FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \,\mathrm{msin}\, A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Trigonometry Unit Assessment

1.(a) Express 4 cos x + 8 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

(b) Express 3 cos x + 8 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

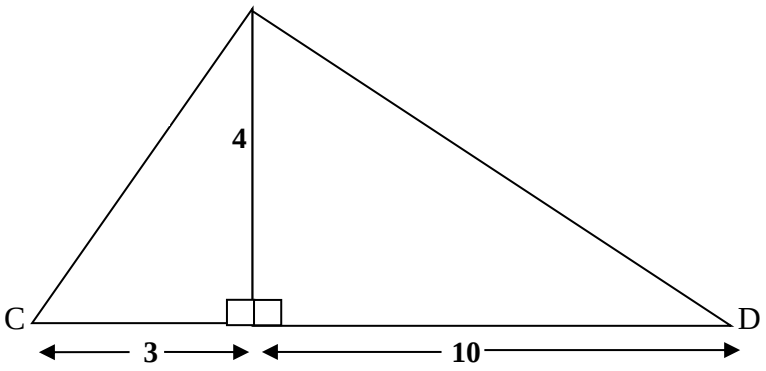
(c) Express 2 cos x + 6 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

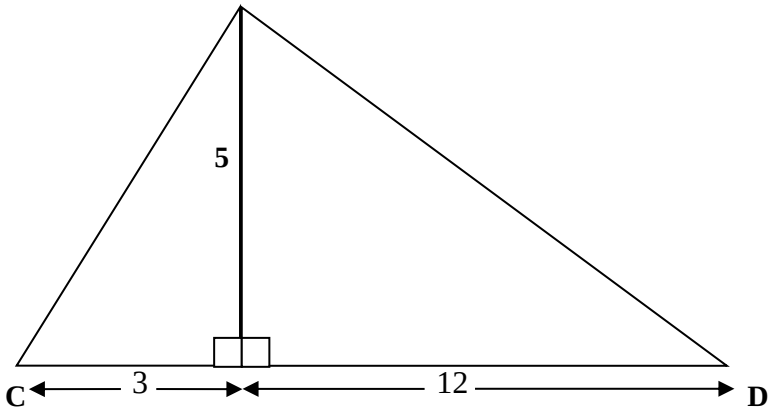
(d) Express 4 cos x + 7 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a. [4]

2. (a) The diagram below shows two right-angled triangles.
Find the exact value of cos (C – D)



(b) The diagram below shows two right-angled triangles.
Find the exact value of cos (C + D)



3. **(a)** Show that $(7 – 4 \sin x)(7 + 4 \sin x) = 16 \cos^2 x + 33$

(b) Show that $(8 – 3 \sin x)(8 + 3 \sin x) = 9 \cos^2 x + 55$

(c) Show that $(3 – 2 \cos x)(3 + 2 \cos x) = 4 \sin^2 x + 5$

(d) Show that $(6 – 5 \sin x)(6 + 5 \sin x) = 25 \cos^2 x + 11$ [2,#2.1]

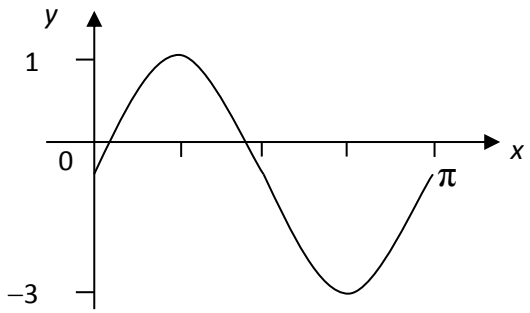
4. **(a)** Sketch the graph of $y = a \cos (x + \pi/3)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

(b) Sketch the graph of $y = b \sin (x + \pi/4)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

(c) Sketch the graph of $y = a \cos (x - \pi/3)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

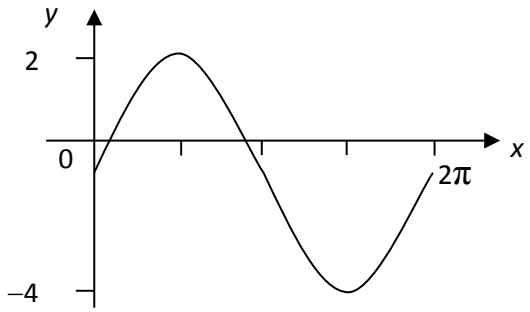
(d) Sketch the graph of $y = b \sin (x - \pi/4)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

5.(a) The diagram below shows the graph of y = a sin (bx) + c



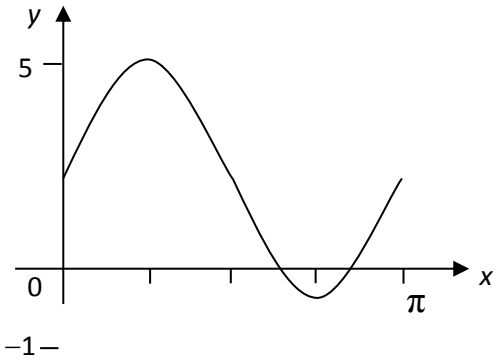
Write down the values of a, b and c.

