FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $\quad a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$

Trigonometric formulae:

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard **derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	
	$\dfrac{1}{a}\sin ax + c$

Geometry Unit Practice

1. (a) A straight line has the equation $5x + y - 3 = 0$.

Write down the equation of the line parallel to the given line, which

passes through the point (4,-8)

(b) A straight line has the equation $y = -4x + 7$.

Write down the equation of the line parallel to the given line, which

passes through the point (3,-12)

(c) A straight line has the equation $3x + y - 1 = 0$.

Write down the equation of the line parallel to the given line, which

passes through the point (6,-4)
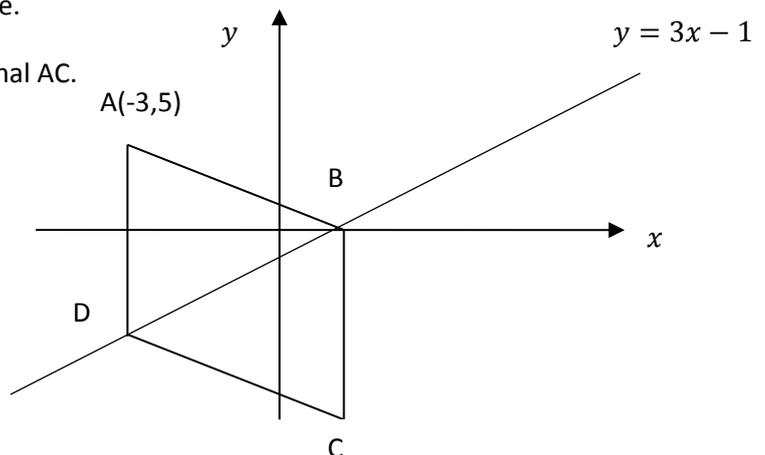
(d) A straight line has the equation $y = -5x + 2$.

Write down the equation of the line parallel to the given line, which

passes through the point (3,-7)

2.(a) ABCD is a rhombus

Diagonal BD has equation $y = 3x - 1$ and point A has coordinates

(-3,5).
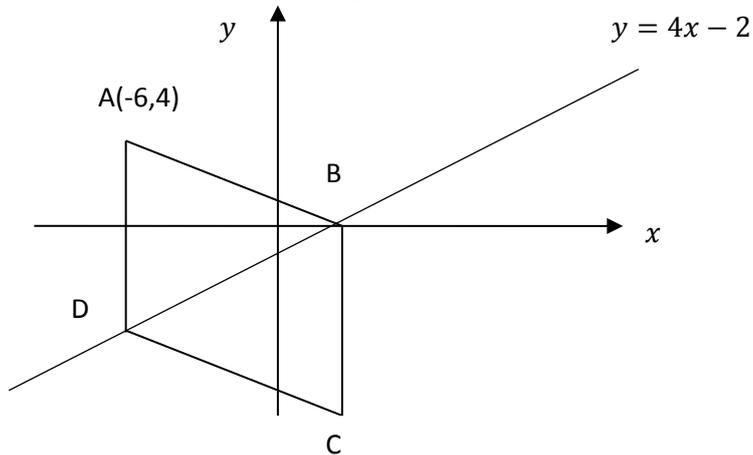Note the diagram is not to scale.

Find the equation of the diagonal AC.

(b)   ABCD is a rhombus

Diagonal BD has equation $y = 4x - 2$ and point A has coordinates

(-6,4).

Note the diagram is not to scale.

Find the equation of the diagonal AC.

(c)   ABCD is a rhombus

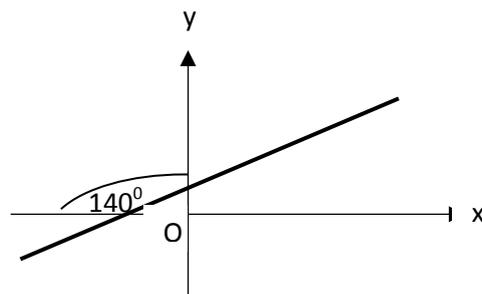Diagonal BD has equation $y = 5x - 3$ and point A has coordinates

A ski slope is categorised by its gradient as shown in the table.

