**Further Calculus**

1. [C9] Find $\int \frac{4}{(2x-3)^3}\,dx$

2. [C1] $f(x) = \sqrt{(2x-1)^3}$ find $f'(5)$.

3. [C5, C6] If $f(x) = cos^3 x$ find the exact value of $f'(\frac{\pi}{6})$.

4. [C13] (a) Show that $(sinx + cosx)^2 = 1 + sin2x$.

   (b) Hence, find $\int (sinx + cosx)^2 dx$.

5. [C5, C6] If $y = 2\sin(2x - \frac{\pi}{3})$, find $\frac{dy}{dx}$.

**Recurrence Relations**

1. [A6, A7] A sequence is modelled by the recurrence relation $U_{n+1} = \frac{4}{5}U_n + 9, U_1 = 25$.

   (a) Determine the values of $U_0, U_2$ and $U_3$.

   (b) Explain why the sequence approaches a limit as $n \to \infty$ and determine the limit.

2. [A6, A7] A recurrence relation is defined by $U_{n+1} = mU_n + c$, where $-1 < m < 1$ and $u_0 = 9$.

   (a) If $U_1 = 21$ and $U_2 = 29$, find the values of $m$ and $c$.

   (b) Find the limit of this recurrence relation as $n \to \infty$.

3. [A6, A7] A loch contains 60 tons of waste.

Each week the Council remove 47% of it but at the end of the week a local factory is allowed to dump 5.5 tons of waste into the loch.

   (a) Set up a recurrence relation in the form $U_{n+1} = aU_n + b$ to describe the state of the loch and find the amount of pollution in the loch during the first three weeks.

   (b) The council has a long-term plan to ensure the maximum amount of waste is 12 tons. Will they reach their objective?