, and has turning points at $(50, -2)$ and $(u, 4)$.

(i) Write down values for p, q, r and u. **(4)**

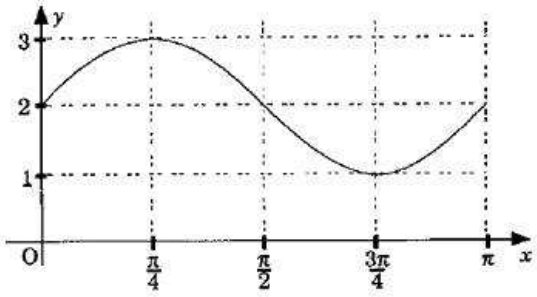
(ii) Find the values for s and t. **(4)**



[SQA] 22. The diagram shows the graph of the function $y = a + b\sin cx$ for $0 \le x \le \pi$.

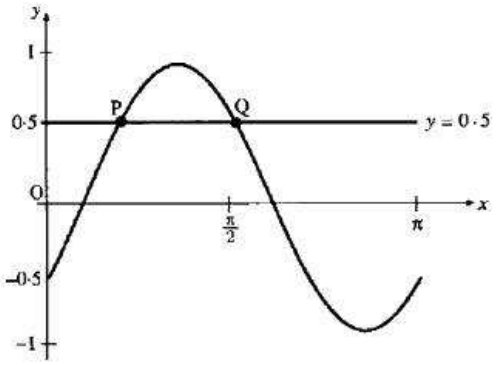
(a) Write down the values of a, b and c. 3

(b) Find algebraically the values of x for which $y = 2{\cdot}5$. 3

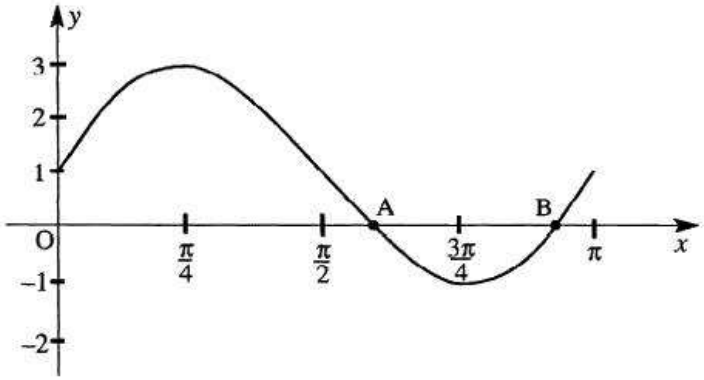


[SQA] 23. The diagram shows a sketch of the graph of $y = \sin\left(2x - \frac{\pi}{6}\right)$, $0 \le x \le \pi$, and the straight line $y = 0{\cdot}5$. These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q. 4



[SQA] 24. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.

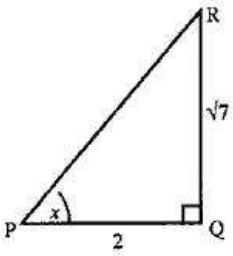


(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(*b*) The points (0, 2) and (π, 0) are joined by a straight line *l*. In how many points does *l* intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line *l*. **(3)**

[SQA] 25. Using triangle PQR, as shown, find the exact value of $\cos 2x$. 3



[SQA] 26. If $\cos\theta = \frac{4}{5}$, $0 \le \theta < \frac{\pi}{2}$, find the exact value of

(*a*) $\sin 2\theta$ **2**

(*b*) $\sin 4\theta$. **3**

[SQA] 27. Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **3**

[SQA] 28. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. **3**

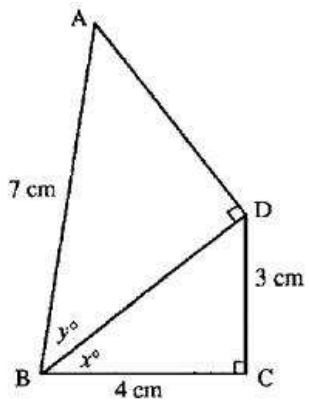
[SQA] 29. Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the exact value of $\sin 2A$. **3**

[SQA] 30. For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$.
Show that the exact value of $\sin(P+Q)$ is $\frac{63}{65}$. **3**

[SQA] 31. Find the exact value of $\sin\theta^\circ + \sin(\theta^\circ + 120^\circ) + \cos(\theta^\circ + 150^\circ)$. **3**

[SQA] 32. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x^\circ + 30^\circ)$ is $\dfrac{4\sqrt{3}+3}{10}$. **3**

[SQA] 33. The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$.
Show that the exact value of $\cos(x+y)^\circ$ is $\dfrac{20-6\sqrt{6}}{35}$. **3**



[SQA] 34. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.
Find the exact value of
(*a*) $\sin 2A$ **2**
(*b*) $\cos 2A$ **1**
(*c*) $\sin(2A+B)$. **2**

[SQA] 35. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

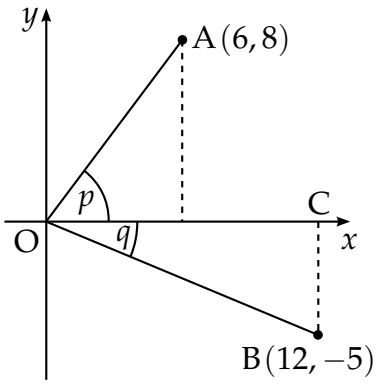
(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

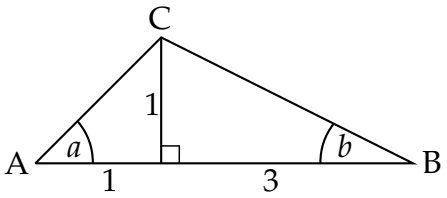
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

[SQA] 36. On the coordinate diagram shown, A is the point $(6, 8)$ and B is the point $(12, -5)$. Angle AOC $= p$ and angle COB $= q$.

Find the exact value of $\sin(p + q)$. **4**



[SQA] 37. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. **4**

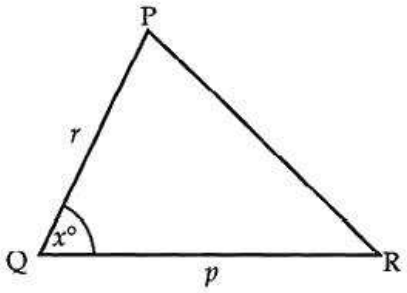


[SQA] 38. The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = $x°$.

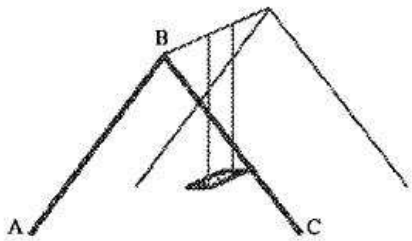
(*a*) Show that $\frac{\sin x°}{p} = \frac{\sin 2x°}{r}$. **(3)**

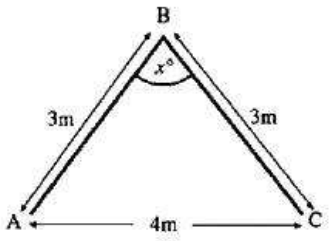
(*b*) (i) State the value of $x°$ when $p = r$.

(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of $x°$. **(5)**



[SQA] 39. **The framework of a child's swing has dimensions as shown in the diagram on the right. Find the exact value of $\sin x°$.** 5





[END OF WRITTEN QUESTIONS]