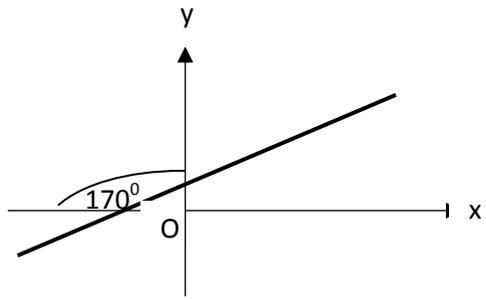| Dry slope category	Gradient (m) of slope
Teaching and general skiing	$0 < m \le 0 \cdot 4$
Extreme skiing	$m > 0 \cdot 4$

2   What is the gradient of each ski slope shown below and which category does each ski slope belong to?
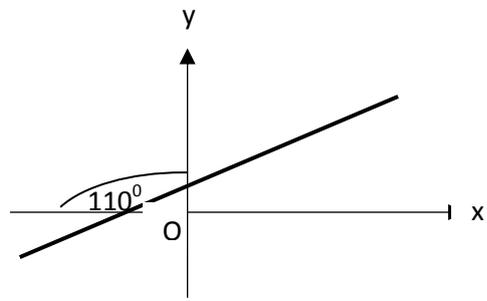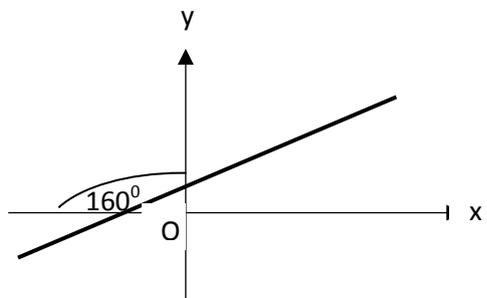   **Explain your answer fully.**
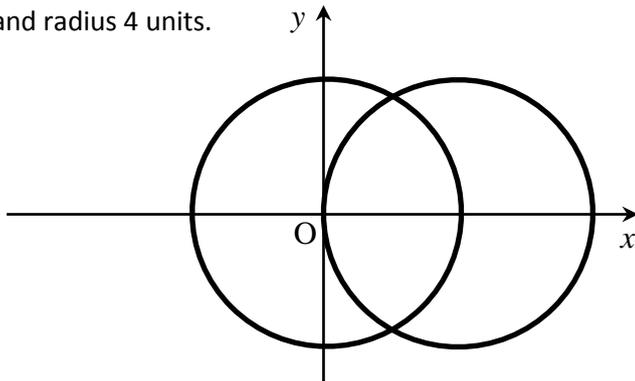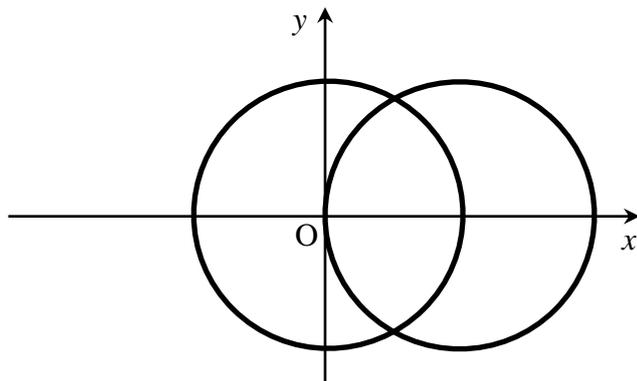
(a)

(b)

(c)

(d)

4. The diagram shows two congruent circles. One circle has centre the origin and radius 4 units.

Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

(b) The diagram shows two congruent circles. One circle has centre the origin and diameter 12 units.

Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

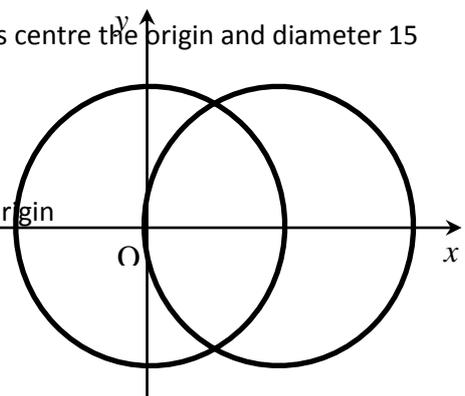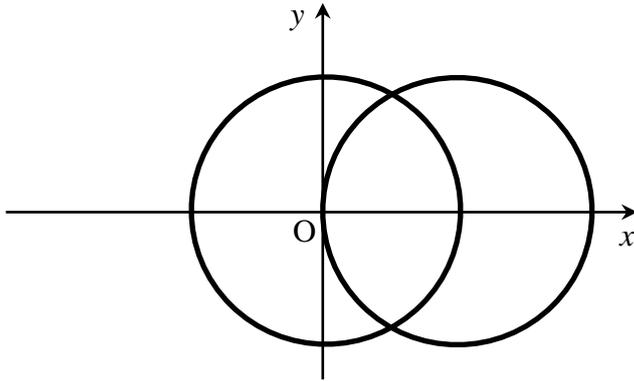(c) The diagram shows two congruent circles. One circle has centre the origin and diameter 15 units.

Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

(d)   The diagram shows two congruent circles. One circle has centre the origin and radius 2.5 units.

Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

5.(a) Determine algebraically if the line $y = x - 1$ is a tangent to the circle

$(x + 4)^2 + (y - 2)^2 = 49$

(b)   Determine algebraically if the line $y = x + 1$ is a tangent to the circle
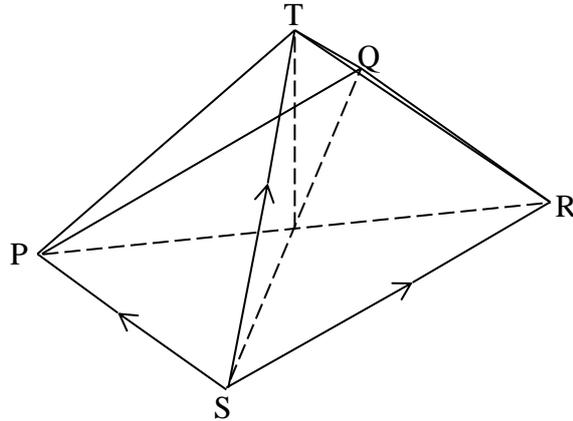
$(x + 1)^2 + (y - 2)^2 = 20$

(c)   Determine algebraically if the line $y = 3x + 10$ is a tangent to the circle

$(x - 4)^2 + (y - 2)^2 = 40$

(d)   Determine algebraically if the line $y = x + 3$ is a tangent to the circle

$(x + 2)^2 + (y - 4)^2 = 9$

12. TPQRS is a pyramid with rectangular base PQRS.

(a) TPQRS is a pyramid with rectangular base PQRS (as above).
If **t**he vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$\overrightarrow{SP} = 3\boldsymbol{i} - 8\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

$\overrightarrow{ST} = -7\boldsymbol{i} + 11\boldsymbol{k}$

Express $\overrightarrow{PT}$ in component form

(b) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)
If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$\overrightarrow{SP} = 4\boldsymbol{i} - 6\boldsymbol{j} - 5\boldsymbol{k}$

$\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

$\overrightarrow{ST} = -6\boldsymbol{i} + 2\boldsymbol{j} + 12\boldsymbol{k}$

Express $\overrightarrow{PT}$ in component form

(c) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)

  If **t**he vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

  $\overrightarrow{SP} = 2\boldsymbol{i} + \boldsymbol{j}$

  $\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

  $\overrightarrow{ST} = -4\boldsymbol{i} + 6\boldsymbol{j} + 5\boldsymbol{k}$

  Express $\overrightarrow{PT}$ in component form

(d) TPQRS is a pyramid with rectangular base PQRS. (see diagram on left)