(b) The diagram below shows the graph of y = a sin (bx) + c



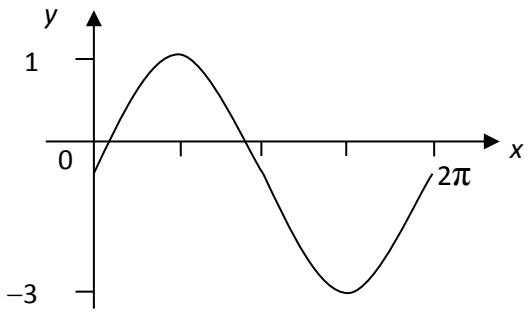
Write down the values of a, b and c.

5. (c) The diagram below shows the graph of $y = a \sin (bx) + c$



Write down the values of a, b and c.

(d) The diagram below shows the graph of $y = a \sin (bx) + c$



Write down the values of a, b and c.

[3]

6.a) Solve $2\cos 2x = \sqrt{3}$, for $0 \leq x \leq 180$

b) Solve $4\sin 2x = 2$, for $0 \leq x \leq 180$

c) Solve $\sqrt{2}\cos 2x = 1$, for $0 \leq x \leq 180$

d) Solve $3\sin 2x = 3$, for $0 \leq x \leq 180$

7. **a)** Solve , $2\sin 2t - \sin t = 0$, for $0 \leq t \leq 180$

b) Solve , $3sin2x + sinx = 0$, for $0 \leq x \leq 180$

c) Solve , $4sin2\alpha - sin\alpha = 0$, *for* $0 \leq \alpha \leq 180$

d) Solve , $5sin2x - sinx = 0$, for $0 \leq x \leq 180$

8.a) Given $\sqrt{3}cosx + sinx = 2\cos(x - 30)^\circ$,

solve $\sqrt{3}cosx + sinx = \sqrt{2}$, for $0 \leq x \leq 360$

b) Given $4cos + 3sinx = 5\cos(x - 36.9)^\circ$,

solve $4cosx + 3sinx = 1.5$, for $0 \leq x \leq 360$

c) Given $2sinx - 5cosx = \sqrt{29}\sin(x - 68.2)^\circ$,

solve $2sinx - 5cosx = 2.5$, for $0 \leq x \leq 360$

d) Given $2sinx + 2cosx = \sqrt{8}\cos(x - 45)^\circ$,

solve $2sinx + 2cosx = 2.7$, for $0 \leq x \leq 360$

Answers

1. (a) $k = 4\sqrt{5}$, a = 333·4° (b) $k = \sqrt{58}$, a = 339·4°

(c) $k = 2\sqrt{10}$, a = 341·6° (d) $k = \sqrt{65}$, a = 330·3°

2. (a) $\frac{23}{5\sqrt{29}}$ (c) $\frac{11}{13\sqrt{34}}$

3. Solution is shown

4. Correct Max and Min, correct x intercepts and correct shape.

5. (a) a = 2, b = 2, c = -1 (b) a = 3, b = 1, c = -1

(c) a = 3, b = 2, c = 2 (d) a = 2, b = 1, c = -1

6. **a)** $x = 15^\circ \; and \; 165^\circ$ **b)** $x = 15^\circ \; and \; 75^\circ$

c) $x = 22.5^\circ and \; 157.5^\circ$ **d)** $x = 45^\circ$

7. **a)** $t = 0^\circ, 75.5^\circ, 180^\circ$ **b)** $t = 0^\circ, 99.6^\circ, 180^\circ$

c) $t = 0^\circ, 82.8^\circ, 180^\circ$ **d)** $t = 0^\circ, 84.3^\circ, 180^\circ$

8. **a)** $x = 75^\circ, x = 345^\circ$ **b)** $x = 109.4^\circ, x = 324.4^\circ$

c) $x = 95.5^\circ, 220.5^\circ$ **d)** $x = 62.3^\circ$