Further Calculus Past Papers Unit 3 Outcome 2

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Differentiate $3\cos\left(2x - \frac{\pi}{6}\right)$ with respect to x.

   A. $-3\sin(2x)$

   B. $-3\sin(2x - \frac{\pi}{6})$

   C. $-6\sin(2x - \frac{\pi}{6})$

   D. $6\sin(2x - \frac{\pi}{6})$

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA]   2. Differentiate $\sin 2x + \dfrac{2}{\sqrt{x}}$ with respect to x.   **4**

[SQA]   3. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.   **3**

[SQA]   4. Given $f(x) = \cos^2 x - \sin^2 x$, find $f'(x)$.   **3**

[SQA]   5. Given that $f(x) = 5(7 - 2x)^3$, find the value of $f'(4)$.   **4**

[SQA]   6. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x.   **4**

[SQA]   7. Find the derivative, with respect to x, of $\dfrac{1}{x^3} + \cos 3x$.   **4**

[SQA]   8. If $f(x) = \cos^2 x - \dfrac{2}{3x^2}$, find $f'(x)$.   **4**

[SQA] 9. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x.   **4**

[SQA] 10. Find $\dfrac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$.   **3**

[SQA] 11. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'(\frac{\pi}{6})$.   **3**

[SQA] 12. Find the equation of the tangent to the curve $y = 2\sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$.   **4**

[SQA] 13. Find $\displaystyle\int \sqrt{1 + 3x}\,dx$ and hence find the exact value of $\displaystyle\int_0^1 \sqrt{1 + 3x}\,dx$.   **4**

[SQA] 14. Differentiate $\sin^3 x$ with respect to x.

Hence find $\displaystyle\int \sin^2 x \cos x\,dx$.   **4**

[SQA] 15. Find $\displaystyle\int \dfrac{1}{(7 - 3x)^2}\,dx$.   **2**

[SQA] 16. Evaluate $\displaystyle\int_{-3}^0 \left(2x + 3\right)^2 dx$.   **4**

[SQA] 17.

     (a) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos 2x\,dx$.   **3**

     (b) Draw a sketch and explain your answer.   **2**

[SQA] 18.

     (a) Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$.   **1**

     (b) Hence find $\displaystyle\int (\cos x + \sin x)^2\,dx$.   **3**

[SQA]   19. Find $\int \left(6x^2 - x + \cos x\right) dx$.   **4**

[SQA]   20.

(a) By writing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$.   **4**

(b) Hence find $\int \sin^3 x \, dx$.   **4**

[SQA]   21.   (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$.   **2**

(b) Hence write down $\int \dfrac{x^2}{(8 - x^3)^{\frac{1}{2}}} \, dx$.   **1**

[SQA]   22. The curve $y = f(x)$ passes through the point $\left(\frac{\pi}{12}, 1\right)$ and $f'(x) = \cos 2x$.

Find $f(x)$.   **3**

[SQA]   23. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$ express y in terms of x.   **4**

[SQA]   24. A curve for which $\dfrac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.

Find y in terms of x.   **4**

[SQA]   25. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$.   **4**

[SQA]   26.   A sketch of part of the graph of $y = \sin 2x$ is shown in the diagram.

The points P and Q have coordinates $(p, 0)$ and $(q, -1)$.

(a) Write down the values of p and q.   **1**

(b) Find the area of the shaded region.   **4**
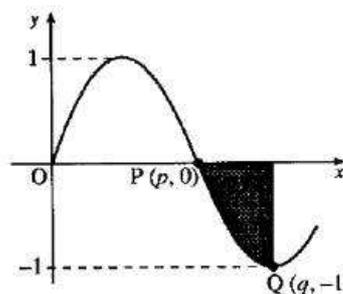
[SQA] 27. An artist has designed a 'bow' shape which he finds can be modelled by
the shaded area below. Calculate the area of this shape. **(6)**
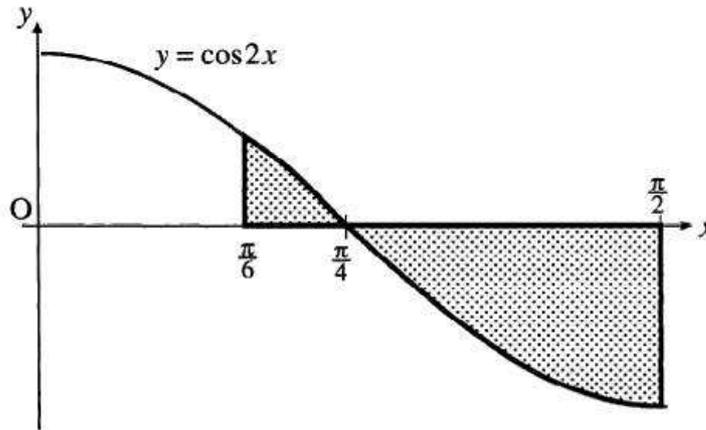
[SQA] 28. Linktown Church is considering designs for a logo for their
Parish magazine. The 'C' is part of a circle and the centre of the
circle is the mid-point of the vertical arm of the 'L'. Since the 'L'
is clearly smaller than the 'C', the designer wishes to ensure that
the total length of the arms of the 'L' is as long as possible.
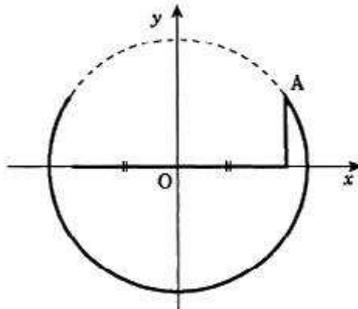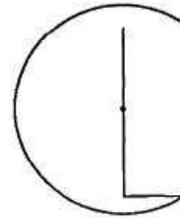
The designer decides to call the point where
the 'L' and 'C' meet A and chooses to draw
co-ordinate axes so that A is in the first
quadrant. With axes as shown, the equation of
the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of

the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(b) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(c) By squaring both sides, solve this equation.

Hence find the greatest length of the arms of the 'L'. **(3)**

[SQA] 29. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.
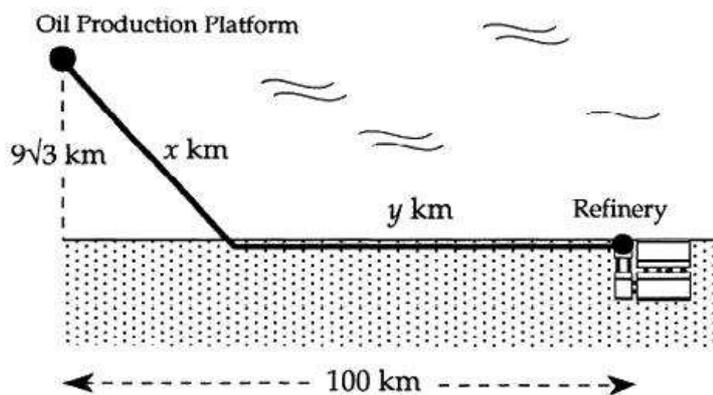
The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(a) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(b) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

[END OF WRITTEN QUESTIONS]

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