  If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

  $\overrightarrow{SP} = 8\boldsymbol{i} + \boldsymbol{j} + 6\boldsymbol{k}$

  $\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

  $\overrightarrow{ST} = -2\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$
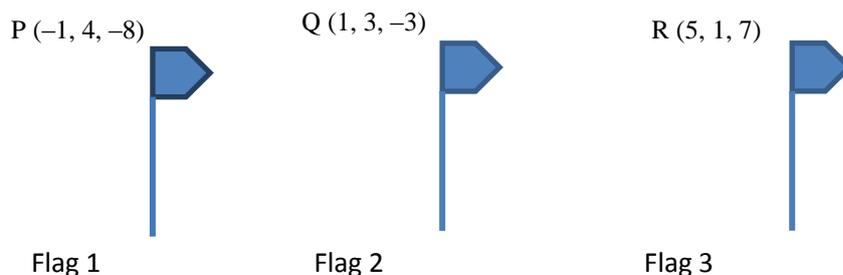
  Express $\overrightarrow{PT}$ in component form

[2]

13. (a) An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

  Relative to suitable axes, the top-left corner of each flag can be represented by the points P (−1, 4, −8), Q (1, 3, −3), and R (5, 1, 7)   respectively. All three flags point vertically upwards.

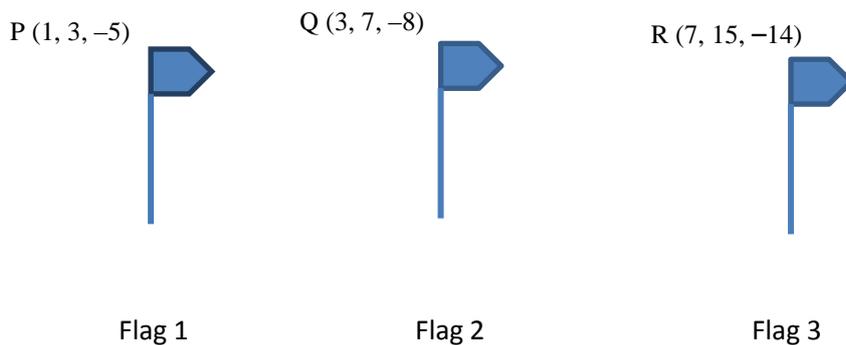P (−1, 4, −8)   Q (1, 3, −3)   R (5, 1, 7)

Flag 1   Flag 2   Flag 3

Has the architect laid the flags correctly? You must justify your answer.

13.   **(b)**   An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (1, 3, −5), Q (3, 7, −8), and R (7, 15, −14) respectively. All three flags point vertically upwards.
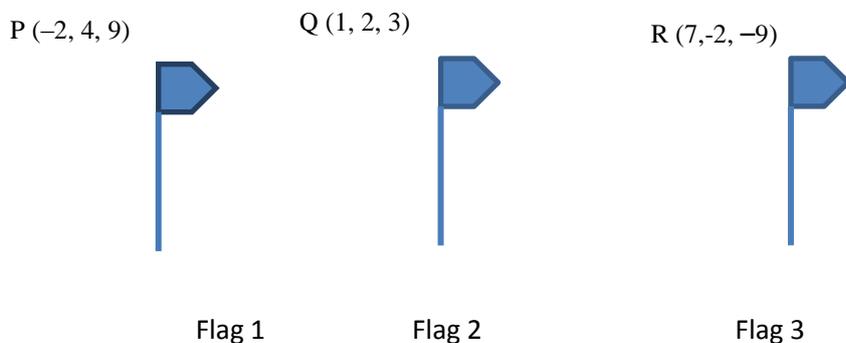
P (1, 3, −5)   Q (3, 7, −8)   R (7, 15, −14)

Flag 1   Flag 2   Flag 3

Has the architect laid the flags correctly? You must justify your answer.

13. (c)   An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be   represented by the points P (−2, 4, 9), Q (1, 2, 3), and R (7, −2, −9)   respectively. All three flags point vertically upwards.

