Higher maths – The discriminant

1. What are the nature of the roots of each function?

(a) $x^2 + 5x + 2 = 0$
(b) $x^2 + 3x + 4 = 0$
(c) $x^2 + 2x + 1 = 0$
(d) $3x^2 – 2x – 1 = 0$
(e) $4x^2 + 5x + 2 = 0$
(f) $-2x^2 + 5x + 3 = 0$
(g) $12 – x – x^2 = 0$
(h) $-5 + x – 2x^2 = 0$
(i) $4 – 4x + x^2 = 0$
(j) $x^2 – 3x = 5$
(k) $4x – 2 = 5x^2$
(l) $x^2 + 1 = 0$
(m) $x^2 = 1$
(n) $-2x^2 = -4x + 1$

2. The function $x^2 + px + 1 = 0$ has equal roots. Calculate the value of p if $p>0$.

3. The function $x^2 + 2rx + r = 0$ has equal roots. What are the possible values of r?

4. The function $(a - 2)x^2 + ax + 2 = 0$ has equal roots, what is the value of a?

5. The roots of the equation $(x + 2)(2x – b) = -2$ are equal. Find the values of b.

6. Show that the roots of the equation $x^2 + px – 4 = 0$ are always real.

7. Show that the roots of $(k – 2)x^2 – (3k – 2)x + 2k = 0$ are always real.

8. Show that the line $y = -2x – 23$ is a tangent to the parabola $y = x^2 - 4x + 15$ and find the point of contact.

9. Show that the line $y = 10x – 2$ is a tangent to the curve $y = 2x(x + 3)$ and find the point of contact.

10. Show that the line $y = 9x - 27$ is a tangent to the curve $y = 3x(x - 3)$ and find the point of contact.

11. Is the line $y = 3x – 2$ a tangent to the curve $y = 3x^2 – 11x + 5$?

12. The roots of the equation $\frac{p-1}{x} + \frac{x}{4} = 1$ are equal. Find the value of p.