Recurrence Relations

1.   For each recurrence relation, state the value of u_5, given $u_0 = 30$.

(a) $u_{n+1} = 0.8u_n + 6$ (b) $u_{n+1} = 1.3u_n - 5$ (c) $u_{n+1} = 0.65u_n + 1.2$
(d) $v_{n+1} = 0.12v_n - 3$ (e) $v_{n+1} = -0.2v_n + 1$ (f) $v_{n+1} = -0.6v_n + 4.2$

2.   A patient is injected with 60 ml of an antibiotic drug. Every 4 hours
      30% of the drug passes out of her bloodstream. To compensate for
      this an extra 15ml of antibiotic is given every 4 hours.
      (a) Find a recurrence relation for the amount of drug in the patient's
      bloodstream.
      (b) Calculate the amount of antibiotic remaining in the bloodstream
      after one day.

3.   A lake next to a chemical factory is found to contain an
      estimated 15 tonnes of pollutant. Through filtration, the
      factory are able to remove 65% of the pollutant annually
      but an extra 4 tonnes is also released into the lake over
      the same period.
      (a) Find a recurrence relation to describe this situation.
      (b) How long would it take for the pollutant to fall below 6.5 tonnes?
      (c) How much pollutant would be in the lake after 10 years?
      (d) What do you notice about the amount of pollutant as time passes?

4.   For the recurrence relation $u_{n+1} = au_n + b$, it is known that $u_o = 6$, $u_1 = 12$
      and $u_2 = 21$.
      (a) Find the values of a and b.
      (b) Hence find the value of u_3.

5.   For the recurrence relation $u_{n+1} = au_n + b$, it is known that $u_o = 10$, $u_1 = 5$
      and $u_2 = 4$.
      (a) Find the values of a and b.
      (b) Hence find the value of u_3.

6.   For the recurrence relation $u_{n+1} = au_n + b$, it is known that $u_1 = 36$, $u_2 = 23$
      and $u_3 = 15.2$.
      (a) Find the values of a and b.
      (b) Hence find the value of u_4.

7.   Given the following information, find the value of u_0.

      (a) $u_{n+1} = 0.8u_n + 2$, $u_1 = 18$ (b) $u_{n+1} = 0.3u_n - 4$, $u_1 = 14$
      (c) $u_{n+1} = 0.65u_n - 6$, $u_2 = 91.5$ (d) $u_{n+1} = 0.45u_n + 30$, $u_2 = 570$
      (e) $u_{n+1} = -0.6u_n + 4$, $u_2 = 16$ (f) $u_{n+1} = -0.2u_n - 6$, $u_2 = -2.4$