P (−2, 4, 9)   Q (1, 2, 3)   R (7,-2, −9)

Flag 1   Flag 2   Flag 3

Has the architect laid the flags correctly? You must justify your answer.

13. **(d)** An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (3, 6, 9), Q (−1, 10, 10), and R (−9, 18, 12) respectively. All three flags point vertically upwards.
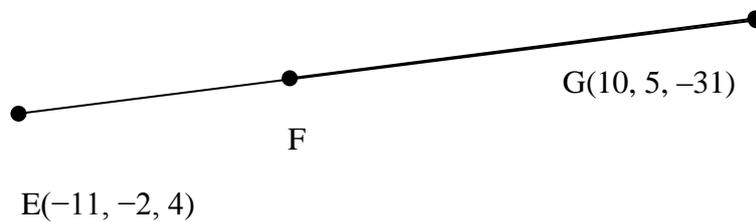
P (3, 6, 9)　　　Q (−1, 10, 10)　　　R (−9,18, 12)

Flag 1　　　　Flag 2　　　　Flag 3
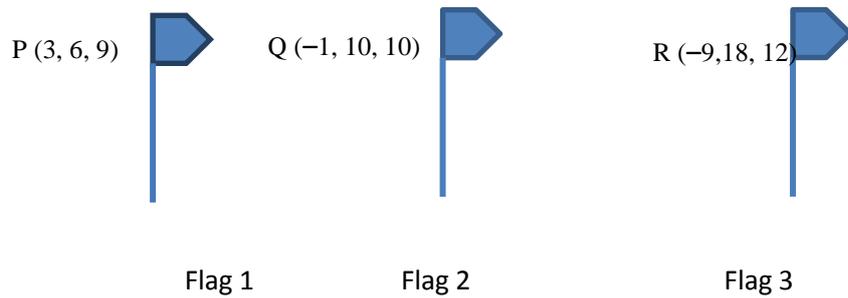
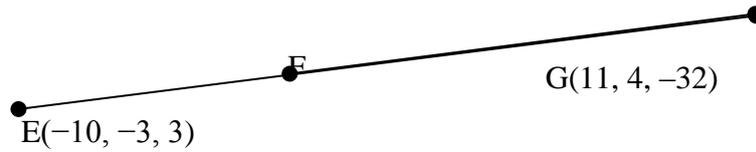Has the architect laid the flags correctly? You must justify your answer.

[#2.1,2,#2.2]

14. **(a)** The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:4.
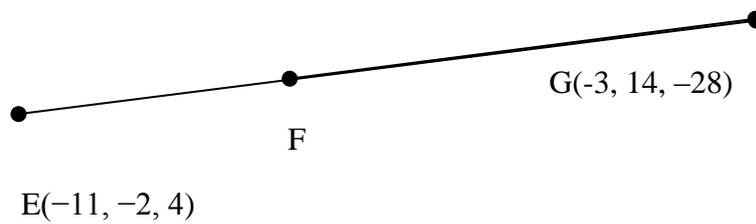
Find the coordinates of F.

G(10, 5, −31)

F

E(−11, −2, 4)

14. (b) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 2:5.

Find the coordinates of F.

F

G(11, 4, −32)

E(−10, −3, 3)
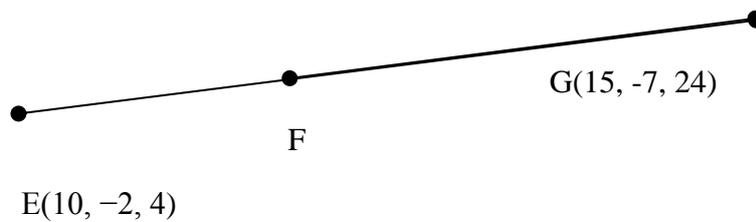
(c) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:5.

Find the coordinates of F.

G(-3, 14, −28)

F

E(−11, −2, 4)

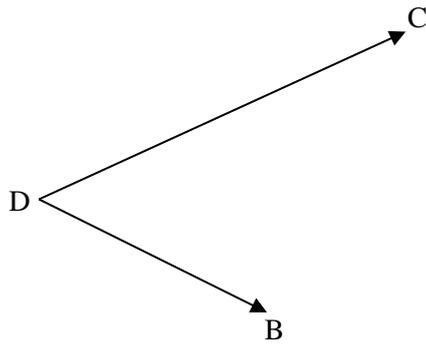**(d)** The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 1:4.
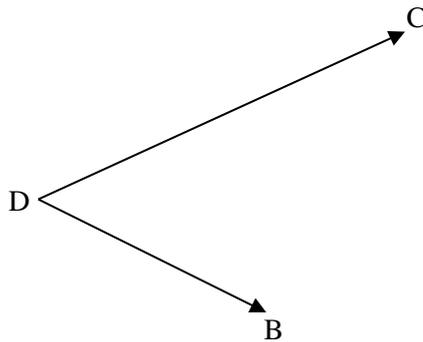
Find the coordinates of F.

G(15, -7, 24)

F

E(10, −2, 4)

15.(a) Points B, C and D have coordinates B(21, −8, 0), C(20, −7, 7) and D(17, −6, 2).

Find the size of the acute angle BDC.

(b) Points B, C and D have coordinates B(15, −7, 4), C(10, −2, 2) and D(18, −1, 3).

Find the size of the acute angle BDC.

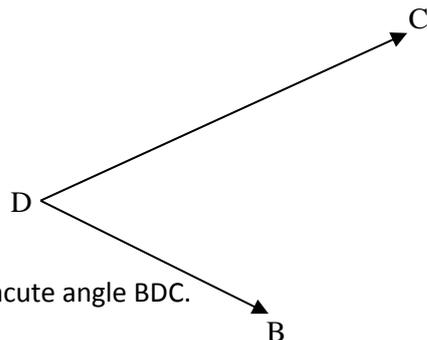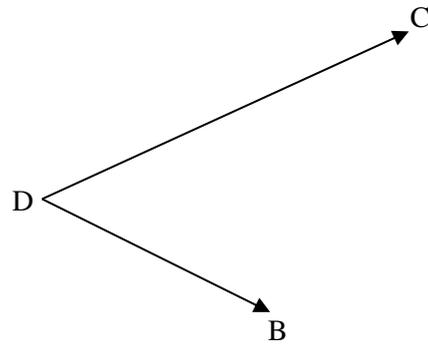**(c)** Points B, C and D have coordinates B(11, −4, 1), C(16, −5, 3) and D(12, −8, 1).

Find the size of the acute angle BDC.

(d) Points B, C and D have coordinates B(15, −4, 2), C(30, −8, 8) and
D(14, −3, 1).

Find the size of the acute angle BDC.

[5]

Answers

1 (a) $y = -5x + 12$ (b) $y = -4x$ (c) $y = -3x$ (d) $y = -5x + 8$

3 (a) $m = 0 \cdot 83$ (b) $m = 0 \cdot 17$ (c) $m = 2 \cdot 75$ (d) $m = 0 \cdot 36$

4 (a) $(x - 4)^2 + y^2 = 16$ (b) $(x - 6)^2 + y^2 = 36$

   (c) $(x - 7.5)^2 + y^2 = 56.25$ (d) $(x - 2.5)^2 + y^2 = 6.25$

5 (a) Line meets at $x = -4, 3$ => Not a tangent.

   (b) Line meets at $x = -3, 3$ => Not a tangent.

   (c) Line meets at $x = -2 (twice)$ => A tangent.

   (d) Line meets at $x = -2, 1$ => Not a tangent.

12.(a)   -10i + 8j + 17k   (b)   -10i + 8j + 17k

       (c)   -6i + 5j + 5k   (d)   -10i + 4j

13.   For each question show they are collinear, interpret ratio and write a suitable solution

14.   (a) F = (-2, 1,-11) (b)   F = (-4, -1,-7) (c)   F = (-8, 4,-8) (d)   F = (11, -3, 8)

15.   (a) 82.1 degrees   (b)   58.2 degrees

       (c) 68.9 degrees   (d)   27.1 degrees